Youth Competition Times

MECHANICAL ENGINEERING CAPSULE

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Mechanics

■ **Newton's law of motion**

First law of motion	It states that everybody continues in the states of rest or of uniform motion, in a straight line, unless it is acted upon by some external force to change that state.
Second law of motion	$F \propto \frac{dP}{dt}$ $\boxed{F = ma}$
Third law of motion	The forces of action and reaction between bodies in contact have same magnitude, same line of action but opposite in direction.

Newton's law of gravitation	Every particle of matter attracts every other particle of matter a force directly proportional to the product of the masses and inversely proportional to the square of the distance between then. $\boxed{F = G\frac{m_1 m_2}{r^2}}$ $G = 6.67 \times 10^{-11}$ N-m^2/kg^2

■ **Types of forces**

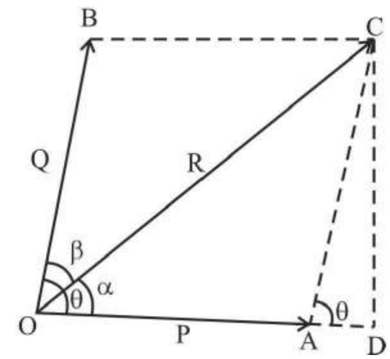
Coplanar forces	Line of action of all forces lying on single plane
None-coplanar forces	Line of action of all forces are not lying on a single plane.
Concurrent forces	Line of action of all forces passes through a single point.
None concurrent forces	Line of action of all forces do not pass through a single point.
Collinear forces	Line of action of all forces passes through a single line.
Parallel forces	Line of action of all forces are parallel to each other.
(a) Like parallel forces	Line of action of all forces are parallel to each other in same direction
(b) Unlike parallel forces	Line of action of all forces are parallel to each other in different direction.

■ **Principle of transmissibility of force–**

When a force acts on a body, this force may be assumed to be acting on all particles of the body which lie on the line of action of the force.

■ **Parallelogram law of forces–**

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$



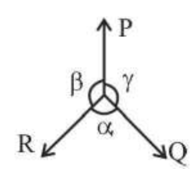
$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}} \qquad \boxed{\tan\beta = \frac{P\sin\theta}{Q + P\cos\theta}}$$

	Case	Resultant
I.	If two forces are like parallel $\theta = 0^o$	$R = P + Q$
II.	If forces are unlike parallel $\theta = 180^o$	$R = P - Q$
III.	If forces are perpendicular $\theta = 90^o$	$R = \sqrt{P^2 + Q^2}$
IV.	If magnitude of two forces are same	$\alpha = \theta/2$

■ **Law of triangle of forces–**

If three forces acting a point are in equilibrium, then their magnitude & directions can be represented by successive sides of a triangle.

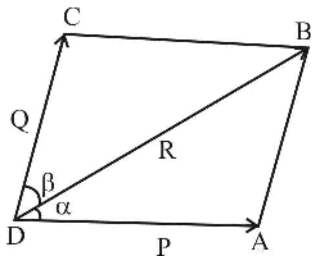
■ **Lami's theorem–**



$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

■ **Law of polygon of forces–** If all the forces acting at a point can be represented by successive sides of a closed polygon, then forces will be in equilibrium.

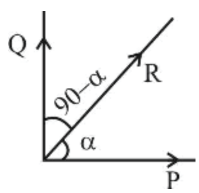
■ **Resolution of forces**



- If magnitude & direction of forces are known then there will be only one resultant of definite magnitude & direction.

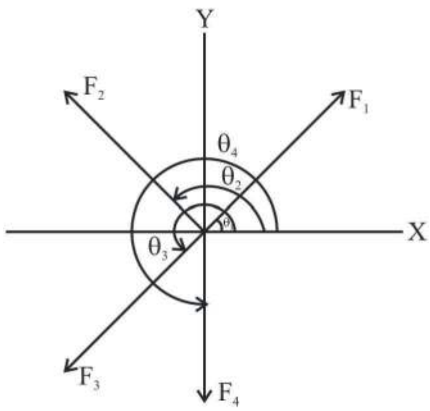
$\boxed{P = \dfrac{R\sin\beta}{\sin(\alpha+\beta)}}$ $\boxed{Q = \dfrac{R\sin\alpha}{\sin(\alpha+\beta)}}$

■ **Resolution of force in two perpendicular direction–**



$\boxed{P = R\cos\alpha}$ $\boxed{Q = R\sin\alpha}$

■ **Resolution of concurrent coplanar forces–**



$$\Sigma F_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 + F_4\cos\theta_4 + ...$$

$$\Sigma F_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 + F_4\sin\theta_4 + ...$$

Resultant force, (R) = $\sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

➢ **Direction of resultant–**

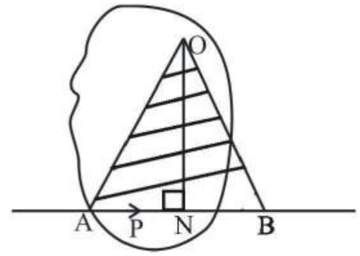
- If resultant is inclined at θ angle with X axis

$\boxed{\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}}$

■ **Moment–**

- Vector quantity
- Moment of force = Force × Perpendicular distance

■ **Geometrical representation of moment of a force–**



Suppose a force 'P' is acting along AB on a body. Body is free to rotate about a fixed point.

$\boxed{\text{Moment of a force} = 2 \times \text{area of } \Delta OAB}$

Varignon's theorem	Algebric sum of moment of two coplanar forces about a point is equal to the moment of resultant force about that point.
Principle of moment	If algebric sum of moments of all forces acting on a body about a point is zero, then body will be in state of rotational equilibrium $\Sigma M = 0$

■ **Lever**

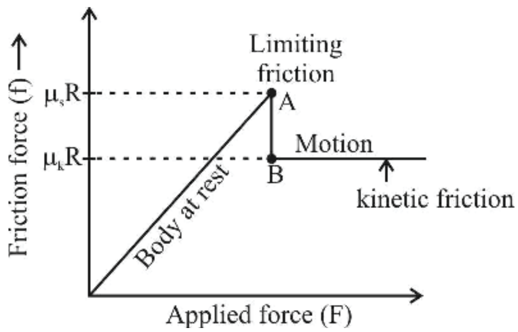
Principle of lever–

An ideal lever works on principle of moments when the lever is in equilibrium.

M.A. of lever ⇒ $\dfrac{\text{Load}}{\text{Effort}} = \dfrac{\text{Effort arm}}{\text{Load arm}}$

Class I lever	• Fulcrum is between effort & load • $\boxed{\begin{matrix} MA \ge 1 \\ MA < 1 \end{matrix} \{\text{may be}\}}$ **Ex.** ⇒ Scissors, see-saw, claw hammer etc.
Class II lever	• Load is in between effort & fulcrum • MA > 1 **Ex.** ⇒ Wheel barrow, lemon crusher, nut cracker, paper cutter.
Class III lever	• Effort is in between fulcrum & load • MA < 1 **Ex.**⇒ Sugar tongs, forearm used for lilting a load.

■ Friction–



Law of static friction	Law of kinetic friction
• Frictional force (f_S) ∝ Normal reaction (R_N). • Frictional force is independent of surface area of contact. • Frictional force depends upon surface roughness. • Friction force depends upon materials of surfaces in contact.	• $\mu_s > \mu_k$ • Force of dynamic friction is independent of relative motion. • Force of friction is opposite to relative motion.

- **Coefficient of friction (μ) =**

$f \propto R_N$, $f = \mu R_N$, $\boxed{\mu = \dfrac{f}{R_N}}$

f = Friction force R_N = Normal reaction

- **Limiting friction–**

$\boxed{f_{lim} = \mu_S \times R_N}$

- **Kinetic friction–**

$\boxed{f_K = \mu_K \times R_N}$ $\boxed{f_S > f_K}$

$\boxed{\mu_S > \mu_K}$

- **Angle of friction–**

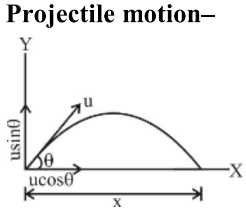
$\boxed{\tan\theta = \mu_S}$ $\boxed{\theta = \tan^{-1}(\mu_s)}$

- **Angle of repose–** It is angle of inclination of the plane to the horizontal, at which the body just begins to move down the plane.

$\boxed{\alpha = \phi}$

Angle of inclination of plane = Angle of friction.

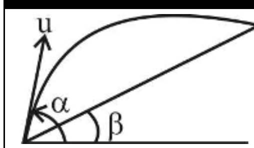
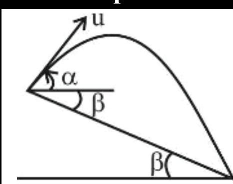
■ Projectile motion–

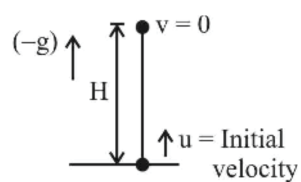
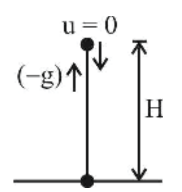


$$\boxed{y = x.\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}}$$

- Path of projectile is parabola.

Time of flight	$(T) = \dfrac{2u\sin\theta}{g}$
Range	$(R) = \dfrac{u^2\sin 2\theta}{g}$
Maximum height	$(H) = \dfrac{u^2\sin^2\theta}{2g}$
Condition for maximum range	$\alpha = 45^o$ $\boxed{R_{max} = \dfrac{u^2}{2g}}$

Body projected upward to inclined plane	Body projected downward inclined plane
 • $T = \dfrac{2u}{g\cos\beta}[\sin(\alpha-\beta)]$ • $R = \dfrac{u^2}{g\cos^2\beta}$ $[\sin(2\alpha-\beta)-\sin\beta]$ • Condition for maximum range– $\boxed{\alpha = 45^o + \dfrac{\beta}{2}}$	 • $T = \dfrac{2u}{g\cos\beta}[\sin(\alpha+\beta)]$ • $R = \dfrac{u^2}{g\cos^2\beta}$ $[\sin(2\alpha+\beta)+\sin\beta]$ • For maximum range $\boxed{\alpha = 45 - \dfrac{\beta}{2}}$

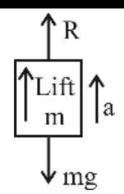
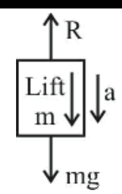
Maximum height obtained by an object thrown in upward	Motion of an object falling freely under gravity
 • Maximum height $\boxed{H = \dfrac{u^2}{2g}}$ • Time taken by object to reach the ground. $\boxed{t = \dfrac{2u}{g}}$	 • Velocity before hitting the ground $\boxed{v = \sqrt{2gH}}$ • Time taken by object to reach the ground. $\boxed{t = \dfrac{2H}{g}}$

■ **System of pulleys–**

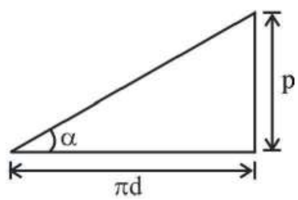
First system of pulleys	Velocity ratio (VR) = 2^n
Second system of pulleys	VR = n
Third system of pulleys	VR = $2^n - 1$

n = number of pulleys

■ **Motion of a lift–**

Lift is moving upward	**Lift is moving downward**
	
• R–mg = ma	• mg – R = ma
• $R = m(g+a)$	• $R = m(g-a)$

■ **Screw jack–**



$$\tan\alpha = \frac{p}{\pi d}$$

Effort required	
•For raising the load	P = W tan (α + ϕ)
• For lowering the load	P = W tan (α – ϕ)

Note–

1. **When friction is neglected then ϕ = 0**

$$P_o = W\tan\alpha$$

2. **The efficiency of screw jack–**

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

3. **The efficiency of screw jack is maximum–**

$$\alpha = 45 - \frac{\phi}{2}$$

$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

i■ **Centroid of regular plane figure**

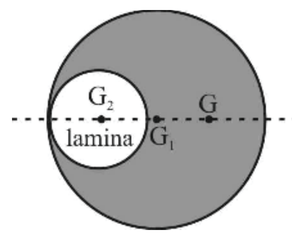
Lamina	**Area**	$\bar{x}$	$\bar{y}$
Right angle Triangle	$\frac{1}{2}b.h$	$\frac{b}{3}$	$\frac{h}{3}$
Rectangle	b.h	$\frac{b}{2}$	$\frac{h}{2}$
Semicircle	$\frac{1}{2}\pi r^2$	r	$\frac{4r}{3\pi}$
Quadrant circle	$\frac{1}{4}\pi r^2$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Three quadrant circle	$\frac{3}{4}\pi r^2$	$\frac{4r}{9\pi}$	$\frac{4r}{9\pi}$

■ **Centre of gravity for given area–**

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + ...}{a_1 + a_2 + a_3 + ...}$$

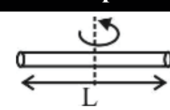
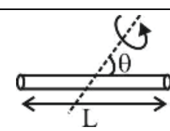
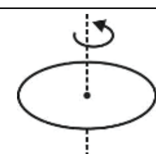
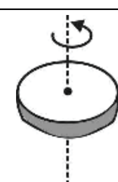
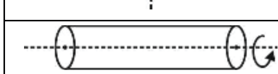
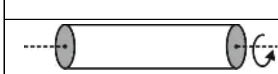
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + ...}{a_1 + a_2 + a_3 + ...}$$

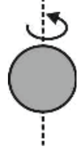
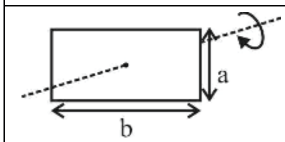
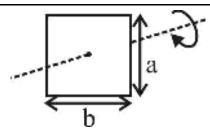
• **Centre of gravity for remains part after cut out a lamina–**



$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} \qquad \bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

■ **Mass moment of inertia–**

Shape	**Name**	**I**
	Rod	$\frac{mL^2}{12}$
	Rod	$\frac{mL^2\sin^2\theta}{12}$
	Ring	mR^2
	Disc	$\frac{mR^2}{2}$
	Hollow cylinder	mR^2
	Solid cylinder	$\frac{mR^2}{2}$

	Spherical shell	$\frac{2}{3}mR^2$
	Solid sphere	$\frac{2}{5}mR^2$
	Rectangular plate	$\frac{m\left(a^2+b^2\right)}{12}$
	Square plate	$\frac{ma^2}{6}$

■ **Area moment of inertia–**

Rectangular section	• About x axis	$I_{XX}=\frac{bd^3}{12}$
	• About y axis	$I_{YY}=\frac{db^3}{12}$
Hallow rectangular section	• About x axis	$I_{XX}=\frac{BD^3}{12}-\frac{bd^3}{12}$
	• About y axis	$I_{YY}=\frac{DB^3}{12}-\frac{db^3}{12}$
Circular section	$I_{XX}=I_{YY}$	$\frac{\pi D^4}{64}$
Triangular section	• About an axis passing through its centre of gravity and parallel to the base	$I_G=\frac{bh^3}{36}$
	• About the base	$I_B=\frac{bh^3}{12}$

■ **Equation of motion–**

For linear motion	For circular motion
• $v = u + at$	• $\omega w = \omega_o + \alpha t$
• $v^2 = u^2 + 2as$	• $\omega^2 = \omega_o^2 + 2\alpha\theta$
• $s = ut + \frac{1}{2}at^2$	• $\theta = \omega_o t + \frac{1}{2}\alpha t^2$

u = Initial velocity of body, v = Final velocity,
t = Time, a = Uniform acceleration,
s = Distance covered, ω = Final angular velocity,
ω_o = Initial angular velocity,
θ = Angular displacement, α = Angular acceleration.

• **Distance covered in n[th] second–**

$$S_n = u + \frac{a}{2}(2n-1)$$

■ **Motion of particle in a plane (2D motion)–**

Velocity	Acceleration
• $(v_x) = \frac{dx}{dt}$ • $v_y = \frac{dy}{dt}$ • $v_{resultant} = \sqrt{v_x^2 + v_y^2}$	• $(a_x) = \frac{d(v_x)}{dt}$ • $a_y = \frac{d(v_y)}{dt}$ • $a_{resultant} = \sqrt{a_x^2 + a_y^2}$

■ **Momentum–** $\boxed{P = mv}$ $\boxed{KE = \frac{P^2}{2m}}$

■ **Impulse momentum theorem–**
Impulse = change in momentum

$$\text{Impulse}(J) = \int F dt = \Delta P = P_f - P_i$$

■ **Law of conservation of momentum–**
If $F_{ext.} = 0$ then,
initial momentum = final momentum

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

■ **Angular momentum–** $\boxed{L = I\omega}$

■ **Conservation of angular momentum–**
If $T_{ext.} = 0$ then,
initial angular momentum = final angular momentum

$$\boxed{I_1\omega_1 = I_2\omega_2}$$

■ **Simple harmonic motion–**

		From origin	From extreme position
Displacement	x	A sin ωt	A cos ωt
Velocity	$v = \pm\omega\sqrt{A^2 - x^2}$	A ω cost ωt	–A ω sin ωt
Acceleration	$-\omega^2 x$	$-A\omega^2$ sin ωt	$-A\omega^2$ cos ωt

■ **Time period for different pendulum–**

Type of pendulum	Time period
Simple pendulum	$T = 2\pi\sqrt{\frac{\ell}{g}}$
Spring-mass system	$T = 2\pi\sqrt{\frac{m}{k}}$
Compound pendulum	$T = 2\pi\sqrt{\frac{k_G^2 + h^2}{g.h}}$
Conical pendulum	$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$

- Time period of second's pendulum is 2 second.
- Equivalent length of compound pendulum is–

$$L = \frac{k_G^2 + h^2}{h}$$

■ **Truss–**

Plane truss	If all members lie in a single plane
Space truss	Consists of members joined together at their ends to form a stable 3D structure.

	Plane truss	Space truss
Statically determinate	$m = 2j - 3$	$m = 3j - 6$
Statically indeterminate	$m > 2J - 3$	$m > 3j - 6$
Unstable truss	$m < 2j - 3$	$m < 3j - 6$

■ **Collision between two bodies**

Perfectly elastic collision	• Initial kinetic energy = Final kinetic energy • $e = 1$ • Velocity of approach = Velocity of separation • $u_1 - u_2 = v_2 - v_1$
Perfectly inelastic collision	• $e = 0$ • $(KE)_{loss} = (KE)_{initial} - (KE)_{final}$
Partially elastic	• $0 < e < 1$ • Velocity of separation = e (velocity of approach) • $v_2 - v_1 = e(u_1 - u_2)$ • Coefficient of restitution (e) = $\frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$

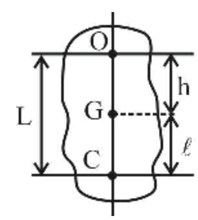
■ **Principle of transmissibility of force–** When a force acts on a body, this force may be assumed to be acting on all particles of the body which lie on the line of action of the force.

■

Principle of virtual work	• It states that for a body to be in equilibrium, the virtual work should be zero. If, $P_1, P_2 \ldots\ldots\ldots P_n$ = force $\delta_1, \delta_2 \ldots\ldots\ldots \delta_n$ = corresponding displacement $M_1, M_2 \ldots\ldots\ldots M_n$ = moment $\delta\theta_1, \delta\theta_2 \ldots\ldots\ldots \delta\theta_n$ = Corresponding angular displacement $P_1\delta_1 + P_2\delta_2 \ldots + M_1\delta\theta_1 + M_2\delta\theta_2 + \ldots = 0$

■ **Centre of percussion–**

- Point at which a blow may be struck on a suspended body on a suspended body so that the reaction at the support is zero.



- Centre of percussion is always below the centre of gravity. $(\ell) = \frac{k_G^2}{h}$
- The distance between the centre of suspension (O) & the centre of percussion (C) is equal to equivalent length (L) of simple pendulum

$$L = \ell + h$$

- Centre of suspension (O) and centre of percussion (C) are interchangeable

■ **D'-Alembert's principle–**

- It is used for analyzing the dynamic problem which can reduce it into a static equilibrium problem.
- It is an alternative form of Newton's second law of motion.
- F = ma (Newton's second law)
- F + (–ma) = 0 (D' Alembert's principle)

Where,
F = Real force,
(–ma) = Inertia force or Fictitious force

Strength of Material

Types of Material

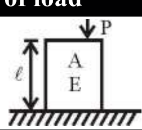
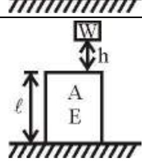
Homogeneous Material	A material which have same elastic properties at any point in a given direction.
Isotropic Material	This material has same identical properties in all direction at a point.
Anisotropic Material	It has different properties in all direction at a point in the body.
Orthotropic Material	A material which has different properties in three mutually perpendicular planes.

Material		**Poission's Ratio**
Cork	-	0
Glass	-	0.02 - 0.03
Cast Iron	-	0.23 - 0.27
Elastic Material	-	0.25 - 0.40
Steel	-	0.27 - 0.33
Rubber	-	0.50
Human Tissues	-	–1
Wrought Iron	-	0.30
Concrete	-	0.10 - 0.20

Elastic Constant

Elastic Constant	Formula
Young's Modulus or Modulus of Elasticity	$E = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}} = \frac{\sigma}{\varepsilon} = \frac{Fl}{\delta l \times A}$
Modulus of Rigidity/ Shear Modulus	$G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$
Poisson's Ratio	$\mu = -\frac{\text{Lateral Strain}}{\text{Linear Strain}} = -\frac{\delta d/d}{\delta l/l}$
Bulk Modulus	$K = \frac{\text{Direct stress}}{\text{Volumetric Strain}} = \frac{\sigma_d}{\varepsilon_v}$

Load with respect time

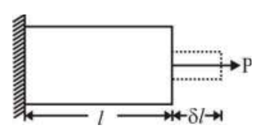
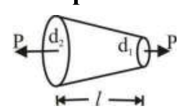
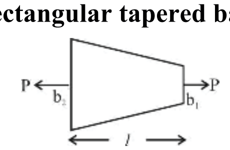
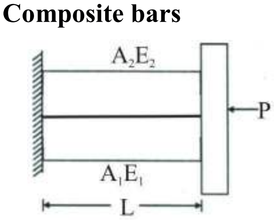
Type of load	Stress
Gradual load 	$\sigma = \frac{P}{A}$
Impact load 	$\sigma_i = \frac{W}{A}\left(1+\sqrt{1+\frac{2hAE}{W\ell}}\right)$
Sudden load	$\boxed{\sigma_{sudden} = 2\sigma}$

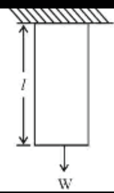
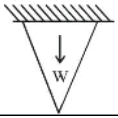
Relation between E, G, K & μ

- $E = 2G(1+\mu)$
- $E = 3K(1-2\mu)$
- $E = \frac{9KG}{3K+G}$
- $\mu = \frac{3K-2G}{6K+2G}$

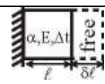
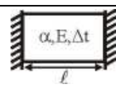
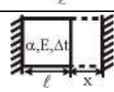
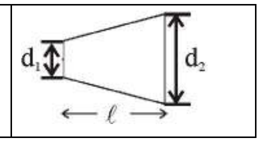
Types of Material	Total number of Elastic Constants	No. of Independent Elastic Constant
Homogeneous and Isotropic	4	2
Orthotropic (wood)	12	9
Anisotropic	∞	21

Axial Elongation in Different Types of Bar–

Type of bar	Elongation due to external load
Prismatic bar 	$\delta l = \frac{Pl}{AE} = \frac{\sigma l}{E}$
Circular tapered bar 	$\delta l = \frac{4Pl}{\pi d_1 d_2 E}$
Rectangular tapered bar 	$\delta l = \frac{Pl \log_e\left(\frac{b_2}{b_1}\right)}{(b_2-b_1)Et}$ t = thickness
Composite bars 	$P = P_1 + P_2$ Change in length $\delta_1 = \delta_2 = \frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$ $P_1 = \frac{A_1E_1}{A_1E_1 + A_2E_2} \times P$ $P_2 = \frac{A_2E_2}{A_1E_1 + A_2E_2} \times P$

Types of bar	Elongation due to self weight
1. Prismatic bar 	$U = \frac{1}{2}\frac{\sigma^2_{max}}{E}$ $= \frac{wl^2}{2E} = \frac{\rho g l^2}{2E}$ (w or $\gamma = \rho g$)
2. Uniform tapering or conical bar 	$\delta l_c = \frac{WL}{6AE} = \frac{\gamma l^2}{6E}$ or $\delta l_c = \frac{1}{3}\times\frac{Wl}{2AE} = \frac{1}{3}\times\delta l_P$
3. Prismatic bar due to external load & self weight	$\delta l = \frac{PL}{AE} + \frac{WL}{2AE}$

Thermal Stress

(1) Case 1 : Free expansion or contraction :- $\sigma_{th} = 0$ (No thermal stress)	
(2) Case 2 : Fully prevented- $\sigma_{th} = E\alpha\Delta T$, $\delta\ell = 0$	
(3) Case 3 : Partially prevented $\sigma_{th} = \frac{E(\ell\alpha\Delta T - x)}{\ell}$	
(4) Case 4 : Taper section $\sigma_{th} = E\alpha\Delta t\frac{d_2}{d_1}$	

■ **True stress and strain & there relation with engineering stress and strain-**

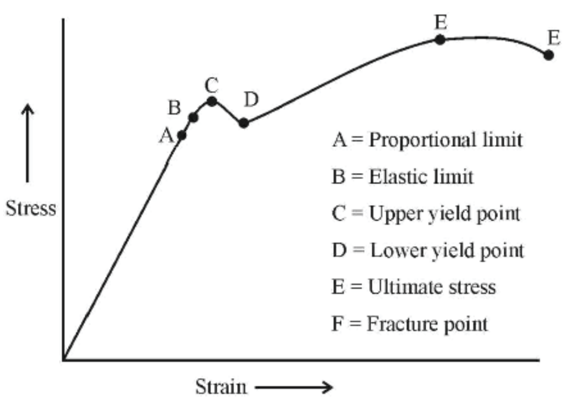
Stress	Strain
$\sigma_T = \frac{P}{A_i}$	$\varepsilon_T = \ell n\left(\frac{\ell_i}{\ell_o}\right)$
$\sigma_T = \sigma(1+e)$	$\varepsilon_T = \ell n(1+e)$

Modulus of Elasticity for different types of Material

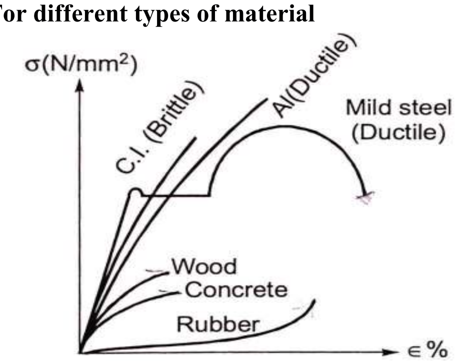
Material	Young's Modulus (E) (MPa)
Steel	2×10^5
Copper	1.17×10^5
Cast Iron	1.7×10^5
Timber (wood)	0.10×10^5
Aluminium	0.70×10^5
Glass	0.80×10^5

Stress strain curve for different material

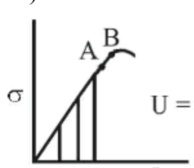
For ductile material



For different types of material

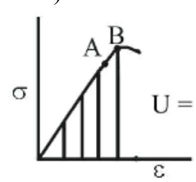


Resilience-(Energy absorbed by body within elastic limit)



$U = \frac{1}{2}\frac{\sigma^2}{E}\times V$

Proof resilience –(Energy absorbed by body upto elastic limit)

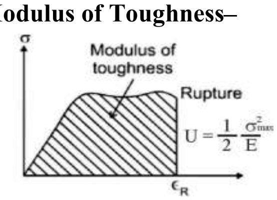


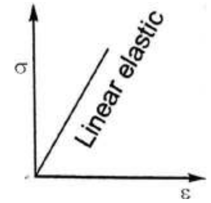
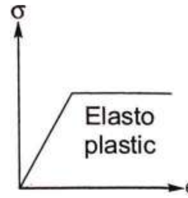
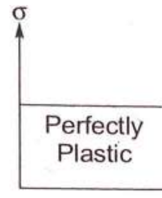
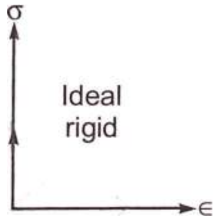
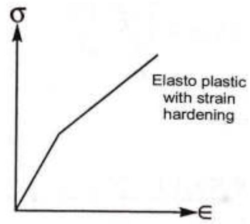
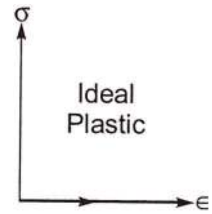
$U = \frac{1}{2}\frac{\sigma^2_{max}}{E}\times V$

Modulus of resilience –(Proof resilience per unit volume)

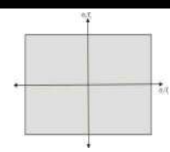
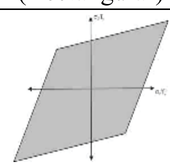
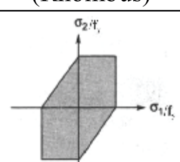
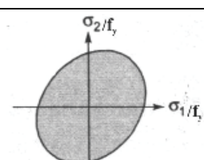
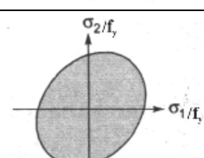
$U = \frac{1}{2}\frac{\sigma^2_{max}}{E}$

Modulus of Toughness–



Theory of failure

Theory	Given by	Suitable for Material	Graphical representation
Maximum Principal Stress or normal stress	Rankine	Brittle	 (Rectangular)
Maximum Principal Strain	St. Venants	Brittle	 (Rhombus)
Maximum Shear Stress	Guest & Trasca's	Ductile	 (Hexagon)
Maximum Strain Energy	High & Beltrami	Ductile	 (Elliptical)
Maximum Shear Strain Energy	Vonmises and Hencky	Ductile	 (Elliptical)

Principle stress/Principal strain

Normal stress & shear stress on any plane :

- Normal stress, (σ_n) =

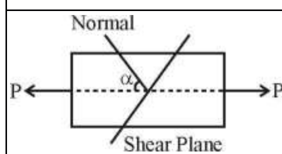
$$\frac{(\sigma_x+\sigma_y)}{2}+\left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos 2\theta+\tau_{xy}\sin 2\theta$$

- Tangential or shear stress, τ =

$$=-\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta+\tau_{xy}\cos 2\theta$$

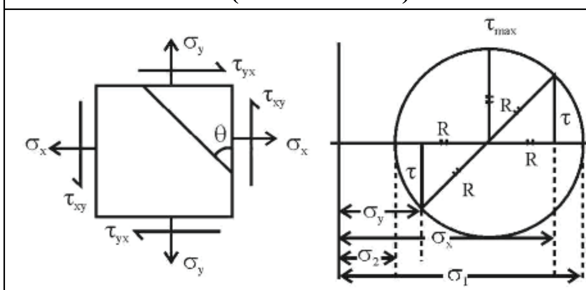
- Principal Plane $\tan 2\theta=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$

Case-1 : Uniaxial or 1 D load :



- Normal Stress$(\sigma_n)=\frac{P}{A}\cos^2\alpha$
- Shear stress$(\tau)=\frac{P}{2A}\sin 2\alpha$
- Resultant stress $(\sigma_r)=\frac{P}{A}\cos\alpha$

Case-2 : 2D- Biaxial (Mohar's Circle)



σ_1 = Major principal stress (normal)

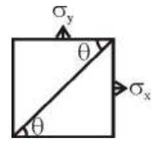
σ_2 = Minor principal stress (normal)

- $\sigma_1=\frac{\sigma_x+\sigma_y}{2}+\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$
- $\sigma_2=\frac{\sigma_x+\sigma_y}{2}-\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$
- Radius of Mohor's circle (τ_{max})

$$=\frac{\sigma_1-\sigma_2}{2}=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

- **Center Mohor's circle** $=\left[\left(\frac{\sigma_x+\sigma_y}{2}\right),0\right]$

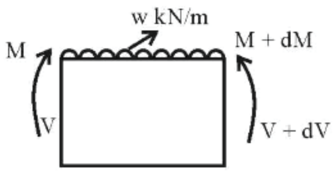
■ **Principal strain**



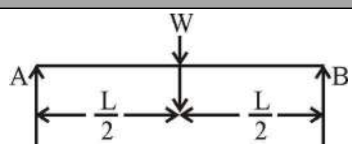
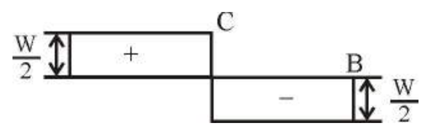
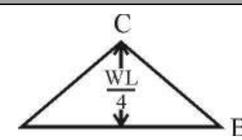
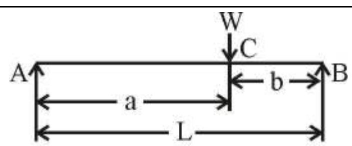
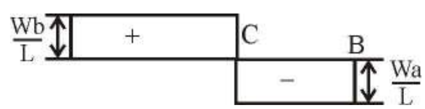
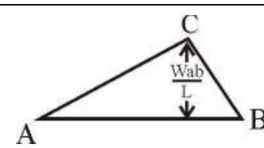
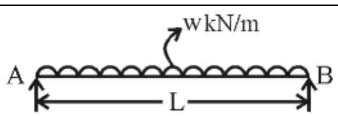
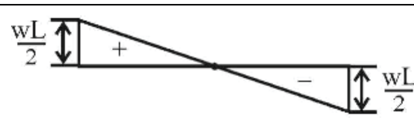
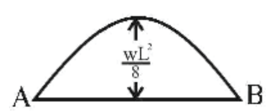
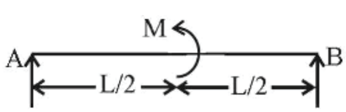
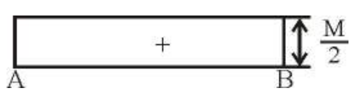
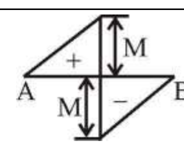
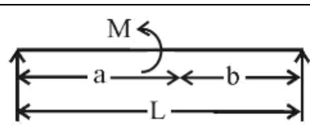
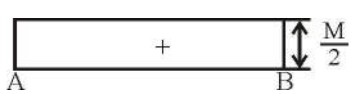
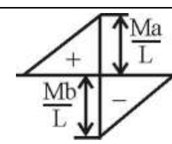
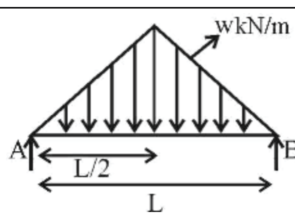
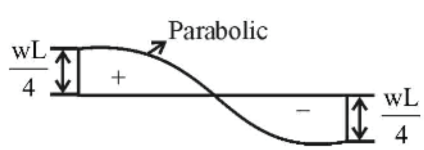
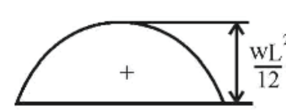
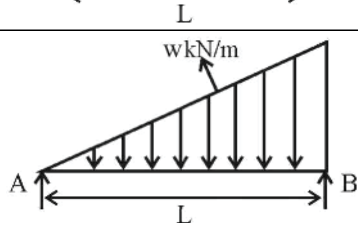
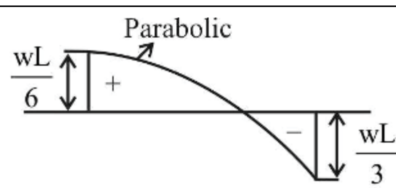
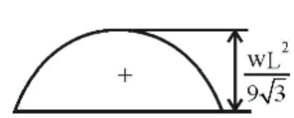
Strain in diagonal due to σ_x	$e_x \cos^2\theta$
Strain in diagonal due to σ_y	$e_y \sin^2\theta$

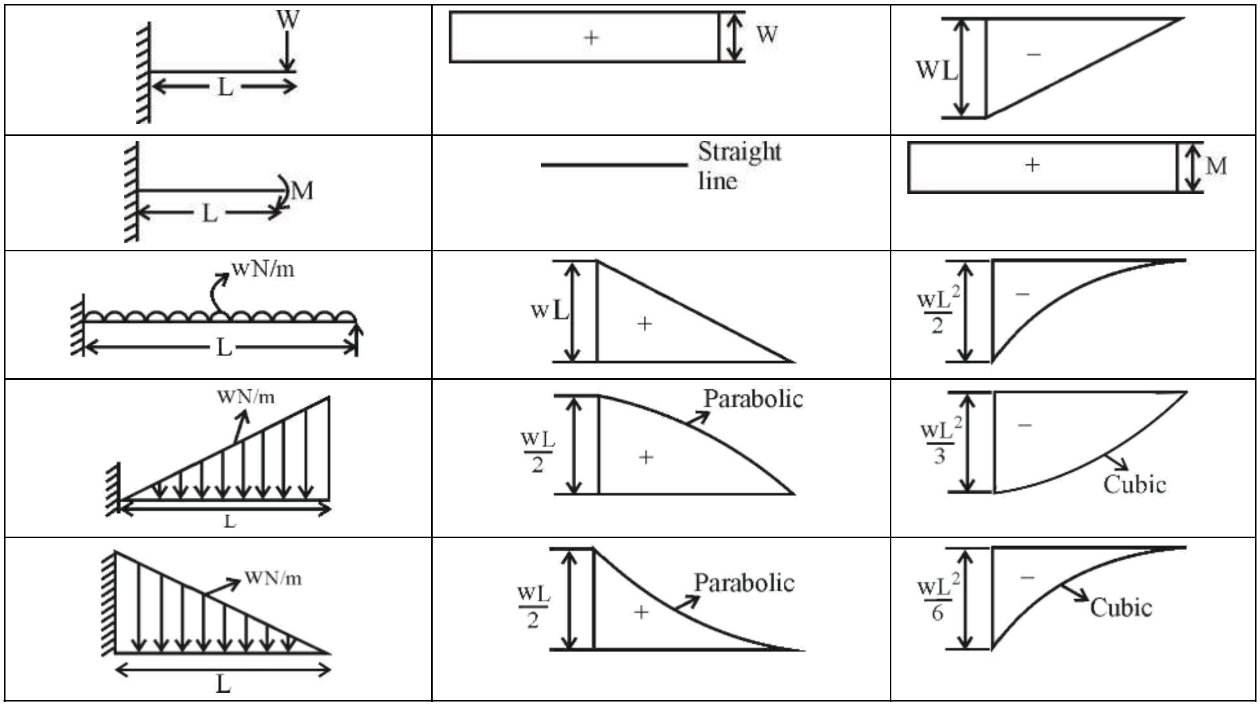
Strain in diagonal due to shear (τ)	$\frac{\phi}{2}\sin^2\theta$
Maximum shear strain $\left(\frac{\phi}{2}\right)_{max}$	$\left(\frac{\phi}{2}\right)_{max} = \left(\frac{e_1 - e_2}{2}\right)$
Principal strain ($e_{1,2}$)	$\frac{e_x + e_y}{2} \pm \sqrt{\left(\frac{e_x - e_y}{2}\right)^2 + \left(\frac{\phi}{2}\right)^2}$

Shear force and Bending moment diagram



- Rate of change of shear force is equal to load $\frac{dV}{dx} = -W$
- Rate of change of bending moment along the length of beam is equal to shear force $\frac{dM}{dx} = V$

Beam	**Shape**	
	SFD	**BMD**
		
		
		
		
		
		
		



Bending of Beam

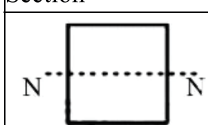
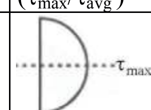
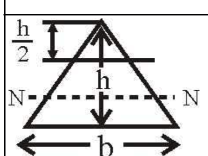
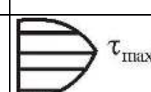
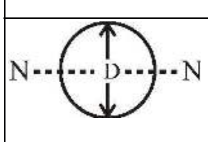
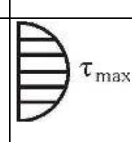
• Bending equation- $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$

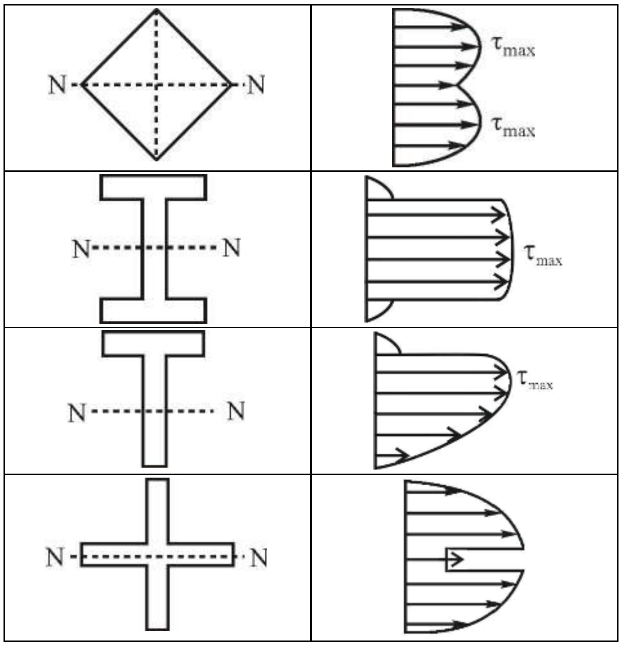
Where, σ_b = Bending stress an any section
y = Distance of any layer from N.A.
M = Resisting bending moment.
I = Area M.O.I. about N.A.
R = Radius of Curvature

Bending stress-	$\sigma_b = \frac{M \times y}{I}$
Section modulus of beam (Z)	$Z = \frac{I}{y}$ if Z ↑→ Strength↑
Radius of curvature (R)	$R = \frac{EI}{M}$
Flexural Rigidity	E × I

Some Important M.O.I. & section modulus

Cross Section	M.O.I	Section Modulus $Z = \frac{I}{y}$
Rectangular section	$I_{XX} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$ $I_{base} = \frac{bd^3}{3}$	$Z = \frac{bd^2}{6}$
Triangular section	$I_{XX} = \frac{bh^3}{36}$ $I_{base} = \frac{bh^3}{12}$ $I_{top} = \frac{bh^3}{4}$	$Z = \frac{bh^2}{24}$
Solid Circular section	$I_{XX} = I_{YY} = \frac{\pi D^4}{64}$	$Z = \frac{\pi D^3}{32}$
Hollow circular section	$I_{xx} = I_{yy} = \frac{\pi(D^4 - d^4)}{64}$	$Z = \frac{\pi}{32D}(D^4 - d^4)$
Diamond section	$I_d = \frac{a^4}{12}$	$Z = \frac{a^3}{6\sqrt{2}}$
Square section	$I_{xx} = I_{yy} = \frac{a^4}{12}$	$Z = \frac{a^3}{6}$

Section	$(\tau_{max}/\tau_{avg}) = r$
	 • $\tau_{avg} = \frac{F}{bh}$ • $\tau_{max} = \frac{3}{2}\frac{F}{bh}$ • $r = \frac{3}{2}$
	 • $\tau_{avg} = \frac{F}{bh/2}$ • $\tau_{max} = \frac{3}{2}\frac{F}{(bh/2)}$ • $\tau_{NA} = \frac{4}{3}\frac{F}{bh/2}$ • $r = \frac{3}{2}$
	 • $\tau_{avg} = \frac{4F}{\pi D^2}$ • $\tau_{max} = \frac{16}{3}\frac{F}{\pi D^2}$ • $r = \frac{4}{3}$



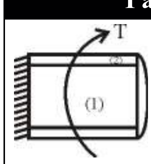
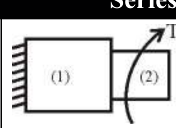
Torsion

- Pure torsion equation- $\frac{T}{J}=\frac{\tau}{R}=\frac{G\theta}{L}$, Where,

T = Torque, J = Polar moment of inertia
τ = Shear stress, R = Radius of shaft
G = Shear modulus, θ = Angle of twist
L = Length of shaft

- Shear stress- $\tau=\frac{T\times R}{J}=\frac{16T}{\pi D^3}$
- Torque (T) = $\frac{\pi}{16}\times\tau\times D^3$
- Power transmitted by shaft $(P)=\frac{2\pi NT}{60\times1000}$ k.W
- Polar section modulus (Z_P) $=\frac{J}{R}$
- Strength of solid shaft- $T_s=\frac{\pi}{16}\times\tau D^3$
- Polar M.O.I of solid shaft- $J=\frac{\pi}{32}d^4$
- Polar M.O.I. of Hollow shaft- $J=\frac{\pi}{32}\left(D^4-d^4\right)$
- Ratio of torque- $\frac{T_{Hollow}}{T_{Solid}}=\frac{D^4-d^4}{D^4}$

Connection of shaft

Parallel	Series
 $T=T_1+T_2$, $\theta_1=\theta_2$	 $T_1=T_2=T$, $\theta=\theta_1+\theta_2$

Design of shaft–

(Design of shaft subjected to combined twisting & bending moment)

According to maximum shear stress theory	According to maximum normal stress theory
• $\tau_{max}=\frac{1}{2}\sqrt{\sigma_b^2+4\tau^2}$ $\tau=\frac{16T}{\pi d^3},\sigma_b=\frac{32M}{\pi d^3}$	• $(\sigma_b)_{max}=\frac{1}{2}\sigma_b+\frac{1}{2}\sqrt{\sigma_b^2+4\tau^2}$ $\tau=\frac{16T}{\pi d^3},\sigma_b=\frac{32M}{\pi d^3}$
• $\tau_{max}=\frac{16}{\pi d^3}\sqrt{M^2+T^2}$	• $\sigma_{b\,max}=\frac{16}{\pi d^3}\left[M+\sqrt{M^2+T^2}\right]$
• $T_e=\sqrt{M^2+T^2}$	• $M_e=\frac{1}{2}\left[M+\sqrt{M^2+T^2}\right]$

Deflection of beam

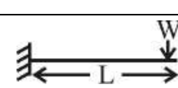
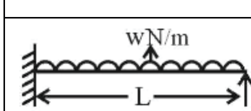
- Relation between loading, S.F., B.M. Slope & deflection –

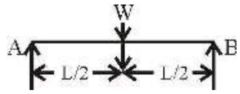
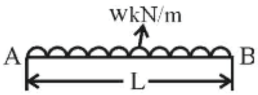
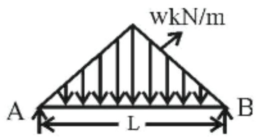
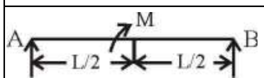
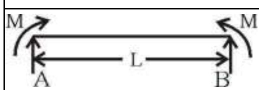
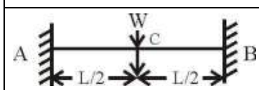
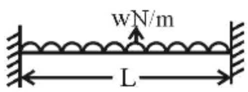
Deflection equation	EI.y
Slope equation	$EI\left(\frac{dy}{dx}\right)$
Moment equation	$EI\left(\frac{d^2y}{dx^2}\right)$
Shear equation	$EI\left(\frac{d^3y}{dx^3}\right)$
Load equation	$EI\left(\frac{d^4y}{dx^4}\right)$

Method to Determine Slope and Deflection–

1. Double Integration Method
2. Macaulay's Method
3. Area Moment Method/ Mohr's Method
4. Strain energy Method
5. Conjugate Beam Method
6. Superposition Method

■ Maximum slope (θ_{max}) & deflection (y_{max}) under different loading condition-

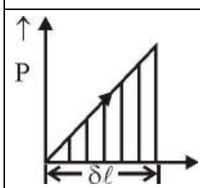
Beam	(θ_{max})	(y_{max})
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$

	$\frac{wL^3}{24EI}$	$\frac{wL^4}{30EI}$
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$
	$\frac{wL^3}{24EI}$	$\frac{5}{384}\frac{wL^4}{EI}$
	$\frac{wL^3}{120EI}$	$\frac{5}{192}\frac{wL^4}{EI}$
	$\frac{ML}{24EI}$	$y_c = 0$
	$\frac{ML}{2EI}$	$(y_c)_{max} = \frac{ML^2}{8EI}$
	$\theta_A = \theta_B = 0$	$(y_{max}) = \frac{WL^3}{192EI}$
	$\theta_A = \theta_B = 0$	$(y_{max}) = \frac{wL^4}{384EI}$

Strain energy

- The energy absorbed or store by the material is called strain energy

- Strain energy under elastic limit-

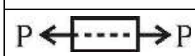


Strain energy = Work done on body

U = Area under curve

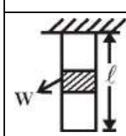
$$U = \frac{1}{2} \times \delta\ell \times P$$

- **Case 1 : Due to axial loading on uniform bar-**



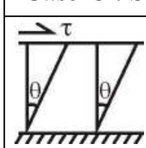
$$U = \frac{P^2L}{2AE} \qquad U = \frac{\sigma^2 V}{2E}$$

- **Case 2 : Uniform bar having under it's own weight-**



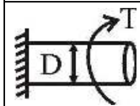
$$U = \frac{w^2 A \ell^2}{6E}$$

- **Case- 3 : Strain energy due to shear load-**



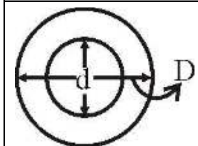
$$U = \frac{\tau^2}{2G} \times V$$

- **Case- 4 : Strain energy due to torsion in solid shaft**



$$U = \frac{1}{2}T\theta = \frac{\tau^2}{4G} \times \text{Volume of Shaft}$$

- **Case-5 : Strain energy due to torsion in hollow shaft-**

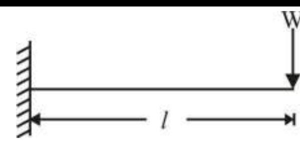
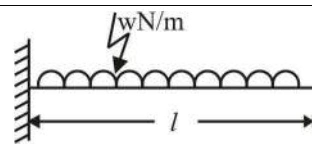
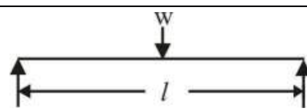
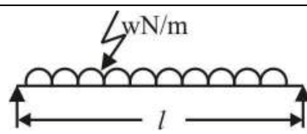
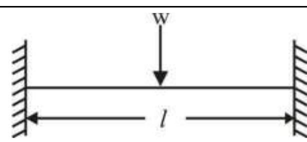
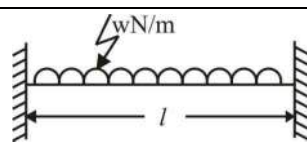
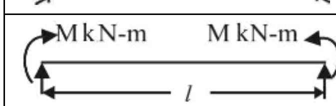


$$U = \frac{\tau^2}{4G} \times \left(\frac{D^2 + d^2}{D^2}\right) \times \text{Volume of shaft}$$

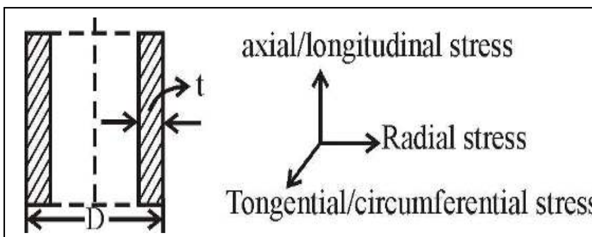
$$V = \frac{\pi}{4}\left(D^2 - d^2\right)L$$

- **Case 6 : Strain energy due to bending-**

$$U = \int \frac{M_x^2 dx}{2EI}$$

Types of Beam	Strain Energy
	$\frac{W^2 l^3}{6EI}$
	$\frac{w^2 l^5}{40EI}$
	$\frac{W^2 l^3}{96EI}$
	$\frac{w^2 l^5}{240EI}$
	$\frac{W^2 l^3}{384EI}$
	$\frac{w^2 l^5}{1440EI}$
	$\frac{M^2 l}{2EI}$

Analysis of thin cylinder



if $\boxed{\frac{t}{D} \le \frac{1}{20}}$ →Thin wall cylinder $\boxed{\frac{t}{D} > 20}$ → Thick wall cylinder

Stress	Strain
1. Hoop stress $\sigma_H = \frac{PD}{2t}$	1. Hoop strain $\varepsilon_h = \frac{PD}{4tE}(2-\mu)$
2. Longitudinal stress $\sigma_L = \frac{PD}{4t}$	2. Longitudinal strain $\varepsilon_L = \frac{Pd}{4tE}(1-2\mu)$
3. Radial stress $\sigma_r = -P$	3. Volumetric strain $\varepsilon_v = \frac{PD}{4tE}(5-4\mu)$
4. Maximum shear stress $(\tau_{max}) = \frac{PD}{8t}$	4. $\boxed{\frac{\varepsilon_h}{\varepsilon_L} = \frac{2-\mu}{1-2\mu}}$
5. Relation between σ_H & σ_L $\sigma_h = 2\sigma_L$	

Analysis of thin sphere

1. Hoop stress/longitudinal stress $\sigma_L = \sigma_H = \frac{PD}{4t}$	1. Hoop strain/longitudinal strain $\varepsilon_L = \varepsilon_h = \frac{PD}{4tE}(1-\mu)$
	2. Volumetric strain $\varepsilon_v = \frac{3PD}{4tE}(1-\mu)$

Column

- Any slender body subjected to axial compressive load is called column.
- Slenderness ratio (S.R.)

$$= \frac{\text{Effective length of Column } (\ell_e)}{\text{Minimum radius of gyration } (K_{min})}$$

- $I = AK^2 \Rightarrow K_{min} = \sqrt{\frac{I_{min}}{A}}$

Classification and failure of Column Based an Slenderness Ratio

S.R	Types of column	Fails in
< 32	Short column	Crushing
32-120	Intermediate column	Combined, crushing and buckling
>120	Long column	Buckling

- **Critical load (P_{cr})/Euler's load (P_b)/Crippling load (P_e)**

$$(P_b, P_{cr}, P_e) = \frac{\pi^2 EI_{min}}{\ell_e^2}$$

Note :

Euler's formula is applicable only for long column.

Effective length of column based on end condition

End Condition	One end Fixed and other end Free	Both end Hinged	Both end Fixed	One end Fixed and other Hinged
Effective length	$l_e = 2L$	$l_e = L$	$l_e = L/2 = 0.5L$	$l_e = \frac{L}{\sqrt{2}} = 0.70L$
Buckling Load/ Euler load $P_e = \frac{\pi^2 EI}{l_e^2}$	$\frac{\pi^2 EI}{4L^2}$	$\frac{\pi^2 EI}{L^2}$	$\frac{4\pi^2 EI}{L^2}$	$\frac{2\pi^2 EI}{L^2}$

Rankine's Formula–

- (Applicable for both medium & long column)
- Column fail due to both crushing & bending

$$\boxed{\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}}$$

Where,
P_c =Crushing load
P_e =Euler load

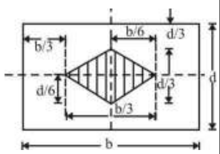
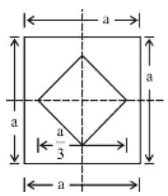
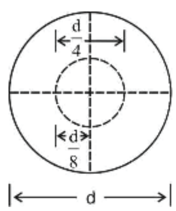
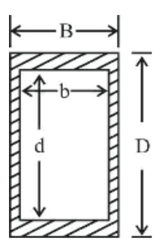
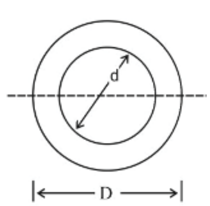
$$\boxed{P_R = \frac{\sigma_c . A}{1 + a\left(\frac{\ell_e}{k}\right)^2}}$$

Where, $a = \frac{\sigma_c}{\pi^2 E}$

σ_c = Compressive stress
A = Cross section of column
a = Rankine constant.

Material	σ_c (N/mm^2)	Rankine's Constant When both ends are hinged
Cast Iron	550	$\frac{1}{1600}$
Wrought Iron	250	$\frac{1}{9000}$
Mild Steel	320	$\frac{1}{7500}$
Strong Timber	50	$\frac{1}{750}$

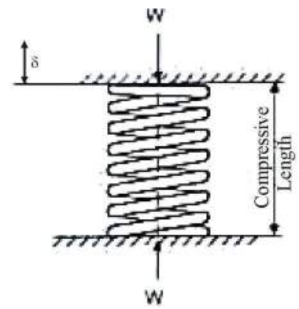
Max. Limit of eccentricity

Section	Max. Eccentricity Limit	Shape of core
Solid Rectangular Section 	$e_{x-x} \le \frac{d}{6}, e_{y-y} \le \frac{b}{6}$ Side of core = $\frac{\sqrt{b^2+d^2}}{6}$ This known as middle third rule.	Rhombus
Square Cross section 	$e \le \frac{a}{6}$ Kernel size, $\frac{a}{3} \times \frac{a}{3}$	Square
Solid Circular Section 	$e_{max} \le \frac{d}{8}$ Dia of core, d/4 Known as middle fourth rule.	Circular
Hollow Rectangular Section 	$e_{x-x} \le \frac{BD^3 - bd^3}{6D(BD-bd)}$ $e_{y-y} \le \frac{DB^3 - db^3}{6B(BD-bd)}$	Rhombus
How Circular Section 	$e_{max} \le \frac{D^2+d^2}{8D}$ Dia of core, $\frac{D^2+d^2}{4D}$	Circular

Spring

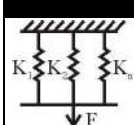
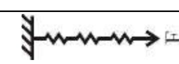
(A) Closed coil helical spring under axial pull :

- Spring are use to absorb energy and restore it slowly or rapidly



Solid Length (L_s)	$n \times d$
Spring Index (C)	$\frac{D}{d}$
Stiffness (S)	$\frac{W}{\delta} = \frac{Gd^4}{8D^3n}$
Axial deflection of spring (δ)	$\frac{8WD^3n}{Gd^4}$
Shear stress in spring (τ_{max})	$\tau_{max} = \frac{8K_wWD}{\pi d^3}$ Where, $K_W \rightarrow$ Wahl's correction factor $K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

Connection of spring

Parallel combination		Series combination
	$F = F_1 + F_2 + ...F_n$ $K_{eq} = K_1 + K_2 ...K_n$	$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} ... + \frac{1}{K_n}$ $F = F_1 = F_2 = F_n$ 

(B) Leaf spring :

$$\sigma = \frac{3}{2}\frac{WL}{nbt^2} \qquad \delta = \frac{3}{8} \times \frac{W\ell^3}{Enbt^3}$$

Where,

W = load

b = width of plate

ℓ = spring span length

n = number of plate

t = thickness of plate

Theory of Machine

Simple Mechanism

Kinematic link	Every part of machine which is having some relative motion with respect to some other machine part.

■ **Type of link**

Rigid link	Deformation are negligible e.g. ⇒ Crank, C.R., Piston etc.
Flexible link	Deformation are not negligible but are in permissible limit, e.g. ⇒ Belt drive, rope drive etc.
Fluid link	Where power is transmitted because of fluid pressure. e.g. ⇒ Hydraulic/Pneumatic system like brake, jack etc.

■ **Kinematic pair/joint–**
Any connection between the two link is known as kinematic pair.

Classification of kinematic pair

(A) According to types of relative motion–

Turning pair	Crank pin, gudgeon pin, pin joint
Sliding pair	Piston inside cylinder of I.C. engine
Rolling pair	Rolling of cylinder on flat surface
Screw pair	Nut-bolt
Cylindrical pair	Two co-axial cylinder in contact
Flat pair	Two flat surface in contact
Spherical pair	Ball and socket joint, open stand, the mirror attachment of vehicles.

(B) According to types of contact–

Lower pair	Surface contact or Area contact	Turning pair, sliding pair screw pair, spherical pair, cylindrical pair
Higher pair	Point or line contact (Zero area contact)	Rolling pair, pair between cam & follower
Wrapping pair	Multiple point contact (Close to higher pair)	• Belt – pulley • Rope – pulley • Chain – sprocket

(C) According to types of closure–

Self closed pair	No external force required to maintain this pair	Turning pair, sliding pair, screw pair etc.
Forced closed pair	Continuous external force required to maintain this pair	• Higher pair between cam & follower • Automatic clutch operating system

■ **Type of relative motion–**

Completely constrained motion	Only one output motion with respect to input **e.g.**– Prismatic pair, shaft with both end collar
Successfully constrained motion	Only one output motion with respect to input. **e.g.**– Foot step bearing, piston-cylinder arrangement in IC engine.
Incompletely constrained motion	More than one output motion with respect to input. **e.g.**– Circular shaft in circular hole

■ **Degree of freedom (DOF)–** No. of independent variables required to define a position (or) motion of the system.

DOF = 6 – No. of restraints (in space)

- If a link of redundant chain is fixed → Structure or locked system is formed.

If DOF is (–ve) ⇒ Super structure

If DOF = 1 ⇒ Constrained chain

DOF > 1 ⇒ Unconstrained chain

■ **Degree of freedom of plane (2D) mechanism (Grubler's criteria)**

Kutzback's equation– $F = 3(L-1) - 2J - h$

Where,

L → No. of link

J → No. of binary joint

h → No. of higher pair

■ **Grubler's equation–**

DOF = 1 & h = 0

Then, $3L - 2J - 4 = 0$

Following relationship–

For a kinematic chain, having lower pairs

$L = 2P - 4$ $J = \frac{3}{2}L - 2$

L.H.S. > R.H.S. ⇒ Locked chain

L.H.S. < R.H.S. ⇒ Incompletely constrained chain

L.H.S. = R.H.S. ⇒ Completely constrained chain

Note–

Minimum no. of link to have a mechanism (1 DOF) with only lower pairs is 4 link. But minimum no. of links to have a mechanism (1 DOF) with both lower & higher pair is 3 link.

1 HP = 2 LP + 1 extra link
L.P. ⇒ 1 D.O.F.
H.P. ⇒ 2 D.O.F.

Mechanism

■ **4-bar mechanism**

Grashof's law– $(S + L) \le (P + Q)$

Here, S = shortest link

L = longest link

P, Q = remaining link

Inversions : No. of inversions ≤ No. of link (N)

Inversion-1 (Frame fixed)	Crank-rocker mechanism	Beam engine
Inversion-2 (Crank fixed)	Double-crank mechanism	Coupling rod mechanism locomotive
Inversion-3 (Coupling fixed)	Crank-rocker mechanism	Beam engine
Inversion-4 (Rocker fixed)	Double-rocker mechanism	Watt's indicator mechanism

■ **Inversion of slider crank mechanism–** 3TP + 1 SP

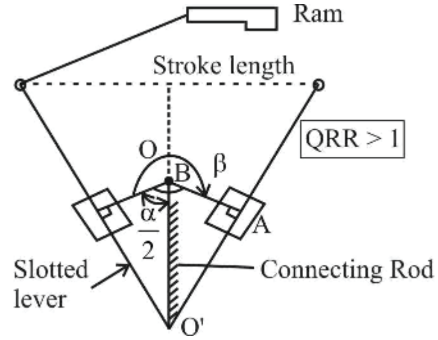
Inversion-1 (Frame fixed)	Crank slider mechanism, reciprocating engine/compressor
Inversion-2 (Crank fixed)	Whitworth quick return mechanism, rotary (radial) engine
Inversion-3 (Connecting rod fixed)	Crank & slotted lever mechanism, oscillating cylinder engine mechanism
Inversion-4 (Slider fixed)	Hand pump, bull engine.

■ **Inversion of double slider crank mechanism–**

2TP + 2SP

Link 1 is fixed	Elliptical trammel
Slider 2 is fixed	Scotch yoke mechanism (follow sine curve) $\text{Rotory} \xrightarrow[\text{converts}]{} \text{reciprocating}$
Link 3 is fixed	Oldham coupling (Used to transmit power between offset shafts) $\omega_{driver} : \omega_{driven} = 1 : 1$

■ **Crank & slotted lever QRMM–**



$$\text{Quick Return Ratio} = \frac{\beta}{\alpha} = \frac{360-\alpha}{\alpha} > 1$$

$$\cos\frac{\alpha}{2} = \frac{OA}{OO'} = \frac{\text{Length of crank}}{\text{Length of connecting rod}}$$

$$\text{Length of stroke} = \frac{2 \times L_{crank} \times L_{Slotted\,bar}}{L_{connecting\,rod}}$$

Approximate straight line mechanism	Watt indicator Modified scott-russel mechanism Grass hopper mechanism
Exact straight line mechanism	Peaucellier mechanism Hart's mechanism Scott-Russel's mechanism

■

Mechanism	No. of link
Hart's mechanism	6 links
Peaucellier mechanism	8 links
Scott Russel's mechanism	3 moving link of which 1 rotating/sliding pair

■ **Mechanical advantage–**

$$MA = \frac{\text{Output force or torque}}{\text{Input force or torque}}$$

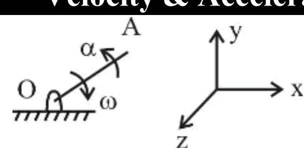
$$MA = \frac{F_o}{F_i} = \frac{T_o}{T_i} = \frac{\text{Load}}{\text{Effort}}$$

➢ Relation between MA and efficiency–

$$\eta = \frac{P_o}{P_i} = \frac{F_o.v_o}{F_i.v_i} = \frac{T_o\omega_o}{T_i.\omega_i}$$

$$\Rightarrow \quad MA = \eta.\frac{v_i}{v_o} = \eta\frac{\omega_i}{\omega_o}$$

Velocity & Acceleration Analysis



$$V_A = OA.\omega$$

O → Centre of rotation

A → Point whose velocity is to be calculated

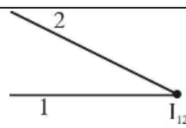
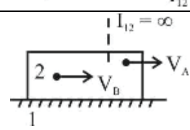
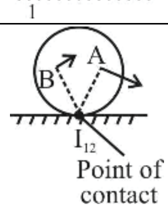
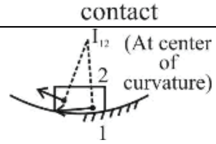
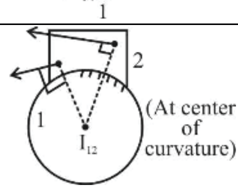
■ **I-centre (Instantaneous centre)–**

It is a point about which a body is said to have pure rotation.

Centrode	The locus of all these instantaneous centre for a particular link.
Axode	The line passing through instantaneous centre & perpendicular to the plane of motion is known as instantaneous axis. It is a surface
No. of I-centre	$I = \frac{n(n-1)}{2} = {}^nC_2$ (Here, n = no. of links)
Kennedy's theorem	If three plane bodies have relative motion among themselves, their I-centre must be lies on a straight line.

Motion of link	Centrode	Axode
General motion	Curve	Curve surface
Pure translation	Straight line	Plane surface
Pure rotation	Point	Line

■ **I-centre of different pair–**

Turning pair	
Sliding pair	
Rolling pair	
Concave surface	
Convex surface	
Rolling with sliding	I-centre lies on the common normal at the point of contact

■ **Angular velocity ratio theorem–**

$$\omega_m(I_{mn}I_{1m}) = \omega_n(I_{mn}I_{1n})$$

If I_{1m} and I_{1n} lies at same side of I_{mn} then sense of $\omega_m \times \omega_n$ will be same.

■ **Velocity of rubbing–**

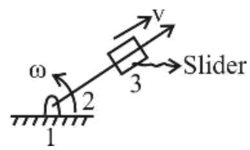
$(\omega_1 \pm \omega_2)r$

ω_1, ω_2 ⇒ Angular velocity of link at joint

(+ve) ⇒ Opposite direction

(–ve) ⇒ Same direction

■ **Acceleration analysis–**

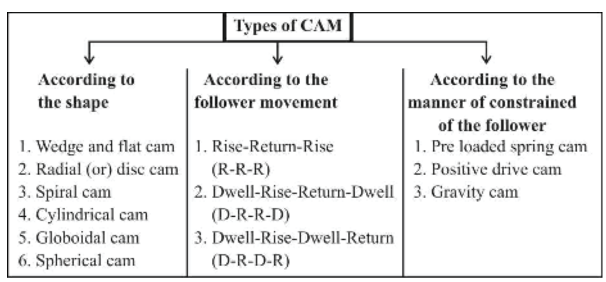
Tangential Acceleration	$a_t = \frac{dv}{dt} = r\alpha$
Centripetal acceleration (or) Radial acceleration	$a_c = \frac{v^2}{r} = \omega^2 r$
Coriolis acceleration component  Motion of slider on rotating link	$\vec{a}_c = 2[\vec{\omega} \times v]$ **Direction of coriolis** 1. Rotate velocity vector by 90° 2. The sense of rotation should be same as ω.

Cams

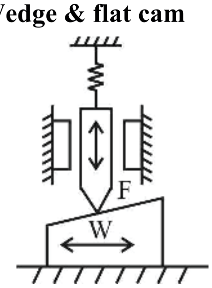
- The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating.
- A cam and follower combination belongs the category of higher pairs.

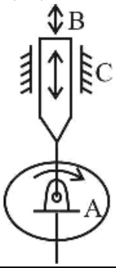
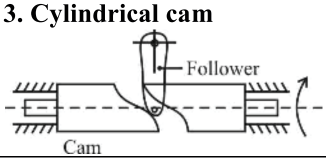
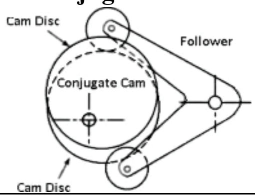
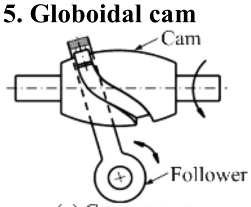
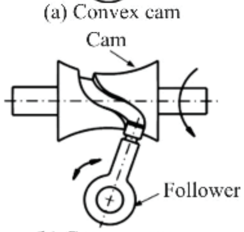
Cam– Cam is the driving link and has a curved (or) straight contact surface.

Follower– It is the driven link and it gets motion by contact with the cam surface.

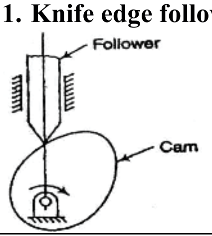
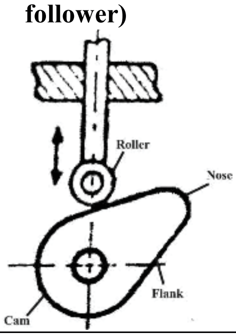
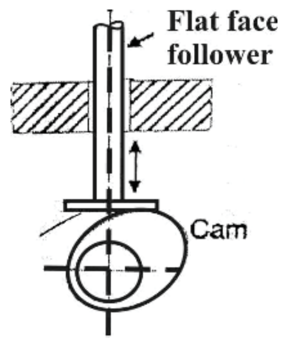
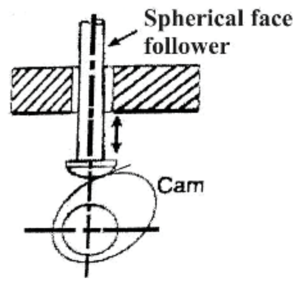


■

1. Wedge & flat cam 	A flat cam has a translational motion & the follower can either translate (or) oscillate

2. Radial (or) disc cam  	• The axis of rotation of cam is perpendicular to line of motion of follower • The axis of the follower passes through the axis of the cam.
3. Cylindrical cam  	Cam has a circumferential contour cut in surface & rotates about its axis.
4. Conjugate cams  	This cam is preferred when the requirement of low wear, low noise, better control of the follower
5. Globoidal cam  (a) Convex cam  (b) Concave cam	• It has two types of surface i.e. convex (or) concave. • Used when moderate speed & angle of oscillation of the follower is large.

■ **Types of follower–**

1. Knife edge follower  	Area of contact is zero Excessive wear ⇓ Wrost follower
2. Roller follower (Best follower)  	• Here because of pure rolling, friction is very low, hence wear is zero Because of roller (3D body) space requirement are high. **Used in–** Gas engines, Aircraft engine, Valve operating mechanism.
3. Flat face follower   	Here contacting area (dA) → 0 Wear is highly reduced as compared to knife edge. • Here surface stress are generated • To minimize surface stress spherical faced follower used

Note– Mushroom follower– Flat face follower in which flat face is in the form of circular disc. It does not create the problem of jamming the cam

■ **According to the location of line of action–**

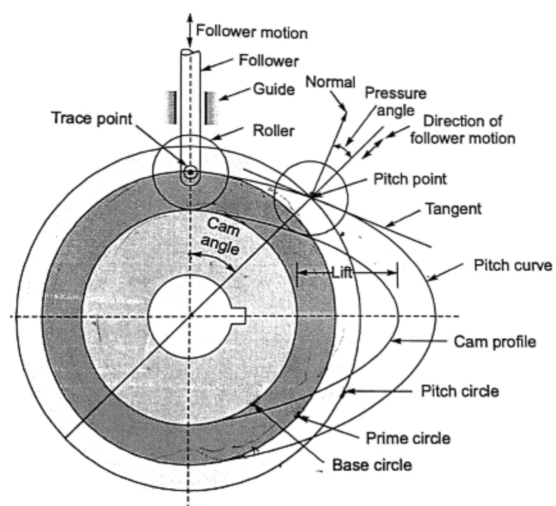
Radial follower– Here line of motion of follower is passing through the centre of rotation of CAM.

Offset follower– Here line of motion of follower is little bit offset from the centre of rotation of CAM.

■ **Purpose of giving offset to follower–**

By offset, pressure angle (ϕ) decreases.

- Less force required to lift the follower
- As result of that, wear side thrust is also little bit reduced.



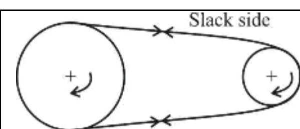
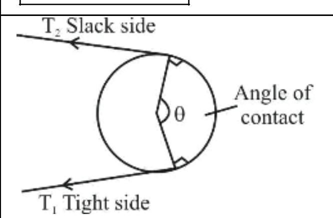
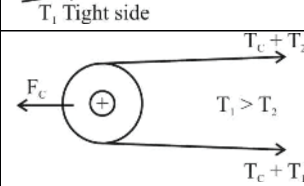
Base circle	It is smallest circle tangent to the cam profile drawn from the centre of rotation of radial cam
Trace point	It is a reference point on the follower to trace cam profile. Trace pt. = Centre of roller (of a roller follower) Trace pt. = Point of contact (in rest follower)
Pressure angle	It is the angle between the normal to the pitch curve at a point and the direction of the follower motion. • A high value of 'ϕ' is not desired as it might jam the follower in the bearing.
Pitch point	It is point on pitch curve at which the pressure angle is maximum
Pitch circle	Circle passing through the pitch point & concentric to base circle
Prime circle	The smallest circle drawn tangent to the pitch curve
Dwell	It is the zero displacement of follower

Note–

1. The size of the cam is specified by the diameter of the base circle, therefore its radius is also known as minimum radius of the cam.
2. Pitch point can be more than one depending upon, on how many points pressure angle is maximum.

		Uniform velocity	**Uniform acceleration**	**SHM**	**Cycloidal**
V_{max}	$\frac{\omega S}{\theta}$	1	$2\left(\frac{\omega S}{\theta}\right)$	$\frac{\pi}{2}\left(\frac{\omega S}{\theta}\right)$	$2\left(\frac{\omega S}{\theta}\right)$
a_{max}	$\frac{\omega^2 S}{\theta^2}$	0	$4\left(\frac{\omega^2 S}{\theta^2}\right)$	$\frac{\pi^2}{2}\left(\frac{\omega^2 S}{\theta^2}\right)$	$2\pi\left(\frac{\omega^2 S}{\theta^2}\right)$
Jerk	$\frac{\omega^3 S}{\theta^3}$	0	$0\,\frac{\omega^3 S}{\theta^3}$	$\frac{\pi^3}{2}\left(\frac{\omega^3 S}{\theta^3}\right)$	$4\pi^2\left(\frac{\omega^3 S}{\theta^3}\right)$
		• Wrost follower • Use for very-very slow speed	• It is next to wrost follower • Used for very slow speed application	• It is a better follower • Used for medium speed	• It is the best follower • Used for high speed application

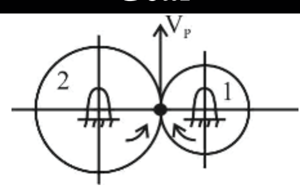
Belt drive

Velocity ratio of belt drive (VR)	$= \frac{\text{Velocity of driven}}{\text{Velocity of driver}}$ $\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$
If belt thickness is (t), VR	$= \boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}}$
Peripheral velocity	$V_1 = \frac{\pi d_1 N_1}{60}\ m/s$ $V_2 = \frac{\pi d_2 N_2}{60}\ m/s$
Total percentage of slip	**(S) = $S_1 + S_2$** % S_1 = Slip between driver & belt % S_2 = Slip between driven & belt $\boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)}$
Power transmitted by Belt	 $\boxed{T_1 > T_2}$ $\boxed{P = (T_1 - T_2)V}$ watt
Ratio of driving tension for flat belt drive	 $\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$
Centrifugal tension	 Max. Tension in the belt $\boxed{T_{max} = T_1 + T_C}$ Cond^n for max. power transmission

	$\rightarrow T_{max} = 3T_C$ $T_C = \frac{T_{max}}{3}$
Velocity of belt for max power	$V = \sqrt{\frac{T_{max}}{3m}}$ $(T_{max} = T_1 + T_C)$ $T_1 = \frac{2}{3}T_{max}$
Initial tension in belt	$T_{initial} = \frac{T_1 + T_2}{2}$ If T_C is given– $T_{initial} = \frac{T_1 + T_2 + 2T_C}{2}$
Creep in belt drive	Differential elongation of belt drive due to difference in tension on two sides of the pulley– $\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$

Note–Included angle in V-belt drive = 30^o to 40^o

Gear



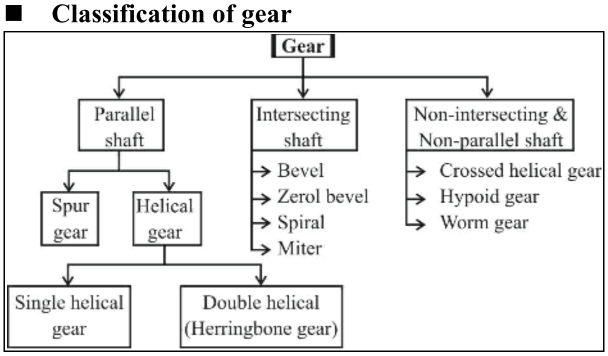
Point P can be assumed on gear 2 (or) gear 1–

$V_p = \omega_2 r_2 = \omega_1 r_1$

$$\frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

- Gear transmits motion by pure rolling at pitch point and partial sliding.

■ **Classification of gear**



■ **Classification of gears–**

(A) Parallel shaft axes–

1. Spur gears	• Straight teeth parallel to the axes of gear. • High impact stresses & excessive noise at high speed
2. Spur rack & pinion	• It converts rotary motion into translatory motion (or) vice-versa • It is made of infinite dia. so that the pitch surface is plane (gear with ∞ radius i.e. rack).
3. Helical gears (or) helical spur gears	• The teeth are inclined to the axis of rotation • They can be used at higher velocity & have greater load carrying capacity. **Draw back** Problem of axial thrust.
4. Double helical (or) Herringbone gears	• It is equivalent to a pair of helical gears. • No axial thrust is present. • Higher load carrying capacity.

(B) Intersecting shaft–

Straight bevel gears	Teeth are straight, radial to the point of inter-section of the shaft axis and vary in cross-section throughout their length.
Mitre gears	Gear of the same size and connecting two shafts at right angle to each other. $(VR)_{mitre\,gear} = 1$
Spiral bevel gears (or) helical bevel gears	• There is gradual load application and low impact stresses. • There exists an axial thrust • Used for the drive to the differential of an automobile
Zerol bevel gear	Spiral bevel gear with curved teeth but with a zero degree spiral angle.

■ **Axes are neither parallel nor intersecting**

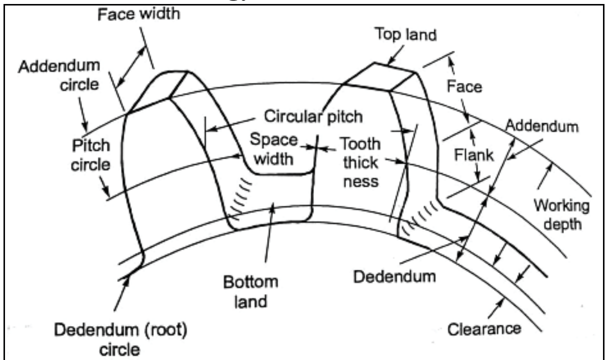
Skew shaft	In case of skew shafts a uniform rotary motion is not possible by pure rolling contact.
(a) Crossed helical gears (or) Spiral gears	• It is limited to light loads. • These gears are used to drive feed mechanism on machine tools, camshafts and oil pumps in I.C. engine.
(b) Worm gears	• Velocity ratio is very high (50:1 to 100:1) (Very large speed reduction ratio).

☛ Sliding velocity of worm gear is higher as compared to other types of gear.

■ **Classification of gear according to peripheral velocity of gear–**

Low velocity gear : 0–3 m/s Medium velocity gear : 3–5 m/s High velocity gear : > 15m/s

■ **Gear terminology**



Pitch circle	It is an imaginary circle drawn in such a way that a pure rolling motion on this circle gives the motion which is exactly similar to the gear motion.
Pressure angle (ϕ)	It is the angle between the pressure line and the common tangent to the pitch circles.
Module (m)	$m = \frac{D}{T} = \frac{\text{Pitch circle diameter (mm)}}{\text{No. of teeth}}$
Circular pitch	It is a distance along a pitch circle from one point on a tooth to the corresponding point on the next tooth. $P_c = \frac{\pi D}{T} = \pi m$
Diametrical pitch	$P_d = \frac{T}{D} = \frac{1}{m}$ $P_c \times P_d = \pi$
Tooth thickness	It is the thickness of tooth measured along pitch circle
Tooth space	It is space between the consecutive teeth measured along the pitch circle
Backlash	Difference between space width and tooth thickness along the pitch circle.
Addendum	It is the radial height of the tooth above the pitch circle. Its standard value is one module. (i.e. 1 A = 1 m)
Dedendum	It is the radial depth of the tooth below the pitch circle. Its standard value is 1.157 module. (i.e. De = 1.157 m)
Clearance	Its standard value is 0.157 m

Face	The surface between the pitch circle and top land
Contact ratio	It shows the average number of teeth in contact during meshing $CR = \frac{\text{Arc of contact}}{\text{Circular pitch}}$ **Note–** For continuous motion transmission contact ratio must be greater than unity (1). (Generally, CR = 1.6)
Full depth of teeth	It is the total radial depth of the tooth space. Full depth = Addendum + dedendum
Working depth of teeth	Working depth = sum of the addendums of the two gears
Gear ratio	$G = \frac{T}{t} > 1 \begin{cases} T \to \text{No. of teeth on the gear} \\ t \to \text{No. of teeth on the pinion} \end{cases}$
Velocity ratio	$V_R = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{t}{T}$ Velocity ratio can be less than one (or) greater than one but G is always greater than 1.

☛ Module is always same for two mating gears.

☛ Velocity ratio $\propto \frac{1}{(\text{Gear train value})}$

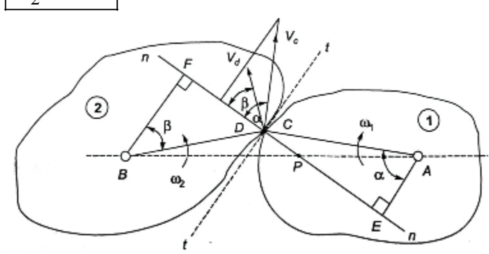
■ **Law of gearing–**

The law of gearing states–

➢ Gear tooth profiles must fulfilled a constant angular velocity ratio between two gears.

➢ For constant angular velocity ratio of the two gear, the common normal at the point of contact of the two mating teeth must pass through the pitch point

$$\frac{\omega_1}{\omega_2} = \frac{BP}{AP}$$



➢ **Velocity of sliding–**

$(\omega_1 + \omega_2)PC$

(sum of angular velocities × distance between the pitch point and point of contact)

Where,

ω_1 = angular velocity of gear 1 (clockwise)

ω_2 = angular velocity of gear 2 (anticlockwise)

- At pitch point, PC = 0
  Sliding velocity = 0
  So, gear have sliding + rolling motion but at pitch point only rolling is there.
- ➢ Common forms of teeth that also satisfy law of gearing–
  → Cycloidal profile teeth
  → Involute profile teeth

Parameter	Cycloidal teeth	Involute teeth
Pressure angle (ϕ)	Varies at each point (Max - zero- max)	Constant at each point
Profile	Double curve profile (epicycloids and hypocycloid)	Single curve profile
Interference	Does not occur	May occur
Strength	More strong due to the wider base	Less strong
Wear	Less	More
Centre distance variation	Not allowed (Exact centre distance is required)	Smaller variation is allow
Application	Suitable for motion transmission (light duty)	Suitable for motion as well as power transssmission

■ **Involute profile–**

- Involute is a curve generated by point on a tangent which rolls on a circle without slipping.
- A normal on any point of involute profile will be tangent to the base circle.
- Tooth profile is always generated from base circle.
- If center distance changes, VR remains the same.

➢ **Base circle =** Pitch circle diameter × cos ϕ

➢ **Path of contact (POC) =** Path of approach + path of recess

➢ **Arc of contact (AOC) =** $\frac{\text{Path of contact}}{\cos\phi}$

➢ **No. of pairs of teeth in contact (or)**

Contact ratio = $\frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\text{AOC}}{\pi m}$

■ **Interference in involute gears**

- Mating of two involute and non-involute profiles results in interference.
- Minimum teeth required to prevent interference

$$t_{min} = \frac{2a_p}{\sqrt{1+G(G+2)\sin^2\phi}-1}$$

$$T_{min} = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$G = \frac{T}{t}$$

Where, a_p, a_w = fractional addendum (addendum of pinion & wheel for 1 mm module)

t_{min} to avoid interference between gear & pinion	G = 1 and 1 m addendum; a_p = 1 $t_{min} = \frac{2}{\sqrt{1+3\sin^2\phi}-1}$ ϕ: 14.5 → t_{min} 23; 20° → 13; 22.5 → 11
Interference between rack & pinion	$t_{min} = \frac{2a_r}{\sin^2\phi}$
When a_r = 1	ϕ: 14.5 → t_{min} 32; 20° → 18; 22.5° → 14

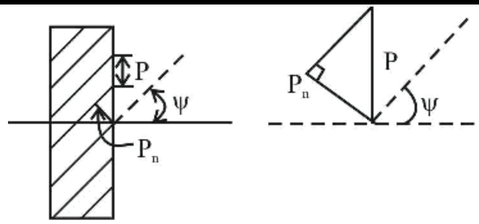
ϕ	t_{min}
14.5	23
20°	13
22.5	11

ϕ	t_{min}
14.5	32
20°	18
22.5°	14

■ **Methods to avoid interference**

Methods	Remarks
Undercutting of gear	Removal of material of non-involute portion below base circle. **Limitation :** Strength of tooth ↓ at the base, so used only in low power transmission.
Increasing 'ϕ' by decrease base circle radius	Non-involute portion is reduced, stronger tooth, contact ratio (↓) interference ↓ **Limitation :** ϕ_{max} = 20° to 25°
Stubbing the teeth	ϕ- No change, stronger tooth, less cost, addendum & addendum radius of wheel ↓, path of contact & contact ratio ↓
Increasing the no. of teeth (best method)	ϕ → No change Addendum & addendum radius ↓ Circular path ↓ Contact ratio ↑ Interference ↓

Helical & spiral gear



Where,
ψ = Helix angle
P = Circular pitch
P_n = Normal pitch

- For two mating gears–

$$\text{Centre distance} = \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]$$

Efficiency–

$$\eta_{max} = \frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}$$

Worm & worm gear

For large speed reduction

Lead (L)– The distance by which the helix advances along the axis of gear for one turn around.

$$L = n \times P_a \qquad \psi + \lambda = 90^o$$

Lead angle (λ)–

- It is the angle at which the teeth are inclined to the normal to the axis of rotation.
- As the shaft of worm (1) and worm gear (2) are at 90^o

$$\psi_1 + \psi_2 = 90^o$$

$$90 - \lambda_1 + \psi_2 = 90^o$$

$$\lambda_1 = \psi_2 \qquad \eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

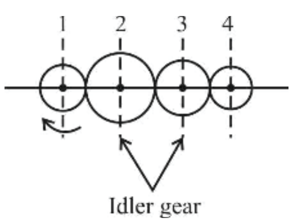
Gear train

Requirement of gear trains

- Large center distance is there
- Very large/very less velocity ratio are required within a small space.
- Multiple velocity ratio are required.

■ **Types of gear trains**

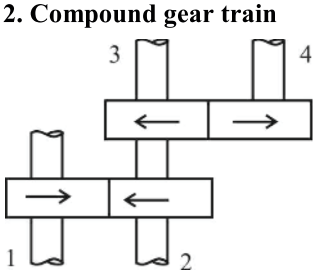
1. Simple gear trains



(Idler gear have no effect on the speed ratio)

- Same module
- A pair of mated external gear always move in opposite direction
- Bevel gear worm & worm wheel are simple gear train.
- $\text{Velocity ratio}(VR) = \frac{N_{driving}}{N_{driven}}$
- $\text{Train value}(TV) = \frac{\text{No. of teeth on driving gear}}{\text{No. of teeth on driven gear}}$
- $\text{Speed ratio (or) Velocity ratio}(SR) = \frac{1}{\text{Train value}}$

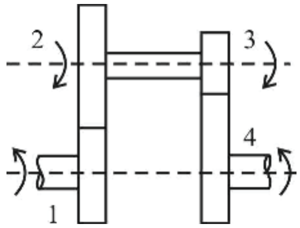
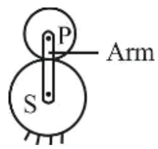
2. Compound gear train



- At least one of the intermediate shaft have more than one gear in use.

$$\frac{N_4}{N_1} = \frac{\text{Product of no. of teeth on driving gear}}{\text{Product of no. of teeth on driven gear}}$$

$$T.V. = \frac{N_4}{N_1} = \frac{T_1 \times T_3}{T_2 \times T_4}$$

3. Reverted gear train 	• The axis of the first and last wheel of a compound gear concide. • Used in clock & in simple lathe $\text{Train Value}(\text{T.V}) = \frac{N_4}{N_1} = \frac{T_1 \times T_3}{T_2 \times T_4}$ $r_1 + r_2 = r_3 + r_4$ If module of all the gears are same– $T_1 + T_2 = T_3 + T_4$
4. Planetary (or) Epicyclic gear train 	• Arm fixed ⇒ Simple gear train • Sun gear fixed ⇒ Planetary gear train • In general, DOF = 2 • Large speed reduction is possible with this gear **Application–** In transmission, computing devices

Sun & Planet gear

- When an annular wheel is added to the epicyclic gear train, then referred as sun & planet gear.
- Used in pre-selective gear box.
- Input is given to either S (or A) or arm. Planet can never be input link.
- More than one planets are there to balance and load distribution.

Differential gear–

It permits the two wheels to rotate at the same speed when driving straight while allowing the wheels to rotate at different speeds when taking a turn.

- An epicyclic gear having two degrees of freedom has been utilized in the differential gear of an automobile.

Flywheel

- Flywheel reduce fluctuation of speed due to cyclic variation of torque.
- It does not control the speed variations caused by the varying load.
- It does not maintain a constant speed also.
- Flywheel controls $\frac{\delta N}{\delta t}$ whereas governor controls δN.

Turning moment diagram–

It is the graphical representation of the turning moment (or) crank effort with crank angle (θ).

Work done per cycle

$$\text{Work done per cycle} = T_{mean} \times \theta$$

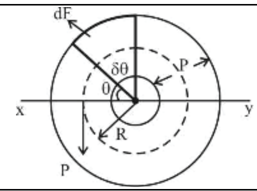
Where,

T_{mean} = mean torque

θ = angle turned in one cycle

$$\theta = 2\pi(\text{for 2 – stroke engine}) \\ = 4\pi(\text{for 4 – stroke engine})$$

Fluctuation of speed (C_S)	$C_S = \frac{N_{max} - N_{min}}{N_{mean}}$
Coefficient of steadiness	$m = \frac{1}{C_S} = \frac{N_{mean}}{N_{max} - N_{min}}$
Maximum fluctuation of energy	ΔE = maximum energy – minimum energy $\Delta E = E_{max} - E_{min}, \quad \Delta E = \frac{1}{2} I(\omega_{max}^2 - \omega_{min}^2)$ $\Delta E = I\omega_{mean}^2 C_S, \quad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$
Coefficient of fluctuation of energy (C_E)	$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$ $C_E = \frac{\Delta E}{\text{Workdone / cycle}}$
Dimension of the flywheel rim	$V = \sqrt{\frac{\sigma}{\rho}}$

	• mass = ρ × V = ρ × circumference × cross section area $m = \rho \times \pi DA$ 

Note–

(i) Flywheel for medium speed → Flywheel with spokes

(ii) Flywheel for high speed → Disc shaped flywheel

(iii) Best flywheel → Rim type flywheel

$I = mk^2$, k → radius of gyration

k = R (for rim type)

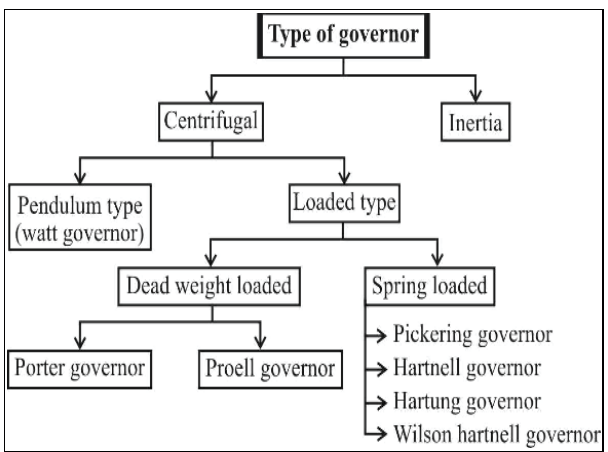
$k = \frac{R}{\sqrt{2}}$ (for disc shape)

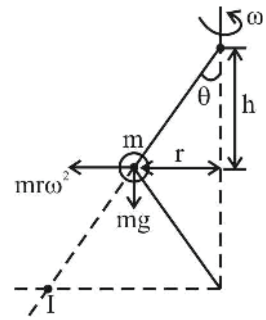
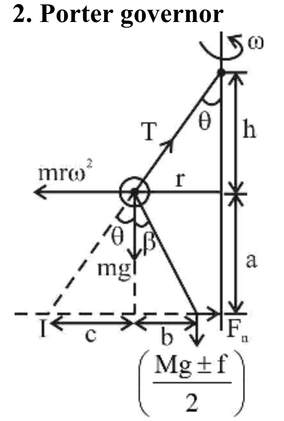
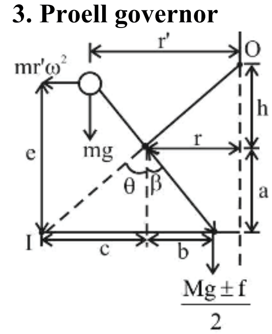
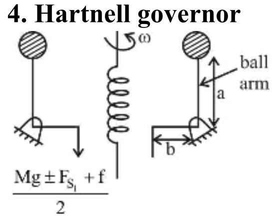
- The rim of a flywheel is subjected to **direct tensile & bending stresses**.
- The spoke of a flywheel is subjected to **direct tensile stress**.

■ Governor–

The function of a governor is to maintain the speed of an engine within specified limits whenever there is a variation of load (i.e. δN).

Difference between flywheel & governor	
Flywheel	**Governor**
Limits cyclic fluctuation due to change in torque during each cycle	Control the speed variation due to loads over a no. of revolution
No influence on mean speed	Controls mean speed by keeping it within specified limits
Has large inertia	Has less inertia
Continuous operation	Intermitted operations
Not used in all type engine	Used in all type of engine as it adjusts the fuel supply as per demand



1. Watt Governor 	• Simplest form of centrifugal governor with a ball or pendulum with links. • It is attached to a sleeve of negligible mass. $h = \frac{g}{\omega^2} = \frac{895}{N^2}$ • Not suitable for high speed. • This governor failed after 60 rpm.
2. Porter governor 	• If the sleeve of watt governor is loaded with a heavy mass. $h = \frac{2mg + (Mg \pm f)(1+k)}{2m\omega^2}$ Where, $k = \frac{\tan\beta}{\tan\theta}$ If k = 1, f = 0 $h = \left(\frac{m+M}{m}\right)\frac{895}{\omega^2}$
3. Proell governor 	$h = \frac{a}{e}\left(\frac{m+M}{m}\right)\frac{895}{N^2}$ $N_{proell} < N_{porter}$ For same N $m_{proell} < m_{porter}$
4. Hartnell governor 	• Sleeve displacement $x = \left(\frac{b}{a}\right)(r_1 - r_2)$ • Spring stiffness $S = 2\left(\frac{a}{b}\right)^2\left[\frac{F_{c_1} - F_{c_2}}{r_1 - r_2}\right]$
5. Pickering governor	It is used in gramophone.

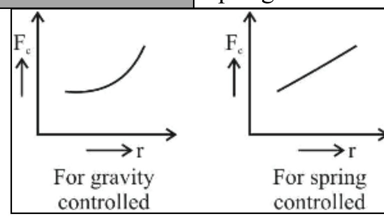
Properties of governor

1. Sensitiveness of governor Where, N_1 = Minimum equilibrium speed corresponding to full load cond[n] N_2 = Maximum equilibrium speed corresponding to no load cond[n]	• When it readily responds to small change of speed. i.e., $\text{Sensitivity} = \frac{N}{N_2 - N_1}$ But when governor is fitted to the engine– $\text{Sensitivity} = \frac{\text{Range of speed}}{\text{Mean speed}} = \frac{N_2 - N_1}{N_{mean}}$
2. Hunting	If a governor is too sensitive
3. Isochronism	• When the equilibrium speed is constant for all radii of rotation, i.e. range is zero. • Isochronism is a stage of ∞ sensitivity.
4. Effort of governor	• Mean force acting on the sleeve to raise (or) lower it for a given change of speed. • At constant speed, the governor is in equilibrium and the resultant force acting on the sleeve is zero. • $\text{Effort} = \frac{1}{2} \times S \times h$ [For Hartnell]
5. Power of governor	• Work done at the sleeve for a given percentage change of speed. Power = Effort of governor × displacement
6. Coefficient of insensitiveness [coefficient of detention] (C.O.D.)	N_1 to N_2 = Range of equilibrium speed within which the sleeve displacement is zero. $\text{C.O.D.} = \frac{N_1 - N_2}{N}$ $N_{mean} = \frac{N_1 + N_2}{2}$ • For porter governor– $\text{C.O.D.} = \frac{f}{(m+M)g}$ • For watt governor ⇒ M = 0 $\text{C.O.D.} = \frac{f}{mg}$

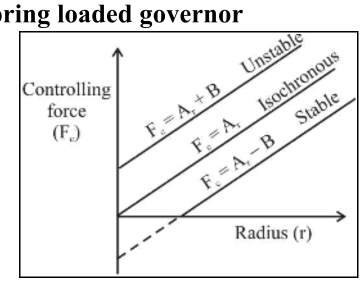
■ **Controlling force–**

- Controlling force is equal and opposite to the centrifugal force and acts radially inward.
- The graph between 'F_c' and 'r' is known as controlling force curve.
- It helps to find stability & sensitiveness & effect of friction.

Governor name	Controlling force Supplied by
Watt	Gravity of mass of ball
Porter & Proell	Gravity of mass of ball and dead weight of sleeve
Hartnell & Hartung	Gravity of ball masses and spring force

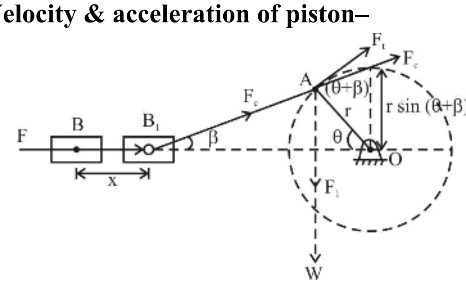


- **For spring loaded governor**



Dynamics force analysis

- **Velocity & acceleration of piston–**



- $\text{Obliquity ratio}(n) = \frac{L}{r}$
- $x = r\left[(1-\cos\theta) + \left(n - \sqrt{n^2 - \sin^2\theta}\right)\right]$

x = Displacement of piston from inner dead centre
L and r = lengths of connecting rod and crank respectively.

For connecting rod–

$x = r(1-\cos\theta)$ $\quad [\text{When}, n^2 >> 1]$

'n' is kept large in order to–

(i) Decrease secondary unbalance force

(ii) Piston excutes SHM

- Velocity of piston, $V = \frac{dx}{dt} = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$
- Acceleration of piston, $a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

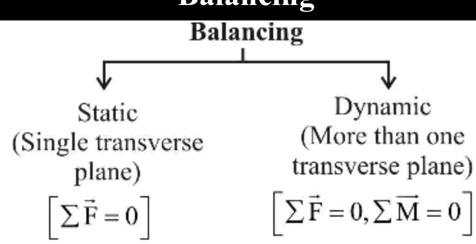
  (Along stroke length)

θ	a	Remarks
0^o (Inner dead centre)	$r\omega^2\left[1+\frac{1}{n}\right]$	Maximum
180^o (Outer dead centre)	$r\omega^2\left[\frac{1}{n}-1\right]$	Minimum

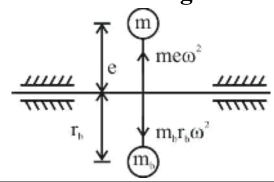
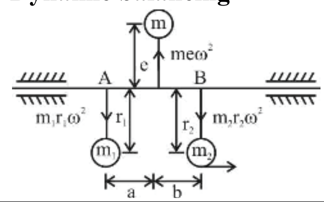
■

Angular velocity and angular acceleration of connecting rod	$\sin\beta = \frac{\sin\theta}{n}$ $\omega_c = \omega\frac{\cos\theta}{\sqrt{n^2-\sin^2\theta}}$ • If $n^2 >> 1$ $\boxed{\omega_c = \omega\frac{\cos\theta}{n}}$
Piston effort (effective driving force) Inertia force	$F_P = P_1A_1 - P_2A_2$ $F_b = ma$ $F_b = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$
Force along the connecting rod	$\boxed{F_c = \frac{F}{\cos\beta}}$
Force (or) thrust to cylinder wall	$F_n = F_c \sin\beta = F\tan\beta$
Radial thrust on crank shaft bearing	$F_r = F_c \times \cos(\theta + \beta)$
Crank effort (F_t)	$F_t = F_c \sin(\theta + \beta)$
Turning moment on crank shaft	$T = F_r \times r$ $= F.r\left[\sin\theta + \frac{\sin^2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$

Balancing



■ **Balancing of single revolving mass–**

Static balancing 	$\left[\sum\vec{F}=0\right]$ $me\omega^2 = m_br_b\omega^2$ $\boxed{me = m_br_b}$
Dynamic balancing 	$\left[\sum\vec{F}=0\right]$ $\left[me = m_1r_1 + m_2r_2\right]$ $\sum M_B = 0$ $\left[m_1r_1(a+b) = meb\right]$

■ **Dynamic balancing–**

- A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

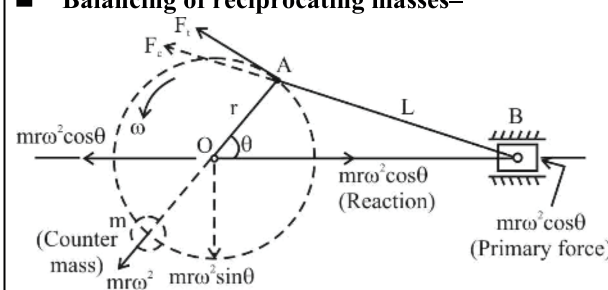
To balance force–

$$\sum m_ir_i + m_{c_1}r_{c_1} + m_{c_2}r_{c_2} = 0$$

To balance couple–

$$\boxed{\sum m_ir_i\ell_i + m_{c_1}r_{c_1}\ell_{c_1} + m_{c_2}r_{c_2}\ell_{c_2} = 0}$$

■ **Balancing of reciprocating masses–**



Force required to accelerate mass 'm'

$$\boxed{F = mr\omega^2\cos\theta + mr\omega^2\frac{\cos 2\theta}{n}}$$

Primary accelerating force	$mr\omega^2\cos\theta$
Secondary accelerating force	$mr\omega^2\frac{\cos 2\theta}{n}$ (Generally 'n' is very high So, secondary force can be neglected for lower speed engine)

■ **Partial balancing of primary forces–**

If 'c' is the fraction of the partial balance reciprocating mass then–

- Partial primary balanced force = $cmr\omega^2\cos\theta$
- Primary unbalanced force = $(1-c)\,mr\omega^2\cos\theta$
- Vertical component unbalanced force = $cmr\omega^2\sin\theta$
- Resultant unbalanced force–

$$= \sqrt{\left[(1-c)mr\omega^2\cos\theta\right]^2 + \left[cmr\omega^2\sin\theta\right]^2}$$