SSC CGL New Pattern

MATHEMATICS HACKBOOK

A CRISP & CONCRETE E-BOOK FOR SSC CGL MATHEMETICS.

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About this Book

Do you find Mathematics difficult?

If your answer is No, then either you're smarter than Albert Einstein or you're talking about something else maybe.

If your answer is Yes, then you must be happy because Albert Einstein once said, “Do not worry about your difficulties in Mathematics. I can assure you mine are still greater."

So the problem is not with the difficulty, I guess.

Also, we must understand that even if this is difficult nothing in the world is worth having or worth doing unless it means effort, pain or difficulty.

Richard Courant, a German American mathematician, best known for his book What is Mathematics?, believed that Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality.

This means that Mathematics requires only the basic virtues of logic, analysis and confidence to be understood and applied.

And if you differ on this then you must understand that without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers. And you do not implement the formulas everywhere I guess.

Enough said on the difficulty and complexities part though. We are there to help as always with our new innovation. (SUPERHACK), which is this beautifully designed eBook especially for aspirants preparing for SSC CGL Mathematics.

The aim of this book is to explain carefully and technically the sort of mathematics, we dealt with at school, presented to you exactly in the same way as asked in SSC CGL. The most fundamental concepts are generic, and readers of this book will emerge with a clearer understanding of paradoxical-sounding concepts and questions such as mixtures and allegations, mensuration and the tricky trigonometric and geometrical concepts. The first few chapters are about general aspects of mathematical thought explained briefly and categorically so that all the basic concepts and formulae are cleared. These are followed by discussions of more specific topics, examples and exercises which will make this section a scoring one for you and the book closes here leaving you with the best understanding and approach required to excel in SSC CGL.

Since, the complete book has been designed with the exact approach required while taking the exam, every section of this book has concepts, questions and tricks covered separately and explained adequately along with examples and references.

Mathematics is all about practice and we have ensured that you get the BEST practice space in the form of this compilation.

SSC CGL Mission Tier 1 & Tier 2 cannot be attained without this section because it is the section that makes a difference in the end due to its complexity over English. And we say this because we have been following the trends from last few years.

Go ahead and explore.

All the Best!

- **SSCHacks Team**

TABLE OF CONTENTS

Since we are aware of the topics we can arrange these in a tabular format so that we can make the most of the leftover time. If a plan is simple and efficient, it becomes a good plan when implemented. Hence, we will provide you this book with a plan. It's upon you to make it a good plan.

According to the previous five years' papers and the latest trends, the topics have been picked keeping in mind their rate of occurrence and difficulty level in SSC CGL. Take a look at the table below because not only does it cater to the preceding statement but also it is the plan that you need to follow for SSC CGL.

TOPICS	**NO. OF QUESTIONS**	**DIFFICULTY**	**TIME TAKEN (in min)**
Percentage	4-6	Easy	5
Ratio and Proportion	6-10	Easy	6
Squares, Square Roots, Averages	8-10	Easy	8
Profit and Loss, Discount	6-8	Medium	8
Simple and Compound Interest	5-8	Medium	5
Mixture and Alligation.	10-12	Difficult	10
Time and Work	4-6	Difficult	6
Speed, Time and Distance	12-15	Difficult	10
Basic Algebraic identities	10-12	Medium	10
Triangles	8-10	Medium	8
Circles	5-8	Medium	6

Quadrilaterals and Polygons	8-10	Medium	8
Mensuration	6-8	Difficult	10
Trigonometry	12-14	Difficult	10
Data Interpretation	10-12	Easy	10
TOTAL	**100**		**120 MINUTES**

These topics have been selected by our team after a detailed research and although we believe that difficulty levels and time taken are subjective factors we took these into consideration just in case you plan to attempt the complete paper after the high cut-offs in Tier 1.

Analyse your weak and strong sections here itself and then begin with this book because once it ends you will definitely have a different say about the difficulty.

Now, we begin with the first topic.

Percentage

In mathematics, a percentage is a number or ratio expressed as a fraction of 100. It is often denoted using the percent sign, "%". Percentage is a pure number.

This is a topic which everyone finds easy and is the easiest scoring section under SSC. On the basis of questions it can be broadly classified into 3 categories:

Type 1: Percentage Increase/Decrease:

If the price of a commodity increases by r%, then the reduction in consumption so as not to increase the expenditure is: [r/ (100 + r)] x 100%

If the price of a commodity decreases by r%, then the increase in consumption so as not to decrease the expenditure is: [r/ (100 – r)] x 100%

Type 2: Results on Population:

Let the population of a town be k now and suppose it increases at the rate of r % per annum, then the:

Population after n years = k (1 + r/100) n

Population n years ago =k / (1 + r/100) n

Type 3: Results on Depreciation:

Let the present value of a machine be PV. Suppose it depreciates at the rate of D% per annum, then the:

Value of the machine after n years = PV (1 – D/100) n

Value of the machine n years ago = PV / [(1 – D/100)] n

If A is R% more than B, then B is less than A by= [D/ (100 + D)] x 100%

If A is R% less than B, then B is more than A by= [D/ (100 – D)] x 100%

Note: For two successive changes of A% and B%,
Net Change = { A + B + (AB/100) } %

Table of commonly used values shown in Percent, Decimal and Fraction		
Percent (%)	**Decimal (.)**	**Fraction (a/b)**
1%	0.01	1/100
5%	0.05	1/20
10%	0.1	1/10
12.5%	0.125	1/8
20%	0.2	1/5
25%	0.25	1/4
33.33%	0.333	1/3
50%	0.5	1/2
75%	0.75	3/4
80%	0.8	4/5
90%	0.9	9/10
99%	0.99	99/100
100%	1	1/1
125%	1.25	5/4
150%	1.50	3/2
200%	2	1/1

This table would come handy for calculations in other questions too and saves a lot of time while taking the test.

Example Questions:

If we have to find 2/5 in %?

In the table we know that 1/5=20% then, if we multiply both by 2 then we have 1/5x2 = 2x20%
Answer is 2/5 = 40%
So, you only to know basic fraction and percentage that given above.

In the inverse,

Ex. If we have to find out 1/15 fraction in percentage%?

In the table we have 1/5 = 20%, then both side multiple by 1/3 we get 1/15 = 6.66%

SOME BASIC QUESTION THAT IS ALWAYS ASKED IN THE EXAMS

TYPE 1.

If there is increase of X% and subsequently X% decrease then there is always loss / decrease in the condition.

Ex. If Rohan's salary is increase by 50% and subsequently decrease by 50%. How much percentage loss?

ans. By trick ---- 50x50/100= 25% decrease/loss

TYPE 2.

If A is P% more than B. then B is less than by A with P/(100+P)x100

Ex. If Radha earning is 25% more than Sita. Then Sita earning is how many percentage less than by Radha?

Ans. By trick ---- 25/ (100+25) x100=20%

TYPE 3.

If A is P% less than B. then B is more than by A with P/ (100-P)x100

Ex. If Golu age is 20% less than Gita than Gita age is how many percentage more than Golu?

Ans. By trick ---- 20/ (100-20) x100=25%

TYPE 4.

If there is p% increase in price. How much % decrease his consumption so that his expenditure on it does not change.

Formula P / (100+P) x 100

Ex. If petrol price increase by 25%. How much percentage a person reduced his consumption so that his expenditure on it does not increase?

ans. 25/ (100+25) x 100=20%

TYPE 5.

If there is p% decrease in price. How much % increase his consumption so that his expenditure on it does not change.

Formula P/ (100-P) x 100

Ex. If there is 30% decrease in eggs price. How much a person increased his consumption so that his expenditure on it does not increase?

ans. 30/ (100-30) x 100=42.85

TYPE 6.

Ex. If Suman need 36% minimum mark to passed an exam but she score 24% mark and fail by 9 marks. What is the total mark?

Ans. *Difference in % = difference in mark*

In that 36%-24%=12%, that is 12% = 9marks

So, total mark = 9/12x100= 75

Some more questions for practice:

1. Three candidates contested an election and received 1136, 7636 and 11628 votes respectively. What percentage of the total votes did the winning candidate got?

A. 55%

B. 56%

C. 57%

D. 58%

Answer: Option **C**

Explanation: Total number of votes polled = (1136 + 7636 + 11628) = 20400
So, Required percentage = 11628/20400 * 100 = 57%

2. Total number of boys and girls in a school is 150. If the number of boys is x, then girls become x% of the total number of students. The number of boys are?

A. 50

B. 60

C. 70

D. 80

Answer: Option **B**

Explanation: Clearly, x% of 150 = 150 - x [as x is number of boys]

=>x + x/100*150 = 150

=>5x/2 = 150

=>x = 60

3. The ratio 5:20 expressed as percent equals to

A. 50 %

B. 125 %

C. 25 %

D. None of above

Answer: Option **C**

Explanation: Actually it means 5 is what percent of 20, which can be calculated as, (5/20)*100 = 5 * 5 = 25

4. What will be the fraction of 4%

A. 1/20

B. 1/50

C. 1/75

D. 1/25

Answer: Option **D**

Explanation: 4*1/100 = 1/25.

Friends, I know it is quite simple, but trust me while solving percentage questions in hurry we use to do these types of mistake only. So I recommend you to have a bit practise of this.

5. Half of 1 percent written as decimal is

A. 5

B. 0.5

C. 0.05

D. 0.005

Answer: Option **D**

Explanation: It will be 1/2(1%) = 1/2(1/100) = 1/200 = 0.005

6. 88% of 370 + 24% of 210 - x = 118

A. 150

B. 250

C. 158

D. 258

Answer: Option **D**

7. In expressing a length of 81.472 km as nearly as possible with the three significant digits, find the percentage error

A. 0.35%

B. 0.34%

C. 0.034%

D. 0.035%

Answer: Option **C**

Explanation: Error = (81.5 - 81.472) = 0.028
required percentage = 0.028 / 81.472×100=0.034

8. A batsman scored 120 runs which included 3 boundaries and 8 sixes. What percent of his total score did he make by running between the wickets?

A. 40%

B. 50%

C. 60%

D. 70%

Answer: Option **B**

Explanation: Number of runs made by running = 110 - (3 x 4 + 8 x 6)
= 120 - (60)
= 60
Now, we need to calculate 60 is what percent of 120.
=> 60/120 * 100 = 50 %

9. One fourth of one third of two fifth of a number is 15. What will be40% of that number?

A. 140

B. 150

C. 180

D. 200

Answer: Option **C**

Explanation: (1/4) * (1/3) * (2/5) * x = 15 then x = 15 * 30 = 450
40% of 450 = 180

10. If x% of y is 100 and y% of z is 200, then find the relation between x and z.

A. z = x

B. 2z = x

C. z = 2x

D. None of above

Answer: Option **C**

Explanation: It is, y% of z = 2(x% of y)
=> yz/100 = 2xy/100
=> z = 2x

11. In an election between two candidates, 75 % of the voters cast their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes.

A. 16800

B. 15800

C. 16700

D. 15700

Answer: Option **A**

Explanation: Let the total number of votes enrolled are x.
Number of votes cast = 75% of x
Valid votes = 98% of 75% of x
Now, as 9261 is the 75% of valid casted votes so,
75% of 98% of 75% of x = 9261 [important]
=> (75 × 98 × 75 × x) / (100×100×100) = 9261

=> x=16800

12. If 15% of 40 is greater than 25% of a number by 2, the number is

A. 14

B. 16

C. 18

D. 20

Answer: Option **B**

Explanation: 15/100 * 40 - 25/100 * x = 2 or x/4 = 4 so x = 16

13. 10% of inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants let the village. The population is then reduced to 4050. Find the original inhabitants

A. 5500

B. 6000

C. 6500

D. 7000

Answer: Option **B**

Explanation: Let the total number is x,
then,
(100-25) % of (100 - 10)% x = 4050
=> 75% of 90% of x = 4050
=> 75/100 * 90/100 * x = 4050
=> x = (4050*50)/27 = 6000

14. Due to an increase in 30% in the price of eggs, 3 eggs less are available for Rs. 7.80. Find the present rate of eggs per dozen.

A. Rs. 9.36

B. Rs. 10.36

C. Rs. 11.36

D. Rs. 12.36

Answer: Option **A**

Explanation: Let the original price per egg be Rs x
Then increased price will be,

$$\left(\frac{130}{100}x\right)$$
$$=> \frac{7.80}{x} - \frac{7.80}{\frac{130}{100}x} = 3$$
$$=> \frac{7.80}{x} - \frac{780}{130x} = 3$$
$$=> 390x = 234$$
$$=> x = 0.6$$
Actual price was Rs 0.6
Present price per dozen will be
$$Rs.\left(12 * \frac{130}{100} * 0.6\right)$$
$$= Rs. 9.36$$

15. Raman's salary was decreased by 50% and subsequently increased by 50%. How much percent does he loses?

A. 75

B. 65

C. 45

D. 25

Answer: Option **D**

Explanation: Let the original salary = Rs. 100

It will be 150% of (50% of 100)
= (150/100) * (50/100) * 100 = 75
So New salary is 75, It means his loss is 25%

16. In a hotel, 60% had vegetarian lunch while 30% had non-vegetarian lunch and 15% had both type of lunch. If 96 people were present, how many did not eat either type of lunch?

A. 27

B. 26

C. 25

D. 24

Answer: Option **D**

Explanation:

$$n(A) = \left(\frac{60}{100} * 96\right) = \frac{288}{5}$$
$$n(B) = \left(\frac{30}{100} * 96\right) = \frac{144}{5}$$
$$n(A \cap B) = \left(\frac{15}{100} * 96\right) = \frac{72}{5}$$

People who have either or both lunch

$$n(A \cup B) = \frac{288}{5} + \frac{144}{5} - \frac{72}{5} = \frac{360}{5} = 72$$

So People who do not have either lunch were = 96 -72 = 24

17. Rahul's Mathematics test had 75 problems, 10 arithmetic, 30 algebra, 35 geometry problems. Although he answered 70% of arithmetic, 40% of arithmetic and 60% of geometry problems correctly, still he got less than 60% problems right. How many more questions he would have to answer more to get passed

A. 5

B. 6

C. 7

D. 8

Answer: Option **A**

Explanation: Number of questions attempted correctly = (70% of 10 + 40% of 30 + 60% of 35) = 7 + 12 + 21 = 40. Questions to be answered correctly for 60% = 60% of total questions = 60 % of 75 = 45. He would have to answer 45 - 40 = 5.

18. Out of 450 students of a school, 325 play football, 175 play cricket and 50 neither play football nor cricket. How many students play both football and cricket?

A. 75

B. 100

C. 125

D. 150

Answer: Option **E**

Explanation: Students who play cricket, n (A) = 325
Students who play football, n (B) = 175
Total students who play either or both games,

$= n(A \cup B) = 450 - 50 = 400$

Required Number, $n(A \cap B)$

$= n(A) + n(B) - n(A \cup B) = 325 + 175 - 400 = 100$

19. A student multiplied a number by 3/5 instead of 5/3. What is the percentage error?

A. 64%

B. 65%

C. 66%

D. 67%

Answer: Option **A**

Explanation: Let the number be x, then,

$$\frac{5}{3} - \frac{3}{5} = \frac{16}{15}x$$

Error% =

$$(\frac{16}{15}x * \frac{3}{5} * 100)$$

20. What is 15 percent of 34?

A. 5.10

B. 4.10

C. 3.10

D. 2.10

Answer: Option **A**

Explanation: It will be 15% of 34
= (15/100) * 34 = 5.10

21. An inspector rejects 0.08% of the meters as defective.How many meters he examine to reject 2 meters?

A. 1200

B. 2400

C. 1400

D. 2500

Answer: Option **F**

Explanation: It means that 0.08% of x = 2

$$=> (\frac{8}{100 \times 100} \times x) = 2$$
$$=> x = \frac{2 \times 100 \times 100}{8}$$
$$=> x = 2500$$

22. A housewife saved Rs. 2.50 in buying an item on sale. If she spent Rs. 25 for the item, approximately how much percent she saved in the transaction?

A. 9%

B. 10%

C. 11%

D. 12%

Answer: Option **A**

Explanation: Actual Price = Rs.25 + Rs.2.50 = Rs.27.5;
Saving = Rs.2.5

$$\text{Percentage Saving} = \frac{2.5}{27.5} \times 100 = \frac{250}{27.5} = \frac{2500}{275} = \frac{100}{11} = 9\frac{1}{11}\% \approx 9\%$$

23. 2.09 can be expressed in terms of percentage as

A. 2.09%

B. 20.9%

C. 209%

D. 0.209%

Answer: Option **C**

Explanation: While calculation in terms of percentage we need to multiply by 100, so 2.09 * 100 = 209.

24. If sales tax is reduced from 5% to 4%, then what difference it will make if you purchase an item of Rs. 1000?

A. 10

B. 20

C. 30

D. 40

Answer: Option **A**

Explanation: Clue: Answer will be 5% of 1000 - 4% of 1000

25. Evaluate 28% of 450 + 45% of 280

A. 232

B. 242

C. 252

D. 262

Answer: Option **C**

Explanation:

(28/100) * 450 + (45/100) * 280
= 126 + 126 = 252

26. If number x is 10% less than another number y and y is 10% more than 125, then find out the value of x?

A. 123.55

B. 123.65

C. 123.75

D. 123.85

Answer: Option **C**

27. In an examination, 34% of the students failed in mathematics and 42% failed in English. If 20% of the students failed in both the subjects, then find the percentage of students who passed in both the subjects.

A. 40%

B. 42%

C. 44%

D. 46%

Answer: Option **C**

Explanation: Failed in mathematics, n(A) = 34
Failed in English, n(B) = 42

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 34 + 42 - 20 = 56$$
Failed in either or both subjects are 56
$$\text{Percentage passed} = (100 - 56)\%$$
$$= 44\%$$

28. How many litres of pure acid are there in 8 litres of a 20% solution

A. 1.5

B. 1.6

C. 1.7

D. 1.8

Answer: Option **B**

Explanation: Question of this type looks a bit typical, but it is too simple, as below. It will be
8 * 20/100 = 1.6

29. Two numbers are less than third number by 30% and 37% respectively. How much percent is the second number less than by the first?

A. 8%

B. 9%

C. 10%

D. 11%

Answer: Option **C**

Explanation: Let the third number is x.
then first number = (100-30)% of x
= 70% of x = 7x/10

Second number is (63x/100)
Difference = 7x/10 - 63x/100 = 7x/10
So required percentage is, difference is what percent of first number
=> (7x/100 * 10/7x * 100)% = 10%

30. 2 is what percent of 50?

A. 2%

B. 4%

C. 6%

D. 8%

Answer: Option **B**

Explanation: 2/50 * 100 = 1/25 * 100 = 4 %

31. Two students appeared at an examination. One of them secured 9 marks more than the other and his marks was 56% of the sum of their marks. The marks obtained by them are

A. 42,30

B. 42,31

C. 42,32

D. 42,33

Answer: Option **D**

Explanation: Let their marks be (x+9) and x.
Then, x+9 = 56/100(x + 9 +x)
=> 25(x+9)
=> 14 (2x + 9)
=> 3x = 99
=> x = 33.
So, their marks are 42 and 33

32. 1/2 is what percent of 1/3

A. 150%

B. 200%

C. 250%

D. 300%

Answer: Option **A**

Explanation: 1/2/1/3 * 100 = 1/2 * 3/1 * 100 = 150 %

Ratio and Proportion

Ratio and proportion though seem similar are two different but easy topics. Just the basic understanding of this section would enable you to score well in SSC CGL. And who would not attempt a section which helps you maximize your scores with minimal efforts?
We do not think anyone wants to miss out on this!

In mathematics, a ***ratio*** is a relationship between two numbers indicating how many times the first number contains the second meaning it can be a fraction as well as a whole number. The numbers compared in a ratio can be any quantities of a comparable kind, such as objects, persons, lengths, or spoonful's. A ratio is written as "a to b" or a: b, or sometimes expressed arithmetically as a quotient of the two. When the two quantities have the same units, as is often the case, their ratio is a dimensionless number.

In the ratio a: b, a is the antecedent and b is the consequent.

There are certain variations under ratio which may be helpful while taking the exam. Take a look:

- Inverse or reciprocal ratio: If a: b be the given ratio, then 1/a: 1/b or b :a .
- Ratio of equality: If the antecedent = the consequent, implying a=b.
- Ratio of greater inequality: If the antecedent >the consequent, implying a>b.
- Ratio of less inequality: If the antecedent < the consequent, implying a<b.

In mathematics, two variables are said to be in ***proportion*** if a change in one is always accompanied by a change in the other, and if the changes are always related by use of a constant multiplier.

In simple words, if two ratios a: b and c:d are equal, then a,b,c,d are said to be in proportion. It can be written as a: b:: c:d, a:b = c:d or a/b = c/d .

The end terms a and d, are called the extremes and the middle terms b and c are called the means. There are two basic tricks to follow while attempting this section. These are:

1) If the respective quantities are in proportion, the product of the extremes is always equal to the product of the means. (ad = bc)

2) Three quantities of the same kind are said to be in continued proportion when the ratio of the first to the second is equal to the ratio of the second the third. (a:b = b:c)

Implement these hacks in the questions that follow below:

Practice Questions:

Q1.Dry grapes contain 10% water while fresh grapes contain 80% water. What will be the weight (in kg) of fresh grapes when it weighs 50 kg as dry?

a. 208

b. 225

c. 235

d. 215

Solution - When dry, 1 kg of grape contains 10% or 0.1 kg of water,

So, 50 kg of dry grape will contain 5kg of water, that is, (50−5) = 45(50−5) = 45 kg of *grape mass* without water.

When fresh, *20% of total weight of the fresh grapes* = this *grape mass* of 45 kg, and rest 80% water.

So when fresh, *100% of total weight of fresh grapes* = 5×45=2255×45=225 kg, 20% is one-fifth, so 100% is five times.

Answer: *Option* ***b: 225.***

Q2.Full tank petrol for Vijay's mobike lasts for 20 days. How many days a full tank will last for him if he increases his daily consumption of petrol by 25%?

a. 16

b. 10

c. 12

d. 14

Solution: Let the original per day consumption of petrol by Vijay be C. So his total consumption in 20 days was, 20C which is the ***capacity of the tank.***

On increase of rate of consumption by 25%, the per day consumption now is, 1.25C. With this consumption rate if the full tank petrol lasts for D number of days, we have, 1.25CD=20C, Or, D = 20 / 1.25 =16.

Answer: Option **a: 16.**

Q3.Number of students in three classes A, B and C in a school were in a ratio 2:3:4 in last year. This year the numbers increased by certain percentages to change the ratio to 11:18:26. If the percentage increase in class C were 30% what is the percentage increase in class B?

a. 25%

b. 20%

c. 10%

d. 15%

Solution: As a first step we reintroduce the common HCF cancelled out between the first ratio terms as xx so that the ratio is transformed to ratio of actual numbers, 2x : 3x : 4x, where the three terms are now the actual numbers of students.

Similarly, after the increase of the individual numbers by certain percentages the changed ratio is expressed as, 11y : 18y : 26y.

The common factor **x** will no longer be the same as the new cancelled out HCF of the second ratio **y**. It won't have been otherwise possible to deduce the relation between **x** and **y** and hence the percent increases for all three numbers unless a key information were provided. The percent increase in number of students in class C is this key information.

Q4.Due to price rise of rice by 25% per kg Labanya could only purchase 20 kg less than what she had purchased at the expense of Rs.400 before the price rise. The increase in price of rice per kg (in Rs.) is,

a. 2

b. 1.5

c. 1

d. 3

Solution: In a purchase, the relation between price, purchased quantity and total cost of purchase is,

P × Q = C, where P is per unit cost or the price, Q is the quantity purchased and C is the total cost of purchase.

If the total cost remains unchanged but the other two change, we say, price is inversely proportional to the quantity purchased in same amount of total cost. We express this as,

$$Q = C\frac{1}{P}.$$

It means, if price increases purchased quantity decrease and vice versa with money to purchase remaining same. In this problem, as price has increased by 25% it has become 1.25P, if P were the original price.

So, $Q_2 = C\frac{1}{1.25P} = C\frac{4}{5P}$, whereas, before the price rise, the quantity that could be purchased was,

$$Q = C\frac{1}{P}.$$

Thus,

$$Q_2 = \frac{4}{5}Q = 0.8Q.$$

It means, if the total cost remains unchanged, for a price rise of 25%, purchasable quantity reduces by a fixed percent of 20% whatever be the total funds or cost of purchase. This is for clarifying the basic concept and also for use in problem solving. Thus, 20 kg less quantity purchased due to price rise is equivalent to 20% of original quantity, which would then be,

5×20=100kg, (100% is five times 20%, and represents the whole amount). So original price = 400/100 = 4 in Rs. per kg and 25% or one-fourth increase of it is, Rs 1 per kg.

Answer: Option **c: 1.**

Q5.Two vessels A and B of equal capacity of 5 litres each are filled with milk and water respectively. If 20% of water is taken out from vessel B, mixed thoroughly in vessel A and then again 20% of the mixture in vessel A is taken out and mixed back into vessel B thoroughly, the relation between the volume of water in milk vessel A and volume of milk in water vessel B is,

a. Water in AA = Milk in BB

b. Water in AA << Milk in BB

c. Water in AA >> Milk in BB

d. cannot be answered with the given information

Solution: This is apparently a liquid mixing problem of a special type that can be solved easily by calculating the requisite portions of milk and water in the exchange.

On looking at the problem closer we find that in the first operation 1 litre water is taken out of vessel B and mixed in 5 litre of vessel A, trying to make the volume of liquid 6 litres in a 5 litre vessel.

It means there will be an overflow in vessel A and the volumes of liquids will be uncertain. Thus the answer cannot be given with the information available.

Answer: Option **d: cannot be answered with the given information.**

Q6.As the price of oil has increased by 32%, a family has managed to decrease its oil consumption in such a way that effective increase in oil price was only 10% for them. If oil consumption were 10 litres before the price rise, what is the consumption (in litres) now?

a. 9

b. $8\frac{1}{3}$

c. $8\frac{3}{4}$

d. $8\frac{1}{2}$

Solution: Before the price rise the expense was, C1=10P, where 10 litres was the consumption and P per litre was the price.The family reduced consumption in such a way that effective price rise was 10%. This means, total cost increased by 10%, that is, it became 11P.So, after the price rise by 32% price P becoming 1.32P,

1.32P×Q2=11P,

Or, $Q_2 = \frac{11}{1.32} = \frac{100}{12} = 8\frac{1}{3}$ litres.

Answer: Option b : $8\frac{1}{3}$.

Q7.A box has 100 red balls, 50 blue balls and 50 black balls. 50% of the blue balls and 25% of the red balls were taken out of the box. What is the percentage of black balls in the box now?

a. 25%

b. 50%

c. $33^{1}/_{3}$%

d. 40%

Solution: 50% of blue balls, that is, one-half of 50, which is 25; 25% of red balls, that is one-fourth of 100, which is 25, a total of 50 balls are taken out from 200. So 50 black balls remains out of 150, that is one-third, a percentage equivalence of $33^{1}/_{3}$%.

Answer: c: **$33^{1}/_{3}$%**

Q8.If 4 litres of water is mixed with a 20 litres of mixture of water and alcohol of 20% alcohol strength, what will be the percentage strength of alcohol after mixing?

a. $33\frac{1}{3}\%$

b. $12\frac{1}{2}\%$

c. $16\frac{2}{3}\%$

d. 25%

Solution: 20% alcohol in 20 liters mixture equals 4 liters of alcohol (one-fifth of 20). When 4 liters of water is added total volume changes to 24 liters out of which 4 litres is alcohol which is a percent of,

$\frac{4}{24} \times 100 = 16\frac{2}{3}$ % alchohol.

Answer: Option c: $16\frac{2}{3}$%.

Q9.If the numerator of a fraction is increased by 20% and the denominator is decreased by 5% then the new fraction becomes 5/2. The original fraction was,

a. $\frac{24}{19}$
b. $\frac{3}{18}$
c. $\frac{48}{95}$
d. $\frac{95}{48}$

Solution: Assuming the original fraction as N/D, after the change in values,

$$\frac{1.2N}{0.95D} = \frac{5}{2},$$

Or, $\frac{N}{D} = \frac{5 \times 0.95}{2 \times 1.2} = \frac{95}{48}$.

Answer: Option d: $\frac{95}{48}$.

Q10.In a village each of the 60% of the families has a cow, each of the 30% of the families has a buffalo and each of the 15% of the families has both a cow and a buffalo. If there are 96 families in the village, how many families have neither a cow nor a buffalo?

a. 28
b. 24
c. 20
d. 26

Solution: Some of families in the group of 30% families each of which has a cow, also has a buffalo. In other words, *60% families each having a cow = families having only a cow + families having both a cow and a buffalo = families having only a cow + 15% of the total number of families*

Similarly for families each having a buffalo,
30% families each having a buffalo = families having only a buffalo + families having both a cow and a buffalo = families having only a buffalo + 15% of the total number of families.

Summing up these two equations adds the15% twice. Subtracting this 15% from the sum we get,

60% families each having a cow + 30% families each having a buffalo - 15% families having both a cow and a buffalo = families having only a cow + families having only a buffalo + families having both a cow and buffalo,

Or,

75% families = families having only a cow + families having only a buffalo + families having both a cow and a buffalo.

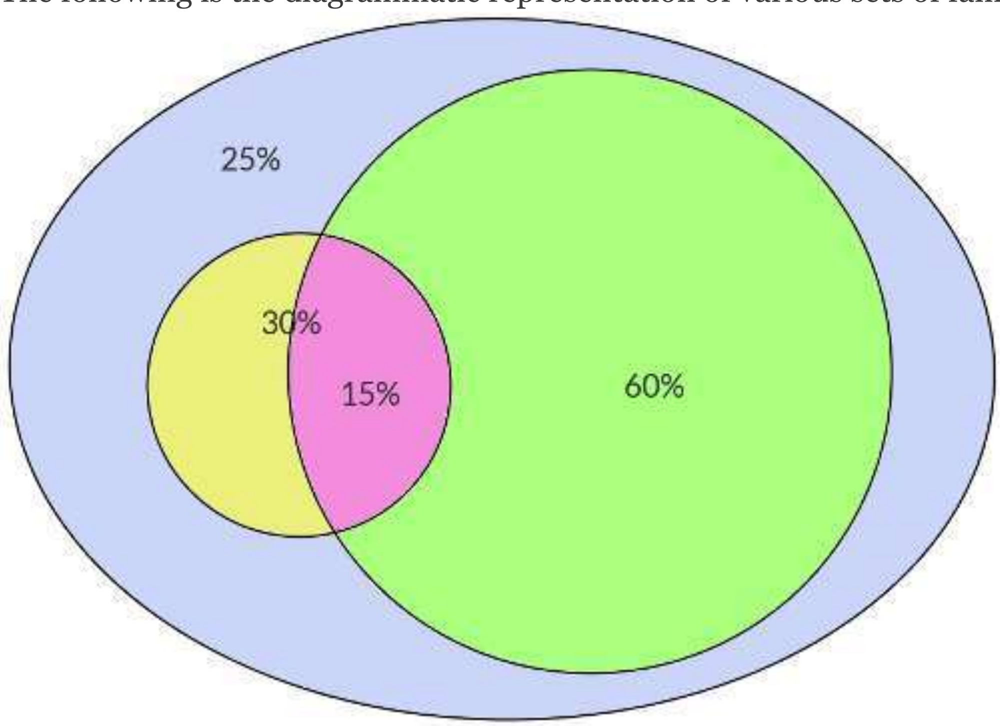
So the percentage of families having neither a cow nor a buffalo = 100% - 75% = 25%.

Total number of families being 96, 25% or one-fourth of which is 24.

Answer: Option **b: 24.**

Explanation of set union with Venn diagram

The following is the diagrammatic representation of various sets of families in the village.



The blue colored whole elliptical region represents the total number of families out of which 60% light green colored region represents families each having a cow and yellow region families each having a buffalo.

But it is not mentioned whether these families of 60% have a buffalo or not. Only when it is mentioned that 15% of families have both a cow and buffalo we immediately understand that this pink colored common region 15% must belong to both the green cow and yellow buffalo regions. In other words, this common region of 15% families is the set intersection of the two sets of cow families and buffalo families.
As this 15% families appear in both 60% and 30%, when we add 60% and 30%, this 15% is included twice. So, 60% + 30% - 15% = 75% gives us the unique families having a cow or a buffalo or both. This is the set union of two sets of cow families and buffalo families.
Thus the rest of the families out of the whole set of families, that is, 100% - 75% = 25% families have neither a cow nor a buffalo.

Q11. Ratio of incomes of A and B as well as B and C are 3 : 2. If one-third of A's income exceeds one-fourth of C's income by Rs.1000, then B's income in Rs. is,

a. 2500
b. 3000
c. 4000
d. 3500

Solution: Let's put down the last statement first, in the form of an expression,

$$\frac{1}{3}A - \frac{1}{4}C = 1000$$

As we need to explore the relation between A and C, let's transform the two ratios to equalize base B to its LCM value 6, giving, $A : B = 9 : 6$ and $B : C = 6 : 4$, so that, joining the two ratios, we have $A : B : C = 9 : 6 : 4$. Eliminating B, $A : C = 9 : 4$ giving, $4A = 9C$.

Now we take up the target expression again,

$\frac{1}{3}A - \frac{1}{4}C = 1000$, or, $4A - 3C = 12000$, Or, $6C = 12000$ or, $C = 2000$.

As $B : C = 3 : 2$, $B = 3000$.

Answer: Option **b: 3000.**

Q12. If the cost of pins reduces by Rs. 4 per dozen, 12 more pins can be purchased for Rs. 48. After reduction, the cost of pins per dozen is,

a. Rs. 12
b. Rs. 8
c. Rs. 20
d. Rs. 16

Solution: As all rates and purchases are by the dozens, let's assume dozen to be a new unit (12 may complicate things). 12 more pins purchased after reduction of price is also a dozen.

So we restate the problem as: due to a reduction in price by Rs. 4 per unit we could purchase 1 more unit with Rs. 48. Note that in both cases, before or after reduction of prices, the total amount being used for purchases has been a fixed amount of Rs. 48.
Whatever be the price per unit, as we are getting whole number of units, the price per unit must be a factor of 48. By this logic, choice value 20 is eliminated.
Testing further with the choice value of starting price as 12, we find it buys 4 units. The ending price would then be 8 buying 6 units - a violation of given condition (1 more unit not 2).
Starting price 16 and ending price 12 leads us to the desired answer quickly.
A sense that the starting price, a factor of 48, should be large enough so that a reduction of Rs. 4 results in only 1 more unit's worth, helps here. Smaller the value of starting price, say 8, the difference in number of purchased units would be larger (12 - 6 = 6).
Here we have used the choice values as a useful problem solving resource.
A more mathematical way to the solution is to assume xx as the items purchased now. So by the problem statement we get the expression of prices as,

$\frac{48}{x} - 4 = \frac{48}{x+1}$

Or, $\frac{12}{x} - 1 = \frac{12}{x+1}$,

Or, $(12 - x)(x + 1) = 12x$

Or, $x^2 + x - 12 = (x + 4)(x - 3) = 0$, giving positive root $x = 3$, old price Rs. 16 and new price Rs. 12.

Answer: Option **a : 12 .**

Q13. A dishonest grocer sells rice at a profit of 10%, but he uses weights 20% less than the marked. His total gain will be,

a. 30.5%
b. 37.5%
c. 40%
d. 35%

Solution: Officially the dishonest grocer earns a profit of 10%. It means, if Rs. 100 per kg were the cost price, he sells 1 kg rice at Rs. 110. But as he uses spurious weight 20% less than marked weight, he actually sells 0.8 kg rice at the price of Rs. 110.

Effectively thus he sells a quantity of rice purchased at a cost of Rs. 80 at Rs. 110. The total gain he accrues is then Rs. 30 on Rs. 80, his cost price. The % gain is,

(3/8)100% = 37.5%
Answer: Option **b: 37.5%.**

Q14. The smallest integer which when subtracted from both the terms of 6 : 7 results in a ratio less than 16 : 21 is,

a. 5
b. 4
c. 3
d. 2

Solution: Before anything we must get an idea of how much the first given fraction 6 : 7 is to reduce to be less than the second fraction 16 : 21. Transforming 6 : 7 by equalizing the base we get, 18 : 21. That is, it needs to be reduced by a very small amount (in comparison to its own value now) to push it below the valued of 16 : 21.

Now we look at the problem definition and find it wants the smallest integer needed to reduce both the numerator and numerator so that the desired state is reached with the changed fraction.
Here we should take recourse to our knowledge of Principle of relative increment (or decrement). It says, in case of fractions, if the two terms are reduced by same amount, the fraction itself will be reduced and will be reduced more as the term reduction is made larger if, the numerator is less than the denominator. Considering this, we decide that if we use choice

value 5 we would go far below 16 : 21 (it will be 1 : 2 or 21 : 42 in comparison to 32 : 42). In other words, this concept urges us to use the lowest choice value 2 to get,

$\frac{6}{7} \Rightarrow \frac{(6-2)}{7-2} = \frac{4}{5} = \frac{16}{20}$ still larger than $\frac{16}{21}$ (numerator equalization technique).

Taking the next choice value 3,

$\frac{6}{7} \Rightarrow \frac{(6-3)}{7-3} = \frac{3}{4} = \frac{63}{84}$, whereas $\frac{16}{21} = \frac{64}{84}$. We have just been able to reduce 6 : 7 to a value lesser than 16 : 21.

By our earlier knowledge we know that if we reduce the two terms further, the fraction will be reduced further. So we stop here with target value 3.
Answer: Option **c: 3.**

Q15. **In a vessel A, milk and water are mixed in ratio 8 : 5 and in a second vessel B, in a ratio 5 : 2. In what ratio these two mixtures are to be mixed to get a mixture containing $69^3/_{13}$ percent milk?**

a. 5 : 3
b. 2 : 7
c. 3 : 5
d. 5 : 7

Solution: Let us first convert the target percentage of milk to ratio of milk.

$$69\frac{3}{13}\% = \frac{69\times13+3}{13\times100} = \frac{900}{13\times100} = \frac{9}{13}$$

This represents milk to total mixture volume ratio as $\frac{9}{13}$.

Let's assume x litres of mixture in vessel A is mixed with y litres of mixture in vesse B to get a mixture C with this target ratio 9 : 13 of milk to total mixture.

Now we will consider only proportion of milk in total mixture without thinking of wat at all because, water proportion will be included or embedded in milk to total mixtur proportion and by this approach we will simplify the whole process of reasoning significantly.

In 1 litre of mixture A milk is $\frac{8}{13}$ litre (as milk : water is 8 : 5, total is 13 and milk : tota becomes 8 : 13).

In x litre of mixture A milk is $\frac{8x}{13}$ litre.

Similarly in y litre of mixture B, milk is $\frac{5y}{7}$ litre.

In $x + y$ litre of the new mixture C then, milk is $\frac{8x}{13} + \frac{5y}{7}$ litre.

Taking up the new mixture C with milk to total mixture volume ratio as 9 : 13 we get,

1 litre C contains $\frac{9}{13}$ litres of milk.

$x + y$ litres of C then contains $\frac{9}{13}(x + y)$ litres of milk.

So equating the two results,

$\frac{8x}{13} + \frac{5y}{7} = \frac{9}{13}(x + y) = \frac{9x}{13} + \frac{9y}{13}$

Or, $\frac{x}{13} = y(\frac{5}{7} - \frac{9}{13})$

Or, $\frac{x}{y} = 13(\frac{5}{7} - \frac{9}{13}) = \frac{65}{7} - 9 = \frac{2}{7}$.

Answer: Option **b-' 2 : 7'.**
Advantage of this approach of using only the very basic concepts of ratio instead of any formula is faster and surer way reach to the solution with greater accuracy.
Less memory load of remembering advanced formula or method relieves the mind from chance of forgetting the formula as well as enables the problem solver to reach the solution quickly because of use of more basic and frequently used simpler relations. This is called ***less facts more procedures approach.***

Q16. Rs. 180 are to be divided among 66 men and women. Men and women receive total amount in the ratio 5 : 4, but individually men and women receive money in the ratio 3 : 2. The number of women is,

a. 42
b. 36
c. 30
d. 46

Solution *As before working from the very basic concepts, amount received by men is*
$5 \times \frac{180}{9} = Rs.100$ (5 portions of total 9 portions, total being 180) and total for women, Rs.80.

If x be number of women, ratio of amount that each man receives to the amount each woman receives is, $\frac{100}{66-x} : \frac{80}{x} = 3 : 2$

Or, $\frac{100x}{80(66-x)} = \frac{3}{2}$

Or, $200x = 240(66 - x)$,

Or, $440x = 240 \times 66$, or, $11x = 6 \times 66$ or, $x = 36$.

Answer: Option **b : 36**.

Q17. Water is $1^2/_5$ times as heavy as a liquid L1 and another liquid L2 is $1^3/_7$ times as heavy as water. The amount of liquid L2 that must be added to 7 litres of liquid L1 so that the mixture may weigh as much as an equal volume of water will be,

a. 5 litres
b. 7 litres
c. $4^2/_3$ litres
d. $5^1/_6$ litres

Solution: Ultimately, weight of a volume of water equals weight of equal volume of mixture.

From first statement, W =7/5 L_1, where W represents weight of 1 litre water and L_1 represents weight of 1 litre of liquid L1. From this we get, 7 litres of liquid L1 is equivalent in weight to 5 litres of water.

Similarly from the second given weight relation we get, $L_2 = 10/7$ W.
With this we know, if we add xx litres of liquid L2 that is heavier than water, it will compensate for the 2 litres weight shortfall plus weight of xx litres water (we need to have weight of 7 litres of water after all, and with 7 litres of liquid L1 we have weight of only 5 litres of water).
So, finally for the mix,

Weight of $(2+x)$ litres of water = Weight of x litres of L2 = Weight of $\frac{10}{7}x$ litres of water

Or, $14+7x=10x$,

Or, $x=\frac{14}{3}=4\frac{2}{3}$ litres.

Answer: Option c: $4\frac{2}{3}$ litres.

Q18. A cricketer with bowling average of 24.85 runs per wicket takes 5 wickets for 52 runs and consequently decreases his average by 0.85. Number of wickets taken by him till last match was,

a. 64
b. 96
c. 72
d. 80

Solution For n wickets average was 24.85 runs per wicket. Taking 5 more wickets makes his total number of wickets n+5. We have then,

Total runs now, $24.85n+52=24(n+5)$

Or, $0.85n=120-52=68$

Or, $n=\frac{100\times68}{85}=80.$

Answer: Option **d: 80.**

Q19. Sum of the two terms of a fraction is 11. If 2 is added to both of its terms, the new fraction exceeds the old by 1/24. Then one term is larger than the other by,

a. 1

b. 3
c. 9
d. 5

Solution. By the principle of relative increment (decrement) of fraction terms, we know that if by adding same value to both terms increases the fraction, the numerator must be smaller than the denominator. In other words, addition of same number to both terms contributes more proportional increase of the numerator (being less than the denominator) resulting an increase of the fraction.

With this knowledge and the fact that the total of numerator and denominator is 11, we can enumerate the possible fractions as,

$$\frac{1}{10}, \frac{2}{9}, \frac{3}{8}, \frac{4}{7}, \frac{5}{6}.$$

From the principle of relative increment (decrement) of fractions, we also know that smaller is the difference between the numerator and denominator, smaller will be the value increase of the fraction by adding the same number to both the terms. In this case the increase 1/24 is quite small compared to the fractions being in consideration.

This urges us to start testing from the largest of the possible solutions, that is, $\frac{5}{6}$.
Thus we get, $\frac{5}{6} \Rightarrow \frac{5+2}{6+2} = \frac{7}{8}$ and $\frac{7}{8} - \frac{5}{6} = \frac{21-20}{24} = \frac{1}{24}$, a success at the very first attempt.

This shows the power of using a basic problem solving principle and its related concepts.
Answer: Option **a: 1**.

Q20. The incomes of A, B and C are in the ratio of 7 : 9 : 12 and their spendings are in the ratio of 8 : 9 : 15. If A saves $^1/_4$ th of his income, then the savings of A, B and C are in the ratio of,

a. 69 : 56 : 99
b. 99 : 56 : 69
c. 56 : 99 : 69
d. 99 : 69 : 56

Solution The first basic concept of ratios says, when forming the ratio of two quantities in a minimized fraction form, HCF of the compared quantities have been eliminated between the numerator and denominator to get the minimized fraction. Thus to get back the original quantities we multiply the two terms of a ratio by their HCF.Thus from the first relation we have the actual incomes as, 7x : 9x :12x, where xx is the HCF that was cancelled out to form the minimized ratio terms.

Similarly from the second relation we get the actual expenditures, 8y : 9y : 15y, where y is the HCF that was eliminated while forming the minimized ratio terms.

For A the saving is, $7x - 8y = \frac{7}{4}x$, or, $x = \frac{32}{21}y$. We will use this value in savings expressions of A, B and C to transform them in terms of y. Accordingly,

Savings of A in terms of y is, $7x - 8y = \frac{32}{3}y - 8y = \frac{8}{3}y$

Savings of B in terms of y is, $9x - 9y = \frac{96}{7}y - 9y = \frac{33}{7}y$

Savings of C in terms of y is, $12x - 15y = \frac{128}{7}y - 15y = \frac{23}{7}y$

So the ratios of savings of A, B and C is,

$\frac{8}{3}y : \frac{33}{7}y : \frac{23}{7}y$

In this expression y cancels out and we multiply each term by $3 \times 7 = 21$, the LCM of the denominators of the fractions. This process is normalization towards an expression in integers.

The target ratio of savings is then, 56:99:69.
Answer: Option **c : 56 : 99 : 69**.

Squares, Square Roots and Averages

In mathematics, a square is the result of multiplying a number by itself. Squaring is the same as raising to the power 2, and is denoted by a superscript 2.

Ex: the square of 3 is written as 4^2, which is equal to the number 16.

In mathematics, a square root of a number x is a number y such that $y^2 = x$. In other words, a number y whose square is x is called a square root.
Ex: 4 and −4 are square roots of 16 because $4^2 = (-4)^2 = 16$.

Here is a list of first 50 numbers with their squares:

Numbers	Squares	Square Roots
1	1	1.000
2	4	1.414
3	9	1.732
4	16	2.000
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162
11	121	3.317
12	144	3.464
13	169	3.606
14	196	3.742
15	225	3.873
16	256	4.000

17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472
21	441	4.583
22	484	4.690
23	529	4.796
24	576	4.899
25	625	5.000
\26	676	5.099
27	729	5.196
28	784	5.292
29	841	5.385
30	900	5.477
31	961	5.568
32	1,024	5.657
33	1,089	5.745
34	1,156	5.831
35	1,225	5.916
36	1,296	6.000
37	1,369	6.083
38	1,444	6.164
39	1,521	6.245
40	1,600	6.325
41	1,681	6.403
42	1,764	6.481

43	1,849	6.557
44	1,936	6.633
45	2,025	6.708
46	2,116	6.782
47	2,209	6.856
48	2,304	6.928
49	2,401	7.000
50	2,500	7.071

You must memorize the squares of the first 50 numbers for sure because they come handy while solving many questions on other topics too. Time has to be managed too so this is a useful trick up your sleeve definitely.

For calculating square roots, the first and foremost thing that we need to remember is the unit digits of all squares from 1 to 10 which is quite easy. This principle must be applied to other numbers with different unit digits as the first step after the hacks that follow:

Square Root of a Four Digit Number: We group the last pair of digits, and the rest of the digits together and then proceed with the steps explained through the four digit number 4096.

Step 1: 40 and 96 be paired separately as these will be worked upon one by one.

Step 2: The unit digit has to be calculated which in this case will be either 4 or 6.

Step 3: Calculate the ten's digit by estimation, as in $6^2 < 40 < 7^2$ and the smaller number would be the ten's digit which is 6 in this case. So we know that the the square root will be either 64 or 66.

Step 4: Consider 6 and 7 now and calculate their product which equals 42 > 40 and hence the lesser of the two values will be selected and the square root will be 64. (if the product is greater than the first two digits we select the greater of the two values.)

Let's try it out with five digit numbers now taking √(14161) as the example!

There's a basic difference here in Step 1. We pair the digits up starting from the right side. Since there is one extra left over after two pairs are formed, we club it with the pair closest to it. The remaining process remains the same as in the case of four digit numbers.

Unit digit will be either 1 or 9.

Now, $11^2 < 141 < 12^2$

So, the square root will either be 111 or 119.

Now, consider 11 and 12,

And calculate 11 * 12 = 132 < 141 implying 141 is greater the 132 so that we pick the larger of the options which is 119.

Solve the questions that follow:

Hope you will be able to move over this topic swiftly and confidently now.

1. Evaluate

$$\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$$

A. 16

B. 8

C. 6

D. 4

Answer :

Option **D**

Explanation:

$$=\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$$

$$= \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{169}}}}$$

$$= \sqrt{10 + \sqrt{25 + \sqrt{108 + 13}}}$$

$$= \sqrt{10 + \sqrt{25 + \sqrt{121}}}$$

$$= \sqrt{10 + \sqrt{25 + 11}}$$

$$= \sqrt{10 + \sqrt{36}}$$

$$= \sqrt{10 + 6}$$

$$= \sqrt{16} = 4$$

2. Evaluate

$$\sqrt{6084}$$

A. 75

B. 77

C. 78

D. 68

Answer :

Option **C**

3. Evaluate

$$\sqrt{1\frac{9}{16}}$$

A. $1\frac{1}{6}$

B. $1\frac{1}{5}$

C. $1\frac{1}{4}$

D. $1\frac{1}{3}$

Answer :

Option **C**

Explanation:

$$= \sqrt{\frac{25}{16}}$$

$$= \frac{\sqrt{25}}{\sqrt{16}}$$

$$= \frac{5}{4}$$

$$= 1\frac{1}{4}$$

4. if a = 0.1039, then the value of

$$\sqrt{4a^2 - 4a + 1} + 3a$$

A. 12.039

B. 1.2039

C. 11.039

D. 1.1039

Answer :

Option **D**

Explanation:

Tip: Please check the question carefully before answering. As 3a is not under the root we can convert it into a formula , lets evaluate now :

$$= \sqrt{4a^2 - 4a + 1} + 3a$$

$$= \sqrt{(1)^2 + (2a)^2 - 2x1x2a} + 3a$$

$$= \sqrt{(1 - 2a)^2} + 3a$$

$$= (1 - 2a) + 3a$$

$$= (1 - 2a) + 3a$$

$$= 1 + a = 1 + 0.1039 = 1.1039$$

5. Find the value of X

$$\sqrt{81} + \sqrt{0.81} = 10.09 - X$$

A. 0.019

B. 0.19

C. 0.9

D. 0.109

Answer :

Option **B**

Explanation:

$$=> \sqrt{81} + \sqrt{0.81} = 10.09 - X$$

$$=> 9 + 0.9 = 10.09 - X$$

$$=> X = 10.09 - 9.9 = 0.19$$

6. Evaluate

$$\sqrt{0.00059049}$$

A. 0.00243

B. 0.0243

C. 0.243

D. 2.43

Answer :

Option **B**

Explanation:

Very obvious tip here is, after squre root the terms after decimal will be half (that is just a trick), works awesome at many questions like this.

7. Evaluate

$$\sqrt[3]{4\frac{12}{125}}$$

A. $1\frac{2}{5}$

B. $1\frac{3}{5}$

C. $1\frac{4}{5}$

D. 1

Answer :

Option **B**

Explanation:

$$= \sqrt[3]{\frac{512}{125}}$$

$$= \left(\frac{8*8*8}{5*5*5}\right)^{\frac{1}{3}}$$

$$= \frac{8}{5} = 1\frac{3}{5}$$

9. Find the value of x

$$\frac{2707}{\sqrt{x}} = 27.07$$

A. 1000

B. 10000

C. 10000000

D. None of above

E. 10,00,000 tonnes

F. 15,00,000 tonnes

G. 20,00,000 tonnes

H. 25,00,000 tonnes

Answer :

Option

Explanation:

$$= \frac{2707}{27.07} = \sqrt{x}$$

$$=> 100 = \sqrt{x}$$

$$=> \frac{2707 \times 100}{2707} = \sqrt{x}$$

$$=> x = 100^2 = 10000$$

We take a look now on the dominant section under this topic, Averages.

In mathematics, the average or arithmetic mean is the sum of a collection of numbers divided by the number of numbers in the collection. The collection is generally a set of results of an experiment, or a set of results from a survey.

Formula:

Average: = (Sum of observations / Number of observations).

The hacks under this category are used mostly to calculate average speeds, age or a specific numeric quantity. Take a look:

- If a person travels a distance at a speed of A km/hr and the same distance at a speed of B km/hr then the average speed during the whole journey will be equal to 1.

- If a person covers X km at A km/hr and Y km at B km/hr and Z km at C km/hr, then the average speed in covering the whole distance is 2.

- When a person leaves the group and another person joins the group in place of that person then :

✓ If the average age is increased, then

Age of new person = Age of separated person + (Increase in average × total number of persons)

✓ If the average age is decreased, then

Age of new person = Age of separated person – (Decrease in average × total number of persons)

- When a person joins the group, then :

✓ In cases where the average increases,

Age of new member = Previous average + (Increase in average × Number of members including new member)

✓ In cases where the average decreases,
Age of new member = Previous average – (Decrease in average × Number of members including new member)

- In an Arithmetic Progression two cases are considered :
  - ✓ When the number of terms is odd, the average will be the middle term
  - ✓ When the number of terms is even,the average will be the average of two middle terms.

- If the average of n numbers is A, then if we add, subtract, multiply or divide x to each term then the new average will be = (A+ x), (A- x), (A * x), (A /x) respectively.

These time saving and thought out hacks are the most adequate requirement to cover Averages for SSC. Once you practice the questions below, this section will be the easiest to attempt while taking the exam.

Solve the questions that follow:

1. The average of four consecutive odd numbers is 24. Find the largest number.

A. 25

B. 27

C. 29

D. 31

Answer :

Option **B**

Explanation:

Let the numbers are x, x+2, x+4, x+6, then

$$=> \frac{x+(x+2)+(x+4)+(x+6)}{4} = 24$$

$$=> \frac{4x+12)}{4} = 24$$

$$=> x+3 = 24 => x = 21$$

So largest number is 21 + 6 = 27

2. A library has an average of 510 visitors on Sundays and 240 on other day. The average number of visitors in a month of 30 days starting with sunday is

A. 280

B. 285

C. 290

D. 295

Answer :

Option **B**

Explanation:

As the month begin with sunday, so there will be five sundays in the month. So result will be:

$$= \left(\frac{510 \times 5 + 240 \times 25}{30}\right) = \left(\frac{8550}{30}\right) = 285$$

3. A batsman makes a score of 87 runs in the 17th match and thus increases his average by 3. Find his average after 17th match

A. 36

B. 37

C. 38

D. 39

Answer :

Option **D**

Explanation:

Let the average after 17th match is x
then the average before 17th match is x-3

so 16(x-3) + 87 = 17x
=> x = 87 - 48 = 39

4. Reeya obtained 65, 67, 76, 82 and 85 out of 100 in different subjects, What will be the average.

A. 70

B. 75

C. 80

D. 85

Answer :

Option **B**

Explanation:

$$\left(\frac{65+67+76+82+85}{5}\right)=75$$

5. Average age of boys in a class is 16 years and average age of girls is 15 years, what is the average age of all

A. 15.5

B. 15

C. 16

D. Cant be computed

Answer :

Option **D**

Explanation:

As number of girls and boys is not given so result cant be computed

6. Average of all prime numbers between 30 to 50

A. 37

B. 37.8

C. 39

D. 39.8

Answer :

Option **D**

Explanation:

Prime numbers between 30 and 50 are:
31, 37, 41, 43, 47

Average of prime numbers between 30 to 50 will be

$$\left(\frac{31+37+41+43+47}{5}\right)=\frac{199}{5}=39.8$$

7. Average of first five multiples of 3 is

A. 9

B. 11

C. 13

D. 15

Answer :

Option **A**

Explanation:

$$Average = \frac{3(1+2+3+4+5)}{5} = \frac{45}{5} = 9$$

8. A motorist travel to a place 150 km away at an avearge speed of 50 km/hr and returns ar 30 km/hr.His average speed for the whole jouney in km/hr is

A. 36.5 km/hr

B. 37.5 km/hr

C. 35.5 km/hr

D. 34.5 km/hr

Answer :

Option **B**

Explanation:

Average speed will be

$$\frac{2xy}{x+y} km/hr$$

= \frac{2(50)(30)}{50+30} km/hr
= 37.5 km/hr

9. The average weight of a class of 24 students is 35 kg. If the weight of the teacher be included, the average rises by 400 g. The weight of the teacher is

A. 45

B. 50

C. 55

D. 60

E. a tax evader who uses deceptive accounting practices to hide her income from auditors

F. an embezzler who steals from her company by pilfering small amounts of money over a long period of time

G. a criminal who argues that it is not he but one of his accomplices who is guilty of a crime

H. a con artist who convinces his victim that it is in the victim's best interest to help him

Answer :
Option

10. The average of six numbers is X and the average of three of these is Y.If the average of the remaining three is z, then

A. $x = y + z$

B. $2x = y + z$

C. $x = 2y + z$

D. $x = y + 2z$

Answer :

Option **B**

Explanation:

$X = ((3y+3z)/6)$
or
$2X = y + z$

11. If the average marks of three batches of 55, 60 and 45 students respectively is 50, 55, 60, then the average marks of all the students is

A. 54.48

B. 54.68

C. 54.60

D. 54.58

Answer :

Option **B**

Explanation:

$$\frac{(55 \times 50) + (60 \times 55) + (45 \times 60)}{55 + 60 + 45}$$

$$\frac{8750}{160} = 54.68$$

12. Average weight of 10 people increased by 1.5 kg when one person of 45 kg is replaced by a new man. Then weight of the new man is

A. 50

B. 55

C. 60

D. 65

Answer :

Option **C**

Explanation:

Total weight increased is 1.5 * 10 = 15.
So weight of new person is 45+15 = 60

13. Average of 10 numbers is zero. At most how many numbers may be greater than zero

A. 0

B. 1

C. 5

D. 9

Answer :

Option **D**

14. Find the sum of first 30 natural numbers

A. 470

B. 468

C. 465

D. 463

Answer :

Option **C**

Explanation:

Sum of n natural numbers

$$= \frac{n(n+1)}{2}$$

$$= \frac{30(30+1)}{2} = \frac{30(31)}{2} = 465$$

15. Find the average of first 10 multiples of 7

A. 35.5

B. 37.5

C. 38.5

D. 40.5

Answer :

Option **C**

Explanation:

$$= \frac{7(1+2+3+\ldots+10)}{10}$$

$$= \frac{7(10(10+1))}{10 \times 2}$$

$$= \frac{7(110)}{10 \times 2} = 38.5$$

16. Average of 10 matches is 32, How many runs one should should score to increase his average by 4 runs.

A. 70

B. 76

C. 78

D. 80

Answer :

Option **B**

Explanation:

Average after 11 innings should be 36

So, Required score = (11 * 36) - (10 * 32)
= 396 - 320 = 76

17. When a student weighing 45 kgs left a class, the average weight of the remaining 59 students increased by 200g. What is the average weight of the remaining 59 students

A. 55

B. 56

C. 57

D. 58

Answer :

Option **C**

Explanation:

Let the average weight of the 59 students be A.
So the total weight of the 59 of them will be 59*A.

The questions states that when the weight of this student who left is added, the total weight of the class = 59A + 45
When this student is also included, the average weight decreases by 0.2 kgs

$$\frac{59A + 45}{60} = A - 0.2$$

=> 59A + 45 = 60A - 12
=> 45 + 12 = 60A - 59A
=> A = 57

18. Find the average of all numbers between 6 and 34 which are divisible by 5

A. 15

B. 20

C. 25

D. 30

Answer :

Option **B**

Explanation:

$$Average = (\frac{10+15+20+25+30}{5}) = \frac{100}{5} = 20$$

19. Average of five numbers is 27. If one number is excluded the average becomes 25. The excluded number is

A. 35

B. 45

C. 55

D. 65

Answer :

Option **A**

Explanation:

Number is (5*27) - (4*25) = 135-100 = 35

20. The average score of a cricketer for ten matches is 38.9 runs. If the average for first six matches is 42, then average for last four matches is

A. 33.25

B. 32.25

C. 34.25

D. 34.50

Answer :

Option **C**

Explanation:

$$= \frac{(38.9 \times 10) - (42 \times 6)}{4}$$

$$= \frac{(1216 - 750)}{4} = 34.25$$

21. The average age of the mother and her six children is 12 years which is reduced by 5 years if the age of the mother is excluded. How old is the mother

A. 40

B. 41

C. 42

D. 43

Answer :

Option **C**

22. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of the husband is

A. 40

B. 35

C. 45

D. 55

Answer :

Option **A**

Explanation:

Sum of the present ages of husband, wife and child = (27 * 3 + 3 * 3) years = 90 years.

Sum of the present ages of wife and child = (20 * 2 + 5 * 2) years = 50 years.

Husband's present age = (90 - 50) years = 40 years

Profit and Loss

The topics we have covered till now are one of the easiest topics covered under the syllabus of SSC CGL. Let's move to the bigger players in the market now beginning with Profit and Loss also covering Discount and Partnership Business at the same time.

There are a few basic terms and formulas which we need to keep in mind while attempting this section. These are :

a) **Cost Price** - The price at which an article is purchased is called its cost price and is represented as C.P.

b) **Selling Price** - The price at which an article is sold is called its selling price and is represented as S.P.

c) **Profit** - If the overall Selling Price exceeds the Cost Price of the buyer then he is said to have incurred profits.
Profit = SP – CP
Profit % = Profit / CP × 100
SP = (100 + gain %)/ 100 × CP
CP = 100 / (100 + gain %) × SP

d) **Loss**: If the overall Cost Price exceeds the selling price of the buyer then he is said to have incurred loss.
Loss = CP – SP
Loss % = LOSS/ (CP) × 100
SP = (100 - loss %) / 100 × CP
CP = 100 / (100 - loss %) × SP

e) **Marked Price** - The normal price of a commodity without a discount is the marked price of that product. Represented as MP, it is generally greater than the SP and their difference is known as Discount, represented as D.
Discount = MP – SP
Discount % = (Discount) / (MP) ×100

These were the basic concepts under Profit and Loss. The questions that are covered under this topic are often framed in relation to either the CP or the SP of the product and hence we will take these relations one at a time and consider all the related cases separately.

A. Based on Cost Price

- To find the percent gain or loss, divide the amount gained or lost by the cost and multiply it by 100.
- To find the loss and the selling price when the cost and the percent loss are given, multiply the cost by the percent and subtract the product from the cost.

B. Based on Selling Price

- To find the profit and the cost when the selling price and the percent profit are given, multiply the selling price by the percent profit and subtract the result from the selling price.
- To find the loss and the cost when the selling price and the percent loss are given, multiply the selling price by the percent loss and subtract the result from the selling price.
- To find the selling price when the cost and the percent loss are given, add the percent loss to 100% and divide the cost by this sum.
- To find the selling price when the profit and the percent profit are given, or to find the selling price when the loss and the percent loss are given, divide the profit or loss by the percent profit or loss.

(This rule should be compared with the one under Profit and Loss Based on Cost. The two rules are exactly similar except that in one case 100% represents cost while in the other case 100% represents selling price.)

- To find the percent profit or loss, divide the amount gained or lost by the selling price.
- To reduce percent loss on cost to percent loss on selling price, divide percent loss on cost by 100% minus percent loss on cost.
- To reduce percent loss on selling price to percent loss on cost, divide percent loss on selling price by 100% plus percent loss on selling price.
- To reduce percent mark-up (percent profit on cost) to percent profit on selling price, divide percent mark-up by 100% plus percent mark-up.

These hacks, once implied in the exercises below, will fade your fears regarding this topic and enable you to score higher than ever before.

Solve the questions that follow:

Q.1 The cost price of 40 articles is the same as the selling price of 25 articles. Find the gain per cent.

(a) 65%

(b) 60%

(c) 15%

(d) 75%

Answer: (b)

Gain per cent =(40-25)/25×100 =15/25×100=60%

Hack:

In Above question We take x = 40, y = 25

Then Gain % = (x –y) x 100/ y

Q.2 Bananas are bought at the rate of 6 for Rs. 5 and sold at the rate of 5 for Rs. 6. Profit per cent is:

(a) 36%

(b) 42%

(c) 44%

(d) 48%

Answer: (c)

To avoid fraction, let the number of bananas boughtLCM of 5 and 6 = 30

CP of 30 bananas = 5 x 5 = Rs. 25

SP of 30 Bananas = 6 x 6 = Rs. 36

Profit = Rs. (36-25) = Rs. 11

Profit % = 11/25×100=44%

Hack

[(6 x 6 -5x 5)/ (5 x 5)] x 100 = 44%

Q.3 A man bought oranges at the rate of 8 for Rs 34 and sold them at the rate of 12 for Rs. 57. How many oranges should be sold to earn a net profit of Rs 45?

(a) 90

(b) 100

(c) 135

(d) 150

Answer: (a)

Let the man buy 24 (LCM of 8 and 12) oranges.

C.P. of 24 oranges = $34/8 \times 24$ = Rs. 102

S.P. of 24 oranges = $27/12 \times 24$ = Rs. 114

Gain = 114 – 102 = Rs. 12

Rs. 12 = 24 oranges

Rs. 45 = $24/12 \times 45$ = 90 oranges

Q4. A shopkeeper earns a profit of 12% on selling a book at 10% discount on printed price. The ratio of the cost price to printed price of the book is ?

(a) 45 : 56

(b) 50 : 61

(c) 90 : 97

(d) 99 : 125

Answer: (a)

C.P. of the book = Rs. x

Printed price = Rs. y

$(y \times 90)/100 = x \times 112/100$

$x/y = 90/112 = 45/56$

Q.5 A dealer sold two types of goods for Rs 10,000 each. On one of them, he lost 20% and on the other he gained 20%. His gain or loss per cent in the entire transaction was

(a) 2% loss

(b) 2% gain

(c) 4% gain

(d) 4% loss

Answer: (d)

Here, S.P. is same, Hence there is always a loss. Loss per cent = (20×20)/100=4%

Hack

Loss % = (n^2)/100= (20)^2/100= 4%

Where n= 20

Q.6 On selling an article for Rs170, a shopkeeper loses 15%. In order to gain 20%, he must sell that article at rupees:

(a) 215.50

(b) 212.50

(c) 240

(d) 210

Answer: (c)

C.P. of article = (200×120)/100 = Rs. 240

Q.7 An article is sold at a loss of 10%. Had it been sold for Rs. 9 more, there would have been a gain of 12 1/2% on it. The cost price of the article is

(a) Rs. 40

(b) Rs. 45

(c) Rs. 50

(d) Rs. 35

Answers: (a)

Let the cost price of the article = Rs. x

S.P. at 10% loss = x×90/100 = Rs. 9x

- P. at 12 1/2 % gain

x × (100+12 1/2)/100 = Rs. 225x/200

According to the question, 9x + 9 = 225x/200

180x + 1800 = 225x

x = Rs. 40

Hack

If sign is not same then, we have to **Add**

If sign is same then, we have to **Subtract**

Here,

– 10 % + 12 ½

22 ½% = 9%

100 % = ?

Formula = (n x 100)/ (difference of loss % or Gain)

Note : where n= 9

Q.8 A sells a suitcase to B at 10% profit. B sells it to C at 30% profit. If C pays Rs 2860 for it, then the price at which a bought it is

(a) 1000

(b) 1600

(c) 2000

(d) 2500

Answer: (c)

If the C.P. of the suitcase for A be Rs. x, then

x ×110/100×130/100=2860

x=(2860×100×100)/(110×130) = Rs. 2000

Q.9 Arun marks up the computer he is selling by 20% profit and sells them at a discount of 15%. Arun's net gain percent is

(a) 4

(b) 2

(c) 3.5

(d) 2.5

Answer: (b)

Hack:

r1 = 20 , r2 = 15

Formula = r1 – r2 – (r1 x r2)/100

(20-15-(20×15)/100)

= 20 -18 = 2%

Q.10 A tradesman sold an article at a loss of 20%. If the selling price had been increased by Rs. 100, there would have been a gain of 5%. The cost price of the

article was:

(a) Rs. 200

(b) Rs. 25

(c) Rs. 400

(d) Rs. 250

Answer: (c)

Let the C.P. of article be Rs. x.

105% of x – 80% of x = Rx. 100

25% of x = Rx. 100

x = Rs. (100×100)/25

= Rs. 400

Q11. If the cost price is 25% of selling price. Then what is the profit percent.

A. 150%

B. 200%

C. 300%

D. 350%

Answer :

Option **C**

Explanation: Let the S.P = 100, then C.P. = 25

Profit = 75

Profit% = 75/25 * 100 = 3005

Q12. Alfred buys an old scooter for Rs. 4700 and spends Rs. 800 on its repairs. If he sells the scooter for Rs. 5800, his gain percent is

A. $\frac{6}{19}\%$

B. $\frac{6}{17}\%$

C. $5\frac{5}{11}\%$

D. $3\frac{5}{11}\%$

Answer : Option **C**

Explanation: Whenever we get this type of question, we should have formula in mind that,
Gain% = (gain/cost)*100
Cost = 4700 + 800 = Rs. 5500
S.P. = Rs. 5800
Profit = 5800 - 5500 = 300

$$
\begin{aligned}
Gain\% &= \frac{300}{5500} * 100 \\
&= 5\frac{5}{11}\%
\end{aligned}
$$

Q13.A shopkeeper expects a gain of 45/2 % on his C.P. If his sale was Rs. 392, then find his profit.

A. Rs. 70

B. Rs. 72

C. Rs. 74

D. Rs. 76

Answer : Option **F**

Explanation: We Know,

$$S.P. = \left(\frac{100 + gain\%}{100} * C.P\right)$$
$$=> C.P. = \frac{100}{122.50} * 392$$
$$= 320$$
$$Profit = 392 - 320 = Rs72$$

Q14. A person incurs a loss of 5% be selling a watch for Rs. 1140. At what price should the watch be sold to earn 5% profit?

A. Rs.1200

B. Rs.1230

C. Rs.1260

D. Rs.1290

Answer: Option **C**

Explanation: Let the new S.P. be x, then.
(100 - loss%):(1st S.P.) = (100 + gain%):(2nd S.P.)

$$=> \left(\frac{95}{1140} = \frac{105}{x}\right)$$
$$=> x = 1260$$

Q15. A shopkeeper sells a transistor at Rs. 840 at a gain of 20% and another for Rs. 960 at the loss of 4%. Find his total gain percent.

A. $5\frac{12}{17}\%$

B. $5\frac{13}{17}\%$

C. $5\frac{14}{17}\%$

D. $5\frac{15}{17}\%$

Answer: Option **D**

Explanation: In this type of question, we will first find total C.P. of items, then total S.P. of items, then we will get gain or loss. From which we can easily calculate its percentage.

So lets solve it now.

$$\text{So, C.P. of 1st transistor} = \left(\frac{100}{120} * 840\right) = 700$$

$$\text{C.P. of 2nd transistor} = \left(\frac{100}{96} * 960\right) = 1000$$

$$\text{Total C.P.} = 1700$$

$$\text{Total S.P.} = 1800$$

$$\text{Gain} = 1800 - 1700 = 100$$

$$\text{Gain\%} = \left(\frac{100}{1700} * 100\right) = 5\frac{15}{17}\%$$

Q16. A shopkeeper fixes the marked price of an item 35% above its cost price. The percentage of discount allowed to gain 8% is

A. 18%

B. 20%

C. 22%

D. 24%

Answer: Option **B**

Explanation: Let the cost price = Rs 100

then, Marked price = Rs 135

Required gain = 8%,

So Selling price = Rs 108

Discount = 135 - 108 = 27

Discount% = (27/135)*100 = 20%

Q17. A material is purchased for Rs. 600. If one fourth of the material is sold at a loss of 20% and the remaining at a gain of 10%, Find out the overall gain or loss percentage

A. $4\frac{1}{2}$

B. $3\frac{1}{2}$

C. $2\frac{1}{2}$

D. $1\frac{1}{2}$

Answer: Option **C**

Explanation: We need to get the Total selling price to solve this question. Because after getting selling price we can get profit or loss, then we can calculate profit% or loss%
So lets solve this:
Price Received by selling one fourth of the material at a loss of 20% =
(1/4) * 600 * (80/100) = Rs. 120

Price Received by remaining material at a gain of 10% =
(3/4) * 600 * (110/100) = Rs. 495 [Note: 1-(1/4) = 3/4]
Total Selling Price = 120 + 465 = Rs. 615
Profit = 615 - 600 = 15

$$\begin{aligned} Profit\% &= \left(\frac{Gain}{Cost} * 100\right)\% \\ &= \left(\frac{15}{600} * 100\right)\% \\ &= \frac{5}{2}\% = 2\frac{1}{2}\% \end{aligned}$$

Q18. A shopkeeper cheats to the extent of 10% while buying and selling, by using false weights. His total gain is.

A. 20%

B. 21%

C. 22%

D. 23%

Answer: Option **B**

Explanation:

$$Gain\% = \left(\frac{(100 + \text{common gain\%})^2}{100} - 100\right)\%$$

$$= \left(\frac{(100 + 10)^2}{100} - 100\right)\%$$

$$= \left(\frac{12100 - 10000}{100}\right)\%$$

$$= 21\%$$

Q19. A book was sold for Rs 27.50 with a profit of 10%. If it were sold for Rs. 25.75, then would have been percentage of profit and loss ?

A. 2% Profit

B. 3% Profit

C. 2% Loss

D. 3% Loss

Answer : Option **B**

Explanation:

Please remember

$$S.P. = \left(\frac{100 + gain\%}{100} * C.P\right)$$
$$\text{So, C.P.} = \left(\frac{100}{110} * 25.75\right)$$
$$\text{When S.P.} = 25.75 \text{ then}$$
$$Profit = 25.75 - 25 = Re. 0.75$$
$$Profit\% = \frac{0.75}{25} * 100 = 3\%$$

Q20. 100 oranges are bought at the rate of Rs. 350 and sold at the rate of 48 per dozen. The percentage of profit is

A. $12\frac{2}{7}\%$

B. $13\frac{2}{7}\%$

C. $14\frac{2}{7}\%$

D. $15\frac{2}{7}\%$

Answer: Option **C**

Explanation: So before solving this question we will get the C.P. and S.P. of 1 article to get the gain percent.

C.P. of 1 orange = 350/100 = Rs 3.50
S.P. of one orange = 48/12 = Rs 4 [note: divided by 12 as 1 dozen contains 12 items]
Gain = 4 - 3.50 = Rs 0.50

$$Gain\% = \frac{0.50}{3.50} * 100$$
$$= \frac{100}{7}\% = 14\frac{2}{7}\%$$

Q21. A man bought an article and sold it at a gain of 5 %. If he had bought it at 5% less and sold it for Re 1 less, he would have made a profit of 10%. The C.P. of the article was

A. Rs 100

B. Rs 150

C. Rs 200

D. Rs 250

Answer: Option **C**

Explanation:

Let original Cost price is x
Its Selling price = 105/100 * x = 21x/20
New Cost price = 95/100 * x = 19x/20
New Selling price = 110/100 * 19x/20 = 209x/200
[(21x/20) - (209x/200)] = 1

=> x = 200

Q22. If the cost price of 12 pens is equal to the selling price of 8 pens, the gain percent is ?

A. 12%

B. 30%

C. 50%

D. 60%

Answer: Option **C**

Explanation: Friends, we know we will need gain amount to get gain percent, right. So let's get gain first.

Let the cost price of 1 pen is Re 1
Cost of 8 pens = Rs 8
Selling price of 8 pens = 12
Gain = 12 - 8 = 4

$$Gain\% = \left(\frac{Gain}{Cost} * 100\right)\%$$
$$= \left(\frac{4}{8} * 100\right)\% = 50\%$$

Q23. A man buys an article for Rs. 27.50 and sells it for Rs 28.60. Find his gain percent

A. 1%

B. 2%

C. 3%

D. 4%

Answer: Option **D**

Explanation:

So we have C.P. = 27.50
S.P. = 28.60

Gain = 28.60 - 27.50 = Rs. 1.10

$$Gain\% = \left(\frac{Gain}{Cost} * 100\right)\%$$
$$= \left(\frac{1.10}{27.50} * 100\right)\% = 4\%$$

Q24. A TV is purchased at Rs. 5000 and sold at Rs. 4000, find the lost percent.

A. 10%

B. 20%

C. 25%

D. 28%

Answer : Option **B**

Explanation: We know, C.P. = 5000

S.P. = 4000

Loss = 5000 - 4000 = 1000

$$Loss\% = \left(\frac{Loss}{Cost} * 100\right)\%$$
$$= \left(\frac{1000}{5000} * 100\right)\% = 20\%$$

Q25. A plot is sold for Rs. 18,700 with a loss of 15%. At what price it should be sold to get profit of 15%.

A. Rs 25300

B. Rs 22300

C. Rs 24300

D. Rs 21300

Answer: Option **A**

Explanation: This type of question can be easily and quickly solved as following:

Let at Rs x it can earn 15% profit
85:18700 = 115:x [as, loss = 100 -15, Profit = 100 +15]
x = (18700*115)/85
= Rs.25300

Q26. The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25% then determine the value of x.

A. 14

B. 15

C. 16

D. 17

Answer: Option **C**

Explanation:

Let the cost price 1 article = Re 1
Cost price of x articles = x
S.P of x articles = 20

Gain = 20 -x

$$=> 25 = \left(\frac{20-x}{x} * 100\right)$$
$$=> 2000 - 100x = 25x$$
$$=> x = 16$$

Q27. The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, find out the value of x

A. 13

B. 14

C. 15

D. 16

Answer: Option **D**

Explanation:

Let the Cost Price of one article = Rs. 1
CP of x articles = Rs. x
CP of 20 articles = 20
Selling price of x articles = 20

Profit = 25% [Given]

$$\Rightarrow \left(\frac{SP-CP}{CP}\right) = \frac{25}{100} = \frac{1}{4} \Rightarrow \frac{(20-x)}{x} = \frac{1}{4}$$
$$\Rightarrow 80 - 4x = x$$
$$\Rightarrow 5x = 80$$
$$\Rightarrow x = \frac{80}{5} = 16$$

Q28. In a certain store, the profit is 320% of the cost. If the cost increases by 25% but the selling price remains constant, approximately what percentage of the selling price is the profit?

A. 70%

B. 80%

C. 90%

D. None of above

Answer: Option **A**

Explanation:

Let C.P.= Rs. 100.
Then, Profit = Rs. 320,
S.P. = Rs. 420.

New C.P. = 125% of Rs. 100 = Rs. 125
New S.P. = Rs. 420.

Profit = Rs. (420 - 125) = Rs. 295

Required percentage = (295/420) * 100
= 70%(approx)

Q29. A producer of tea blends two varieties of tea from two tea gardens one costing Rs 18 per kg and another Rs 20 per kg in the ratio 5 : 3. If he sells the blended variety at Rs 21 per kg, then his gain percent is

A. 12%

B. 13%

C. 14%

D. 15%

Answer: Option **A**

Explanation: Suppose he bought 5 kg and 3 kg of tea.
Cost Price = Rs. (5 x 18 + 3 x 20) = Rs. 150.
Selling price = Rs. (8 x 21) = Rs. 168.
Profit = 168 - 150 = 18

So, Profit % = (18/150) * 100 = 12%

Q30. A fruit seller sells mangoes at the rate of Rs.9 per kg and thereby loses 20%. At what price per kg, he should have sold them to make a profit of 5%

A. Rs 8.81

B. Rs 9.81

C. Rs 10.81

D. Rs 11.81

Answer: Option **D**

Explanation: 85 : 9 = 105 : x
x= (9×105/85)
= Rs 11.81

Q31. The cash difference between the selling prices of an article at a profit of 4% and 6% is Rs 3. The ratio of two selling prices is

A. 51:52

B. 52:53

C. 53:54

D. 54:55

Answer: Option **B**

Explanation: Let the Cost price of article is Rs. x

Required ratio =

$$\frac{104\% \text{ of } x}{106\% \text{ of } x}$$

$$= \frac{104}{106} = \frac{52}{53} = 52 : 53$$

Q32. If the manufacturer gains 10 %, the wholesale dealer 15 % and the retailer 25 %, then find the cost of production of a table if the retail price was Rs 1265

A. Rs. 750

B. Rs. 800

C. Rs. 850

D. Rs. 900

Answer: Option **B**

Explanation:

Let the cost of Production = Rs. P
Then, as per question,

$$=> \left(\frac{125}{100} * \frac{115}{100} * \frac{110}{100} * P\right) = 1265$$
$$=> \frac{253}{160}P = 1265$$
$$=> P = 800$$

Q33. A man buys an item at Rs. 1200 and sells it at the loss of 20 percent. Then what is the selling price of that item

A. Rs. 660

B. Rs. 760

C. Rs. 860

D. Rs. 960

Answer: Option **D**

Explanation: Here always remember, when ever x% loss,
it means S.P. = (100 - x)% of C.P
when ever x% profit,
it means S.P. = (100 + x)% of C.P

So here will be (100 - x)% of C.P.
= 80% of 1200
= 80/100 * 1200
= 960

Q34. Akhil purchased 70kg vegetable at Rs. 420, then sold them at the rate of Rs. 6.50 per kg, find the profit percent.

A. $8\frac{1}{3}\%$

B. $7\frac{1}{3}\%$

C. $6\frac{1}{3}\%$

D. $5\frac{1}{3}\%$

Answer: Option **A**

Explanation: Please note in this type of questions, get the value of 1 kg to solve this question, lets solve it.

C.P. of 1 Kg = 420/70 = Rs. 6
Selling Price = 6.50
Gain = Rs. 0.50

$$Gain\% = \frac{.50}{6} * 100$$
$$= 8\frac{1}{3}\%$$

Q35. A shopkeeper sold an article for Rs 2564.36. Approximately what was his profit percent if the cost price of the article was Rs 2400?

A. 4%

B. 5%

C. 6%

D. 7%

Answer: Option **D**

Explanation:

Gain % = (164.36*100/2400) = 6.84 % = 7% approx

Q36. If the cost price of 12 items is equal to the selling price of 16 items, the loss percent is?

A. 20%

B. 25%

C. 30%

D. 35%

Answer: Option **B**

Explanation: Let the Cost Price of 1 item = Re. 1
Cost Price of 16 items = 16
Selling Price of 16 items = 12

Loss = 16 - 12 = Rs 4
Loss % = (4/16)* 100 = 25%

Q37. A man gains 20% by selling an article for a certain price. If he sells it at double the price, the percentage of profit will be.

A. 130%

B. 140%

C. 150%

D. 160%

Answer: Option **B**

Explanation:

Let the C.P. = x,
Then S.P. = (120/100)x = 6x/5
New S.P. = 2(6x/5) = 12x/5

Profit = 12x/5 - x = 7x/5

Profit% = (Profit/C.P.) * 100
=> (7x/5) * (1/x) * 100 = 140 %

Q38. A pair of articles was bought for Rs. 37.40 at a discount of 15%. What must be the marked price of each of the articles ?

A. Rs15

B. Rs 20

C. Rs 22

D. Rs 25

Answer: Option **C**

Explanation:

As question states that rate was of pair of articles,

So rate of One article = 37.40/2 = Rs. 18.70

Let Marked price = Rs X
then 85% of X = 18.70

=> X = 1870/85 = 22

Q39. In terms of percentage profit, which among following the best transaction.

A. C.P. 36, Profit 17

B. C.P. 50, Profit 24

C. C.P. 40, Profit 19

D. C.P. 60, Profit 29

Answer : Option **D**

Explanation:

Hint: Calculate profit percent as

Profit% = (profit/cost) * 100

Q40. Sahil purchased a machine at Rs 10000, then got it repaired at Rs 5000, then gave its transportation charges Rs 1000. Then he sold it with 50% of profit. At what price he actually sold it.

A. Rs. 22000

B. Rs. 24000

C. Rs. 26000

D. Rs. 28000

Answer: Option **B**

Explanation:

Question seems a bit tricky, but it is very simple.
Just calculate all Cost price, then get 150% of CP.

C.P. = 10000 + 5000 + 1000 = 16000

150% of 16000 = 150/100 * 16000 = 24000

Simple and Compound Interest

Interest is a payment from a borrower or deposit-taking financial institution to a lender or depositor for the use of money lent. The amount borrowed is called the principal sum. An interest rate or rate of interest is applied on the amount of interest due per period, as a proportion of the principal sum. This interest is generally calculated on a ' per annum' or yearly basis. The sum of the principal and interest is called the Amount represented as A.

Interest can be classified as:

- **Simple Interest :** When interest is calculated on the original principal for any length of time it is called simple interest. It is calculated numerically by multiplying the principal by the number of years and the rate per cent and dividing the result by 100.
  Formula : SI = (P × R × T) / 100
  where SI = Simple Interest, P = principle, T = number of years, R = % rate

This is a topic which has less concepts but complicated questions.Keep these hacks in mind and it will be a cake walk for you to solve Interest questions.

1. In case of successive but different interest rates a,b,c applied after time intervals of x,y,z units the Simple Interest can be calculated by the formula,
   SI = P (ax + by + cz) / 100
2. For questions where the principle is multiplied (2P, 3P, 4P etc.) after a certain period of time, use the formula,
   Rate × Time = 100 (Multiplier - 1)

Practice the questions that follow for complete control on Simple Interest :

1. At what rate percent per annum will the simple interest on a sum of money be 2/5 of the amount in 10 years

A. 1%

B. 2%

C. 3%

D. 4%

Answer: Option **D**

Explanation: Let sum = x
Time = 10 years.
S.I = 2x /5, [as per question]
Rate =((100 * 2x) / (x*5*10))%
=> Rate = 4%

2. A man took a loan at rate of 12% per annum simple interest. After 3 years he had to pay 5400 interest. The principal amount borrowed by him was.

A. Rs 14000

B. Rs 15000

C. Rs 16000

D. Rs 17000

Answer: Option **B**

Explanation:

$$\text{S.I.} = \frac{P * R * T}{100}$$
$$=> P = \frac{S.I. * 100}{R * T}$$
$$=> P = \frac{5400 * 100}{12 * 3} = Rs15000$$

3. What will the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years.

A. 1:2

B. 2:1

C. 2:2

D. 2:3

Answer: Option **D**

Explanation:

Let the principal be P and rate be, then

$$\text{ratio} = [\frac{(\frac{P*R*6}{100})}{(\frac{P*R*9}{100})}]$$
$$= \frac{6PR}{9PR} = 2:3$$

4. There was simple interest of Rs. 4016.25 on a principal amount at the rate of 9%p.a. in 5 years. Find the principal amount

A. Rs 7925

B. Rs 8925

C. Rs 7926

D. Rs 7925

Answer: Option **B**

$$P = \frac{S.I.*100}{R*T}$$

Explanation:

So by putting values from our question we can get the answer

$$P = \frac{4016.25 * 100}{9 * 5} = 8925$$

5. A sum of money at simple interest amounts to Rs. 2240 in 2 years and to Rs. 2600 in 5 years. What is the principal amount

A. 1000

B. 1500

C. 2000

D. 2500

Answer: Option **C**

Explanation:
SI for 3 year = 2600-2240 = 360
SI for 2 year 360/3 * 2 = 240
principal = 2240 - 240 = 2000

6. Reema took a loan of Rs 1200 with simple interest for as many years as the rate of interest. If she paid Rs. 432 as interest at the end of the loan period, what was the rate of interest?

A. 5%

B. 6%

C. 7%

D. 8%

Answer: Option **B**

Explanation:

Let rate = R% then Time = R years.

$$=> \frac{1200 * R * R}{100} = 432$$
$$=> R^2 = 36$$
$$=> R = 6\%$$

7. Find the rate at Simple interest, at which a sum becomes four times of itself in 15 years.

A. 10%

B. 20%

C. 30%

D. 40%

Answer: Option **B**

Explanation: Let sum be x and rate be r%
then, (x*r*15)/100 = 3x [important to note here is that simple interest will be 3x not 4x, beause 3x+x = 4x]

=> r = 20%

8. Find the simple interest on Rs 7000 at 50/3 % for 9 months

A. Rs. 1075

B. Rs. 975

C. Rs. 875

D. Rs. 775

Answer: Option **C**

Explanation:

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

So, by putting the values in the above formula, our result will be.

$$\text{Required result} = \frac{7000 \times 50 \times 9}{3 \times 12 \times 100} = 875$$

[Please note that we have divided by 12 as we converted 9 months in a year format]

9. The simple interest on a certain sum of money at the rate of 5% p.a. for 8 years is Rs. 840. At what rate of intrest the same amount of interest can be received on the same sum after 5 years.

A. 5%

B. 6%

C. 7%

D. 8%

Answer: Option **D**

Explanation:

Here firstly we need to calculate the principal amount, then we can calculate the new rate.

$$P = \frac{S.I. * 100}{R * T}$$
$$P = \frac{840 * 100}{5 * 8}$$
$$P = 2100$$
$$\text{Required Rate} = \frac{840 * 100}{5 * 2100}$$
$$R = 8\%$$

10. At 5% per annum simple interest, Rahul borrowed Rs. 500. What amount will he pay to clear the debt after 4 years?

A. 750

B. 700

C. 650

D. 600

Answer: Option **D**

Explanation:

We need to calculate the total amount to be paid by him after 4 years, So it will be Principal + simple interest.

So, $$=> 500 + \frac{500 * 5 * 4}{100} => Rs. 600$$

11. If a sum of money doubles itself in 8 years at simple interest, the rate percent per annum is

A. 12

B. 12.5

C. 13

D. 13.5

Answer: Option **B**

Explanation:

Let sum = x then Simple Interest = x

Rate = (100 * x) / (x * 8) = 12.5

12. A sum of money amounts to Rs 9800 after 5 years and Rs 12005 after 8 years at the same rate of simple interest. The rate of interest per annum is

A. 9%

B. 10%

C. 11%

D. 12%

Answer: Option **D**

Explanation:

We can get SI of 3 years = 12005 - 9800 = 2205

SI for 5 years = (2205/3)*5 = 3675 [so that we can get principal amount after deducting SI]

Principal = 12005 - 3675 = 6125

So Rate = (100*3675)/(6125*5) = 12%

13. A financier claims to be lending money at simple interest, But he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes.

A. 10.25%

B. 10%

C. 9.25%

D. 9%

Answer: Option **A**

Explanation:

Let the sum is 100.

As financier includes interest every six months., then we will calculate SI for 6 months, then again for six months as below:

SI for first Six Months = (100*10*1)/(100*2) = Rs. 5

Important: now sum will become 100+5 = 105

SI for last Six Months = (105*10*1)/(100*2) = Rs. 5.25

So amount at the end of year will be (100+5+5.25)
= 110.25

Effective rate = 110.25 - 100 = 10.25

14. What is the present worth of Rs. 132 due in 2 years at 5% simple interest per annum?

A. 110

B. 120

C. 130

D. 140

Answer: Option **B**

Explanation: Let the present worth be Rs.x
Then, S.I = Rs.(132 - x)

=> (x*5*2/100) = 132 - x
=> 10x = 13200 - 100x
=> 110x = 13200
x= 120

15. A sum of Rs 12,500 amounts to Rs. 15,500 in the 4 years at the rate of simple interest. Find the rate percent?

A. 6 %

B. 7 %

C. 8 %

D. 9 %

Answer: Option **A**

Explanation:

$$\text{S.I.} = \frac{P*R*T}{100}$$
$$=> R = \frac{S.I.*100}{P*T}$$

So, S.I = 15500 - 12500 = 3000.

$$=> R = \frac{3000*100}{12500*4} = 6\%$$

16. Sahil took a loan for 6 years at the rate of 5% per annum on Simple Interest, If the total interest paid was Rs. 1230, the principal was

A. 4100

B. 4200

C. 4300

D. 4400

Answer: Option **A**

Explanation:

$$\text{S.I.} = \frac{P*R*T}{100}$$
$$=> P = \frac{S.I.*100}{R*T}$$

By applying above formula we can easily solve this question, as we are already having the simple interest.

$$=> P = \frac{1230*100}{6*5}$$
$$=> P = 4100$$

17. Sachin borrows Rs. 5000 for 2 years at 4% p.a. simple interest. He immediately lends money to Rahul at 25/4% p.a. for 2 years. Find the gain of one year by Sachin.

A. 110.50

B. 111.50

C. 112.50

D. 113.50

Answer: Option **C**

Explanation: Two things need to give attention in this question, First we need to calculate gain for 1 year only.
Second, where we take money at some interest and lends at other, then we use to subtract each other to get result in this type of question. Lets solve this Simple Interest question now.

$$\text{Gain in 2 year} = \left[\left(5000 \times \frac{25}{4} \times \frac{2}{100}\right) - \left(\frac{5000 \times 4 \times 2}{100}\right)\right]$$
$$= (625 - 400) = 225$$
$$\text{So gain for 1 year} = \frac{225}{2} = 112.50$$

18. In how many years Rs 150 will produce the same interest at 8% as Rs. 800 produce in 3 years at 9/2% ?

A. 8

B. 9

C. 10

D. 11

Answer: Option **B**

Explanation: ***Clue***-Firstly we need to calculate the SI with prinical 800,Time 3 years and Rate 9/2%, it will be Rs. 108

Then we can get the Time as ,
Time = (100*108)/(150*8) = 9

19. Rs. 800 becomes Rs. 956 in 3 years at a certain rate of simple interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years?

A. Rs 1052

B. Rs 1152

C. Rs 1252

D. Rs 1352

Answer: Option **A**

Explanation: S.I. = 956 - 800 = Rs 156

$$R = \frac{156 * 100}{800 * 3}$$
$$R = 6\frac{1}{2}\%$$
$$\text{New Rate} = 6\frac{1}{2} + 4$$
$$= \frac{21}{2}\%$$
$$\text{New S.I.} = 800 \times \frac{21}{2} \times 3100$$
$$= 252$$

Now amount will be 800 + 252 = 1052

20. If A lends Rs. 3500 to B at 10% p.a. and B lends the same sum to C at 11.5% p.a., then the gain of B (in Rs.) in a period of 3 years is ?

A. Rs. 154.50

B. Rs. 155.50

C. Rs. 156.50

D. Rs. 157.50

Answer: Option **D**

Explanation:

We need to calculate the profit of B.
It will be, SI on the rate B lends - SI on the rate B gets

Gain of B

$$= \frac{3500 \times 11.5 \times 3}{100} - \frac{3500 \times 10 \times 3}{100} = 157.50$$

- **Compound Interest:** When the interest earned is reinvested instead of paying out, the interest in the next period is earned on the principal sum plus previously-accumulated interest. The total interest in this case is called the Compound Interest and is represented as C.I.
  Formula : $A = P [1 + R / 100]^n$
  where P = Principle, n years = Time, R % per annum = Rate and A is the total amount at the end of n years.

This is a tricky topic with complicated calculations sometimes. The hacks that follow would blow away your doubts though:

1. If at an annual rate of R% per annum the Compound Interest is to be calculated for n years,then

   1.1. On a half yearly basis then the new rate = R/2 and new time = 2n are to be changed before the calculations.

   1.2. For a quarterly basis, the new rate = R/4 and new time = 4n are to be changed in this manner.

2. When rates are different for different years, say X, Y, Z for 1st, 2nd, 3rd year respectively, then
   Amount = $P (1 + X /100) (1 + Y /100)(1 + Z /100)$

3. Present worth of Rs.C due n years hence is calculated by the formula,
   Present Worth = $C / (1 + R/100)^n$

Relation between Compound and Simple Interest

1. ONE YEAR OF SIMPLE INTEREST = ONE YEAR OF COMPOUND INTEREST.

2. For two years, the ratio of CI / SI = (200+r) / 200 and
   Difference between CI & SI = $PR^2/100^2$

3. For three years, the difference between CI & SI = $PR^2(300+R)/100^3$

- ✓ **Note: When the time is given in days or in years and days, 365 days are assumed to a year.**

When the time is given in months and days, 12 months form a year and 30 days a month.
While counting, the day on which the money is paid back is included but not the day on which it is borrowed, i.e., the first day is omitted.

Now you are ready to begin with the exercises on this topic. Start practicing below:

1. A man saves Rs 200 at the end of each year and lends the money at 5% compound interest. How much will it become at the end of 3 years.

A. Rs 662

B. Rs 662.01

C. Rs 662.02

D. Rs 662.03

Answer :

Option **C**

Explanation:

$$= [200(\frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}) + 200(\frac{21}{20} \times \frac{21}{20}) + 200(\frac{21}{20})] = 662.02$$

2. In what time will Rs.1000 become Rs.1331 at 10% per annum compounded annually

A. 2 Years

B. 3 Years

C. 4 Years

D. 5 Years

Answer :

Option

Explanation:

Principal = Rs.1000;
Amount = Rs.1331;
Rate = Rs.10%p.a.

Let the time be n years then,

$$1000(1+\frac{10}{100})^n = 1331$$
$$(\frac{11}{10})^n = \frac{1331}{1000}$$
$$(\frac{11}{10})^3 = \frac{1331}{1000}$$

So answer is 3 years

3. The difference between simple and compound interests compounded annually on a certain sum of money for 2 years at 4% per annum is Rs 1. Find the sum

A. Rs 600

B. Rs 625

C. Rs 650

D. Rs 675

E. Rs. 543.44 lakhs

F. Rs. 544.44 lakhs

G. Rs. 545.44 lakhs

H. Rs. 546.44 lakhs

Answer :

Option **F**

Explanation:

Let the Sum be P

$$
\begin{aligned}
S.I. &= \frac{P*4*2}{100} = \frac{2P}{25} \\
C.I. &= P(1+\frac{4}{100})^2 - P \\
&= \frac{676P}{625} - P \\
&= \frac{51P}{625} \\
\text{As, C.I. - S.I} &= 1 \\
=> \frac{51P}{625} - \frac{2P}{25} &= 1 \\
=> \frac{51P-50P}{625} &= 1 \\
P &= 625
\end{aligned}
$$

4. Find the compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.

A. Rs. 610

B. Rs. 612

C. Rs. 614

D. Rs. 616

Answer :

Option **B**

Explanation:

$$Amount = [7500 \times (1 + \frac{4}{100})^2]$$
$$= (7500 \times \frac{26}{25} \times \frac{26}{25})$$
$$= 8112$$

So compound interest = (8112 - 7500) = 612

5. What will be the compound interest on Rs after 3 years at the rate of 12 % per annum

A. Rs 10123.20

B. Rs 10123.30

C. Rs 10123.40

D. Rs 10123.50

Answer :

Option **A**

Explanation:

$$(25000 \times (1 + \frac{12}{100})^3)$$
$$=> 25000 \times \frac{28}{25} \times \frac{28}{25} \times \frac{28}{25}$$
$$=> 35123.20$$

So Compound interest will be 35123.20 - 25000 = Rs 10123.20

6. Albert invested amount of 8000 in a fixed deposit for 2 years at compound interest rate of 5 % per annum. How much Albert will get on the maturity of the fixed deposit.

A. Rs. 8510

B. Rs. 8620

C. Rs. 8730

D. Rs. 8820

Answer :

Option **D**

Explanation:

$$=> (8000 \times (1 + \frac{5}{100})^2)$$
$$=> 8000 \times \frac{21}{20} \times \frac{21}{20}$$
$$=> 8820$$

7. Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually

A. Rs 312

B. Rs 412

C. Rs 512

D. Rs 612

Answer :

Option **D**

Explanation:

Please apply the formula

$$Amount = P(1 + \frac{R}{100})^n$$
$$\text{C.I.} = \text{Amount - P}$$

8. Simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on Rs. 4000 for 2 years at 10% per annum. The sum placed on simple interest is

A. Rs 1650

B. Rs 1750

C. Rs 1850

D. Rs 1950

Answer :

Option **B**

Explanation:

$$
\begin{aligned}
C.I. &= (4000 \times (1+\frac{10}{100})^2 - 4000) \\
&= 4000 * \frac{11}{10} * \frac{11}{10} - 4000 \\
&= 840 \\
\text{So S.I.} &= \frac{840}{2} = 420 \\
\text{So Sum} &= \frac{S.I. * 100}{R * T} \\
&= \frac{420 * 100}{3 * 8} \\
&= Rs 1750
\end{aligned}
$$

9. Find the compound interest on Rs.16,000 at 20% per annum for 9 months, compounded quarterly

A. Rs 2520

B. Rs 2521

C. Rs 2522

D. Rs 2523

Answer :

Option **C**

Explanation:

Please remember, when we have to calculate C.I. quarterly then we apply following formula if n is the number of years

$$Amount = P(1+\frac{\frac{R}{4}}{100})^{4n}$$

Principal = Rs.16,000;
Time=9 months = 3 quarters;
Rate = 20%, it will be 20/4 = 5%

So lets solve this question now,

$$
\begin{aligned}
Amount &= 16000(1+\frac{5}{100})^3 \\
&= 18522 \\
C.I &= 18522 - 16000 = 2522
\end{aligned}
$$

10. The present worth of Rs.169 due in 2 years at 4% per annum compound interest is

A. Rs 155.25

B. Rs 156.25

C. Rs 157.25

D. Rs 158.25

Answer :

Option **B**

Explanation:

In this type of question we apply formula

$$Amount = \frac{P}{(1+\frac{R}{100})^n}$$
$$Amount = \frac{169}{(1+\frac{4}{100})^2}$$
$$Amount = \frac{169*25*25}{26*26}$$
$$Amount = 156.25$$

11. On a sum of money, simple interest for 2 years is Rs 660 and compound interest is Rs 696.30, the rate of interest being the same in both cases.

A. 8%

B. 9%

C. 10%

D. 11%

Answer :

Option **D**

Explanation:

Difference between C.I and S.I for 2 years = 36.30
S.I. for one year = 330.
S.I. on Rs 330 for one year = 36.30

So R% = \frac{100*36.30}{330*1} = 11%

12. A sum of money invested at compound interest to Rs. 800 in 3 years and to Rs 840 in 4 years. The rate on interest per annum is.

A. 4%

B. 5%

C. 6%

D. 7%

Answer :

Option **B**

Explanation:

S.I. on Rs 800 for 1 year = 40

Rate = (100*40)/(800*1) = 5%

13. What will be the difference between simple and compound interest @ 10% per annum on the sum of Rs 1000 after 4 years

A. Rs 62.10

B. Rs 63.10

C. Rs 64.10

D. Rs 65.10

Answer :

Option **C**

Explanation:

$$S.I. = \frac{1000 * 10 * 4}{100} = 400$$
$$C.I. = [1000(1 + \frac{10}{100})^4 - 1000]$$
$$= 464.10$$

So difference between simple interest and compound interest will be 464.10 - 400 = 64.10

14. The least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled is

A. 4 years

B. 5 years

C. 6 years

D. 7 years

E. Jayesh

F. Subodh

G. Alok

H. Bhagat

Answer :

Option

Explanation:

As per question we need something like following

$$P(1+\frac{R}{100})^n > 2P$$
$$(1+\frac{20}{100})^n > 2$$
$$(\frac{6}{5})^n > 2$$
$$\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5} > 2$$

So answer is 4 years

15. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years

A. 3%

B. 4%

C. 5%

D. 6%

Answer :

Option **D**

Explanation:

Let Rate will be R%

$$1200(1+\frac{R}{100})^2 = \frac{134832}{100}$$
$$(1+\frac{R}{100})^2 = \frac{134832}{120000}$$
$$(1+\frac{R}{100})^2 = \frac{11236}{10000}$$
$$(1+\frac{R}{100}) = \frac{106}{100}$$
$$=> R = 6\%$$

16. If the simple interest on a sum of money for 2 years at 5% per annum is Rs.50, what will be the compound interest on same values

A. Rs.51.75

B. Rs 51.50

C. Rs 51.25

D. Rs 51

Answer :

Option **C**

Explanation:

$$S.I. = \frac{P*R*T}{100}$$
$$P = \frac{50*100}{5*2} = 500$$
$$Amount = 500(1+\frac{5}{100})^2$$
$$500(\frac{21}{20}*\frac{21}{20})$$
$$= 551.25$$
$$C.I. = 551.25 - 500 = 51.25$$

17. Effective annual rate of interest corresponding to nominal rate of 6% per annum compounded half yearly will be

A. 6.09%

B. 6.10%

C. 6.12%

D. 6.14%

Answer :

Option **A**

Explanation:

Let the amount Rs 100 for 1 year when compounded half yearly, n = 2, Rate = 6/2 = 3%

$$Amount = 100(1 + \frac{3}{100})^2 = 106.09$$

Effective rate = (106.09 - 100)% = 6.09%

Mixture and Alligations

A very important topic under SSC, Mixture and Alligations is one of the toughest and trickiest topics in mathematics itself. This is because you get a variety of questions which vary hugely in their approach and solution.

The only good thing with difficult things is that only few people attempt to do them. And hence this section becomes a game changer if you maximise the attempts with the right approach.

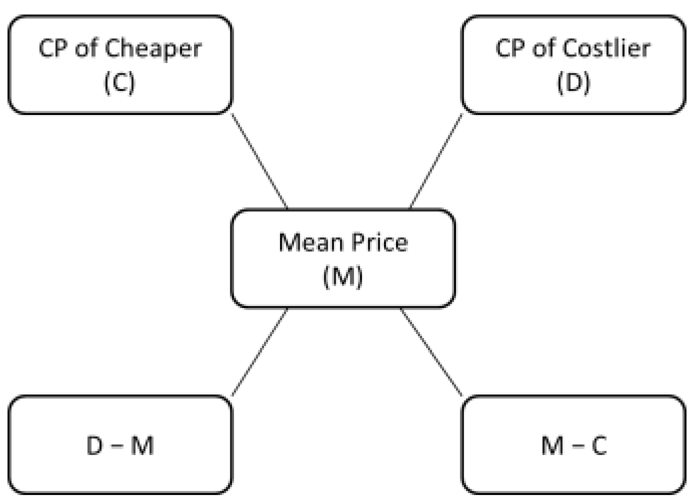
Alligation is a rule that helps us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price. In simpler words, it is a process or rule for the solution of problems concerning the compounding or mixing of ingredients differing in price or quality.

Formula : If two ingredients X and Y with prices C and D respectively are mixed and the mean price of the resultant mixture is M then the ratio (R) in which the ingredients are mixed is given by, the rule of allegation,

R = (D - M) / (M - C)

M = Mean price = The cost price of a unit quantity of the resultant mixture.

The above formula can be understood with the help of this diagram below and it is easier to remember too:



Thus, the required ratio will be,

R = Cheaper Quantity: Costlier Quantity = (D – M): (M – C)

It seems difficult till here but once you read these explained concepts and hacks, we believe you will have no issues thereafter:

- If n different vessels of equal size are filled with the mixture of P and Q in the ratio $p_1 : q_1$, $p_2 : q_2$, ……, $p_n : q_n$ and content of all these vessels are mixed in a large vessel, then

$$\frac{\text{Quantity of P}}{\text{Quantity of Q}} = \frac{\frac{p_1}{p_1 + q_1} + \frac{p_2}{p_2 + q_2} + \cdots + \frac{p_n}{p_n + q_n}}{\frac{q_1}{p_1 + q_1} + \frac{q_2}{p_2 + q_2} + \cdots + \frac{q_n}{p_n + q_n}}$$

- If n different vessels of sizes $x_1, x_2, \ldots, x_n$ are filled with the mixture of P and Q in the ratio $p_1 : q_1$, $p_2 : q_2$, ……, $p_n : q_n$ and content of all these vessels are mixed in a large vessel, then

$$\frac{\text{Quantity of P}}{\text{Quantity of Q}} = \frac{\frac{p_1 x_1}{p_1 + q_1} + \frac{p_2 x_2}{p_2 + q_2} + \cdots + \frac{p_n x_n}{p_n + q_n}}{\frac{q_1 x_1}{p_1 + q_1} + \frac{q_2 x_2}{p_2 + q_2} + \cdots + \frac{q_n x_n}{p_n + q_n}}$$

- If a vessel contains "x" litres of liquid A and if "y" litres be withdrawn and replaced by liquid B, then if "y" litres of the mixture be withdrawn and replaced by liquid B, and the operation is repeated 'n' times in all, then

$$\frac{\text{Quantity of liquid A after } n^{\text{th}} \text{ operation}}{\text{Initial quantity of liquid of A}} = \left[\frac{x - y}{x}\right]^n = \left[1 - \frac{y}{x}\right]^n$$

- If p gram of ingredient solution has a% ingredient in it then to increase the ingredient content to b% in the solution, the quantity of ingriient needed is calculated by the formula,

$$\text{Quantity of ingredient need to be added} = \frac{p(b-a)}{100-b}$$

- For mixture of more than two elements, the questions may seem a little tricky at first, but it is a similar concept applied repeteadly.In order to calculate the final ratio of ingredients when a mixture contains more than two ingredients, you need to follow these steps:

1. **Step 1** - Take two ingredients such that first ingredient is LOWER than the mean price and the other one is HIGHER than the mean price.
2. **Step 2** - Calculate the ratio of ingredients.
3. **Step 3** - Repeat step 2 for all possible pairs.
4. **Step 4**- Final ratio is the ratio obtained from step 2 (if an ingredient is common in the ratios, add values for this particular ingredient for the final answer)

Note: It's best to simplify the intermediate ratios at the end, else the answer may vary.

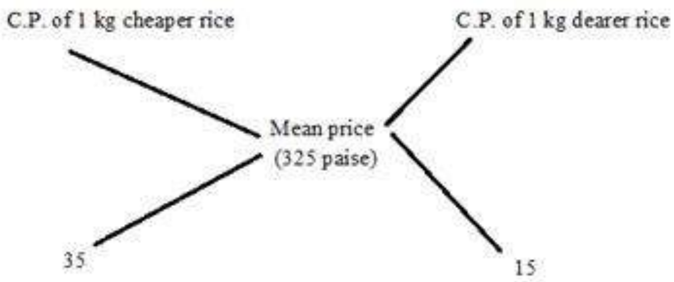
This hack only sounds complex and once you will apply it in the questions and exercises, this section would be no big deal for you.

Ex: Mixture containing 3
Ex: Mixture containing 4

Solve the questions that follow:

Ex1: In what proportion must rice at Rs3.10 per kg be mixed with rice at Rs3.60 per kg, so that the mixture be worth Rs3.25 a kg?

Solution:



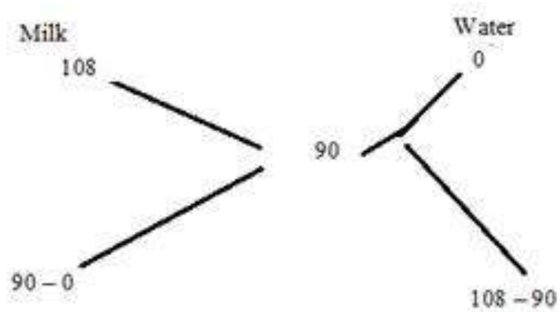
$$\frac{\text{Quantity of cheaper rice}}{\text{Quantity of dearer rice}} = \frac{35}{15} = \frac{7}{3}$$

By the alligation rule : They must be mixed in the ratio 7 :3.

Ex2: A mixture of certain quantity of milk with 16 liters of water is worth 90 P per liter. If pure milk be worth Rs1.08 per liter, how much milk is there in the mixture?

Solution:

The mean value is 90 P and price of water is 0 P.

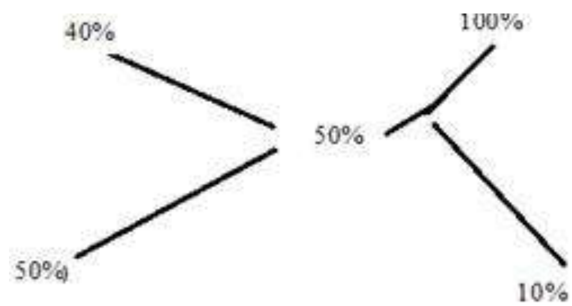


By the Alligation Rule, milk and water are in the ratio of 5 :1. Quantity of milk in the mixture = 5 ×16 = 80 liters.

Ex3: 300 gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?

Solution:

The existing solution has 40% sugar, and sugar is to be mixed; so the other solution has 100% sugar. So, by alligation method; the two mixture should be added in the ratio 5 :1



Therefore, required sugar = $\frac{300}{5}\times 1 = 60gm$

Direct formula:

Quantity of sugar added = $\frac{\text{Solution (required\% value - present\% value)}}{(100 - \text{required\% value})}$

In this case,

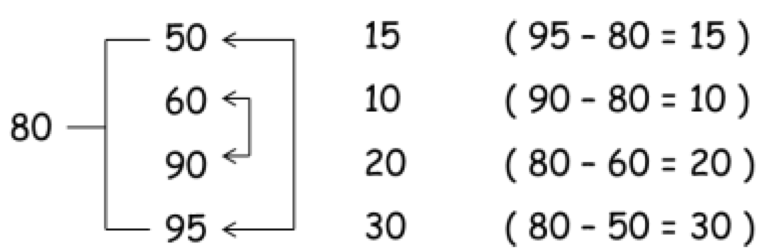
Ans = $\frac{300(50-40)}{100-50} = 60gms$

Example:

How must a shop owner mix 4 types of rice worth Rs 95, Rs 60, Rs 90 and Rs 50 per kg so that he can make the mixture of these sugars worth Rs 80 per kg?
Sol:

Here the prices of sugars are 95, 60, 90 and 50.
And the mean price is 80.

Now read the above paragraph and the image given below to understand this method.



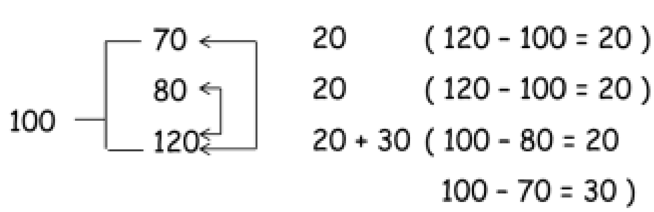
So the proportion of sugar is

50 : 60 : 90 : 95 = 15 : 10 : 20 : 30 or
50 : 60 : 90 : 95 = 3 : 2 : 4 : 6

Example

In what ratio must a person mix three kind of tea each of which has a price of 70, 80 and 120 rupees per kg, in such a way that the mixture costs him 100 rupees per kg?
Sol:

Here the prices of tea are 70, 80 and 120. And mean price is 100, so



So the proportion of tea is; **70 : 80 : 120 = 20 : 20 : 50 or**
70 : 80 : 120 = 2 : 2 : 5

Example:
Three equal buckets containing the mixture of milk and water are mixed into a bigger bucket. If the proportion of milk and water in the glasses are 3:1, 2:3 and 4:2 then find the proportion of milk and water in the bigger bucket.
Sol:

Let's say P stands for milk and Q stands for water,
So, $p_1:q_1 = 3:1$
$p_2:q_2=2:3$
$p_3 : q_3=4:2$

$$\frac{\text{Quantity of P}}{\text{Quantity of Q}} = \frac{\frac{p_1}{p_1+q_1}+\frac{p_2}{p_2+q_2}+\cdots+\frac{p_n}{p_n+q_n}}{\frac{q_1}{p_1+q_1}+\frac{q_2}{p_2+q_2}+\cdots+\frac{q_n}{p_n+q_n}}$$

$$\frac{\text{Quantity of Milk}}{\text{Quantity of Water}} = \frac{\frac{3}{3+1}+\frac{2}{2+3}+\frac{4}{4+2}}{\frac{1}{3+1}+\frac{3}{2+3}+\frac{2}{4+2}}$$

$$\frac{\text{Quantity of Milk}}{\text{Quantity of Water}} = \frac{109}{71}$$

So in bigger bucket,
Milk : Water = 109 : 71

Example:

Three buckets of size 2 liter, 4 liter and 5 liter containing the mixture of milk and water are mixed into a bigger bucket. If the proportion of milk and water in the glasses are 3:1, 2:3 and 4:2 then find the proportion of milk and water in the bigger bucket.

Sol: Let's say P stands for milk and Q stands for water,
So, $p_1:q_1 = 3:1$, $x_1 = 2$
$p_2:q_2=2:3$, $x_2 = 4$
$p_3 : q_3=4:2$ $x_3 = 5$, so

$$\frac{\text{Quantity of P}}{\text{Quantity of Q}} = \frac{\frac{p_1 x_1}{p_1 + q_1} + \frac{p_2 x_2}{p_2 + q_2} + \cdots + \frac{p_n x_n}{p_n + q_n}}{\frac{q_1 x_1}{p_1 + q_1} + \frac{q_2 x_2}{p_2 + q_2} + \cdots + \frac{q_n x_n}{p_n + q_n}}$$

$$\frac{\text{Quantity of Milk}}{\text{Quantity of Water}} = \frac{\frac{3 \times 2}{3 + 1} + \frac{2 \times 4}{2 + 3} + \frac{4 \times 5}{4 + 2}}{\frac{1 \times 2}{3 + 1} + \frac{3 \times 4}{2 + 3} + \frac{2 \times 5}{4 + 2}}$$

$$\frac{\text{Quantity of Milk}}{\text{Quantity of Water}} = \frac{193}{137}$$

So in bigger bucket,
Milk : Water = 193 : 137

Example:

A container is containing 80 liter of wine. 8 liter of wine was taken out from this container and replaced by water. This process was further repeated two times. How much wine is there in the container now?
Sol:

Here x = 80, y = 8 and n = 3, so

$$\frac{\text{Quantity of wine after } 3^{rd} \text{ operation}}{80} = \left[\frac{80 - 8}{80}\right]^3$$

Quantity of wine after 3rd operation = 58.32 litres.

Example:

125 liter of mixture of milk and water contains 25% of water. How much water must be added to it to make water 30% in the new mixture?

Sol:

Let's say p = 125, b = 30, a = 25

So from the equation

$$\text{Quantity of water need to be added} = \frac{125(30-25)}{100-30}$$

Quantity of water need to be added = 8.92 liter.

Q1. A jar contained a mixture of two liquids A and B in the ratio 4:1. When 10 litres of the mixture was taken out and 10 litres of liquid B was poured into the jar, this ratio became 2:3. The quantity of liquid A contained in the jar initially was

a) 4 litres b) 8 litres c) 16 litres d) 40 litres

Q2. In a mixture of 75 llitres, the ratio of milk to water is 2:1. The amount of water to be further added to the mixture so as to make the ratio of the milk to water 1:2 will be

a) 45 litres b) 60 litres c) 75 litres d) 80 litres

Q3. Two types of alloys posses gold and silver in the ratio of 7:22 and 21:37. In what ratio should these alloys be mixed so as to have a new alloy in which gold and silver would exist in the ratio 25:62?

a) 13:8 b) 8:13 c) 13:12 d) 6:9

Q4. A mixture of 40 litres of milk and water contains 10% of water. How much water must be added to make the water 20% in the new mixture?

a) 10 litres b) 7 litres c) 5 litres d) 3 litres

Q5. A mixture contains wine and water in the ratio 3:2 and another mixture contains them in the ratio 4:5. How many litres of latter must be mixed with 3 litres of the former so that the resulting mixture may contain equal quantities of wine and water?

a) 27/5 litres b) 27/3 litres c) 9/2 litres d) 15/4 litres

Alligation Questions

Q6. The ratio in which tea costing Rs 192 per kg is to be mixed with tea costing Rs 150 per kg so that the mixed tea, when sold for rs 194.40 per kg, gives a profit of 20%, is

a) 2:5 b) 3:5 c) 5:3 d) 5:2

Q7. A shopkeeper bought 15 kg of rice at the rate of Rs 29 per kg and 25 kg of rice at the rate of rs 20 per kg. He sold the mixture of both types of rice at the rate of rs 27 per kg. His profit in this transaction is

a) Rs 125 b) Rs 150 c) Rs 140 d) Rs 145

Q8. A can contains a mixture of two liquids A and B in the ratio 7:5. When 9 litres of mixture are drained off and the Can is filled with B, the ratio of A and B becomes 7:9. How many litres of liquid A was contained by the Can initially?

a) 10 b) 20 c) 21 d) 25

Q9. A container contains 60 kg of milk. From this container 6 kg of milk was taken out and replaced by water. This process was repeated further two times. The amount of milk left in the container is

a) 34.24 kg b) 39.64 kg c) 43.74 kg d) 47.6 kg

Q10. Two vessels A and B contain milk and water mixed in the ratio 8:5 and 5:2 respectively. The ratio in which these two mixtures be mixed to get a new mixture containing 900/13% milk is:

a) 3:5 b) 5:2 c) 5:7 d) 2:7

Alligation Questions: Answer key

1:d , 2:c , 3:a , 4:c , 5:a , 6:a , 7:d , 8:c , 9:c , 10:d

Time and Work

Time here implies time taken to complete an assigned job and work is the amount of work done actually. Time and work can both be individual as well as collective.

Work and Time is a tricky but scoring topic. This is important because every year many questions appear from this section in SSC which become difficult only when you do not understand the question. The most important thing in this section is understand the question and after that any of these hacks can be used to answer that. Use these while practicing to be comfortable with the type of questions asked and the hacks needed to answer them in SSC CGL.

- If 1/n of work is done by A in one day, then A will take n days to complete the full work(ONE).

- If A can do a particular amount of work in X days and B can do the same work in Y days, then A and B together will do the same work in XY/(X+Y) days.

- If A, B and C can complete a work in X, Y and Z days working alone respectively, then A,B and C will together complete the work in XYZ/(XY+YZ+ZX) days.

- If A does 1/nth of a work in t hours, then to complete the full work A will take n x t hours.

- If A and B together finish a piece of work in X days, B and C together in Y days and C and A together in Z days, then

a) A, B and C together will finish the job in (2XYZ/XY+YZ+ZX) days.

b) A will finish the job in (2XYZ/XY+YZ- ZX) days.

c) B will finish the job in (2XYZ/ZX+XY- YZ) days.

d) C will finish the job in (2XYZ/ZX+YZ- XY) days.

- If A can finish a work in X days and B is n times efficient than A, then time taken by A and B working together to complete the work is X/(1+n).

- If A and B working together finish a work in X days and B is n times efficient than A, then time taken by A working alone to complete the work is (n+1)X and B working alone to complete the work is (n+1/n)X.

Practice Questions:

Q1. Shambhu is twice as good as workman as Bablu and together they finish a piece of work in 18 days. Find the total number of days in which Bablu can finish the work.

a) 27 days

b) 54 days

c) 56 days

d) 68 days

Answer: Option **B**

Explanation: As per question, Shambhu does twice the work as done by Bablu. So A:B = 2:1. Also (Shambhu+Bablu) one day work is 1/18

To get days in which Bablu will finish the work, lets calculate work done by Bablu in 1 day =(118 ∗13)=154. So Bablu will complete the work in 54 days

Q2. Ritu can complete a piece of work in 5 days, but with the help of her son she can do it in 3 days. Find the time taken by the son alone to complete the work.

a) 7.5 days

b) 13 days

c) 11 days

d) 9 days

Answer: Option **A**

Explanation:- In this type of question, where we have one person work and together work done. Then we can easily get the other person work just by subtracting them. As,

Son's one day work = (1/3 −1/5)=(5−3/15)=2/15

So son will do whole work in 2/15 days

which is = 7.5 *days*

Q3. Two pipes can fill the cistern in 10 hr and 12 hr respectively, while the third pipe can empty it in 20 hr. Simultaneously, if all the pipes are opened then the cistern will be filled in

a) 7.5 hr

b) 8 hr

c) 5 hr

d) 10 hr

Answer – **(A)**

Explanation:- Work done by all the tanks working together in 1 hour.

⇒ 1/10+1/12-1/20=2/15. Hence, tank will be filled in 15/2= 7.5 hour.

Q4. Mr. Chawla is on tour and he has Rs 360 for his expenses. If he exceeds his tour by 4 days he must cut down daily expenses by Rs 3. The number of days of Mr. Chawla's tour programme is

a) 28 Days

b) 24 Days

c) 22 Days

d) 20 Days

Answer – **(D)**

Explanation:- Let Mr. Chawla under takes a tour of x days.

Then, expenses for each day =360/x360/x+4=360/x−3

⇒ x=20 and −24. Hence, x= 20 days.

More Questions:

1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do a job. How long should it take both A and B, working together to do same job.

A. $\frac{4}{9}$

B. $2\frac{4}{9}$

C. $3\frac{4}{9}$

D. $4\frac{4}{9}$

Answer :

Option **D**

Explanation:

In this type of questions, first we need to calculate 1 hours work, then their collective work as,

A's 1 hour work is 1/8
B's 1 hour work is 1/10

(A+B)'s 1 hour work = 1/8 + 1/10
= 9/40

So both will finish the work in 40/9 hours
=

$$4\frac{4}{9}$$

2. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete the work, how much time shall B take to do it

A. 40 days

B. 35 days

C. 30 days

D. 25 days

Answer :

Option **C**

Explanation:

Suppose B takes x days to do the work.
As per question A will take

$$2 * \frac{3}{4} * x = \frac{3x}{2} days$$

(A+B)s 1 days work= 1/18
1/x + 2/3x = 1/18 or x = 30 days

3. A can do a piece of work in 15 days and B alone can do it in 10 days. B works at it for 5 days and then leaves. A alone can finish the remaining work in

A. 5 days

B. 6 days

C. 7.5 days

D. 8.5 days

Answer :

Option **C**

Explanation:

B's 5 days work =

$$\begin{aligned} \frac{1}{10} * 5 &= \frac{1}{2} \\ \text{Remaining work } &= 1 - \frac{1}{2} \\ &= \frac{1}{2} \\ \text{A can finish work } &= 15 * \frac{1}{2} \\ &= 7.5 days \end{aligned}$$

4. A is twice as good as workman as B and together they finish a piece of work in 18 days. In how many days will B alone finish the work.

A. 27 days

B. 54 days

C. 56 days

D. 68 days

Answer :

Option **F**

Explanation:

As per question, A do twice the work as done by B.
So A:B = 2:1
Also (A+B) one day work = 1/18

To get days in which B will finish the work, lets calculate work done by B in 1 day =

$$= \left(\frac{1}{18} * \frac{1}{3}\right)$$
$$= \frac{1}{54}$$

[Please note we multiplied by 1/3 as per B share and total of ratio is 1/3]

So B will finish the work in 54 days

5. A is thrice as good a workman as B and takes 10 days less to do a piece of work than B takes. B alone can do the whole work in

A. 15 days

B. 10 days

C. 9 days

D. 8 days

Answer :

Option **A**

Explanation:

Ratio of times taken by A and B = 1:3
Means B will take 3 times which A will do in 1 time

If difference of time is 2 days, B takes 3 days
If difference of time is 10 days, B takes (3/2) * 10 =15 days

6. A piece of work can be done by 6 men and 5 women in 6 days or 3 men and 4 women in 10 days. It can be done by 9 men and 15 women in how many days ?

A. 3 days

B. 4 days

C. 5 days

D. 6 days

Answer :

Option **A**

Explanation:

To calculate the answer we need to get 1 man per day work and 1 woman per day work.

Let 1 man 1 day work =x
and 1 woman 1 days work = y.
=> 6x+5y = 1/6
and 3x+4y = 1/10
On solving, we get x = 1/54 and y = 1/90

(9 men + 15 women)'s 1 days work =
(9/54) + (15/90) = 1/3

9 men and 15 women will finish the work in 3 days

7. A can do a piece of work in 4 hours . A and C together can do it in just 2 hours, while B and C together need 3 hours to finish the same work. In how many hours B can complete the work ?

A. 10 hours

B. 12 hours

C. 16 hours

D. 18 hours

Answer :

Option **B**

Explanation:

Work done by A in 1 hour = 1/4

Work done by B and C in 1 hour = 1/3

Work done by A and C in 1 hour = 1/2

Work done by A,B and C in 1 hour = (1/4)+(1/3) = 7/12

Work done by B in 1 hour = (7/12)–(1/2) = 1/12

So together they can complete work in 6 days.

8. A can finish a work in 18 days and B can do same work in half the time taken by A. then working together, what part of same work they can finish in a day

A. 1\5

B. 1\6

C. 1\7

D. 1\8

Answer :

Option **B**

Explanation:

Please note in this question, we need to answer part of work for a day rather than complete work. It was worth mentioning here because many do mistake at this point in hurry to solve the question

So lets solve now,
A's 1 day work = 1/18
B's 1 day work = 1/9 [because B take half time than A]

(A+B)'s one day work =

$$\left(\frac{1}{18}+\frac{1}{9}\right)$$
$$=\left(\frac{1+2}{18}\right)$$
$$=\frac{1}{6}$$

So in one day 1/6 work will be done.

9. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work ?

A. 5 days

B. 6 days

C. 7 days

D. 8 days

Answer :

Option **B**

Explanation:

Firstly we will find 1 day work of both A and B, then by adding we can get collective days for them,
So,
A's 1 day work = 1/10
B's 1 day work = 1/15

(A+B)'s 1 day work =

$$\left(\frac{1}{10}+\frac{1}{15}\right) = \left(\frac{3+2}{30}\right) = \frac{1}{6}$$

10. 5 men and 2 boys working together can do four times as much work as a man and a boy. Working capacity of man and boy is in the ratio

A. 1:2

B. 1:3

C. 2:1

D. 2:3

Answer :

Option **C**

Explanation:

Let 1 man 1 day work = x
1 boy 1 day work = y

then 5x + 2y = 4(x+y)
=> x = 2y
=> x/y = 2/1
=> x:y = 2:1

11. To complete a work A and B takes 8 days, B and C takes 12 days, A,B and C takes 6 days. How much time A and C will take

A. 24 days

B. 16 days

C. 12 days

D. 8 days

Answer :

Option **D**

Explanation:

A+B 1 day work = 1/8
B+C 1 day work = 1/12
A+B+C 1 day work = 1/6

We can get A work by (A+B+C)-(B+C)
And C by (A+B+C)-(A+B)

So A 1 day work =

$$\frac{1}{6}-\frac{1}{12} = \frac{1}{12}$$

12. A man can do a piece of work in 5 days, but with the help of his son he can do it in 3 days. In what time can the son do it alone ?

A. $7\frac{1}{2}days$

B. $6\frac{1}{2}days$

C. $5\frac{1}{2}days$

D. $4\frac{1}{2}days$

Answer :

Option **A**

Explanation:

In this type of question, where we have one person work and together work done. Then we can easily get the other person work just by subtracting them. As,

Son's one day work =

$$\left(\frac{1}{3}-\frac{1}{5}\right) = \left(\frac{5-3}{15}\right) = \frac{2}{15}$$

So son will do whole work in 15/2 days which is =

$$7\frac{1}{2}days$$

13. 4 men and 6 women finish a job in 8 days, while 3 men and 7 women finish it in 10 days. In how many days will 10 women working together finish it ?

A. 30 days

B. 40 days

C. 50 days

D. 60 days

Answer :

Option **B**

Explanation:

Let 1 man's 1 day work = x
and 1 woman's 1 days work = y.
Then, 4x + 6y = 1/8
and 3x+7y = 1/10
solving, we get y = 1/400 [means work done by a woman in 1 day]

10 women 1 day work = 10/400 = 1/40

10 women will finish the work in 40 days

14. A alone can do a piece of work in 6 days and B alone in 8 days. A and B undertook to do it for Rs. 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C

A. Rs. 300

B. Rs. 400

C. Rs. 500

D. Rs. 600

Answer :

Option **B**

Explanation:

C's 1 day's work =

$$\begin{aligned} \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) \\ = \left(\frac{1}{3} - \frac{7}{24}\right) \\ = \frac{1}{24} \\ A : B : C = \frac{1}{6} : \frac{1}{8} : \frac{1}{24} \\ = 4 : 3 : 1 \\ C'sShare = \frac{1}{8} * 3200 \\ = 400 \end{aligned}$$

If you are confused how we multiplied 1/8, then please study ratio and proportion chapter, for small information, it is the C ratio divided by total ratio.

15. A and B can together complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work ?

A. 4 days

B. 5 days

C. 6 days

D. 7 days

Answer :

Option **C**

Explanation:

(A+B)'s 1 day work = 1/4

A's 1 day work = 1/12

B's 1 day work =

$$\left(\frac{1}{4} - \frac{1}{12}\right) = \frac{3-1}{12} = \frac{1}{6}$$

So B alone can complete the work in 6 days

16. 10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?

A. 6 days

B. 7 days

C. 8 days

D. 9 days

Answer :

Option **F**

Explanation:

1 woman's 1 day's work = 1/70
1 Child's 1 day's work = 1/140
5 Women and 10 children 1 day work =

$$\left(\frac{5}{70}+\frac{10}{140}\right) = \frac{1}{7}$$

So 5 women and 10 children will finish the work in 7 days.

17. A tyre has two punctures. The first puncture alone would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat ?

A. $3\frac{1}{5}min$

B. $3\frac{2}{5}min$

C. $3\frac{3}{5}min$

D. $3\frac{4}{5}min$

Answer :

Option **C**

Explanation:

Do not be confused, Take this question same as that of work done question's. Like work done by 1st puncture in 1 minute and by second in 1 minute.
Lets Solve it:

1 minute work done by both the punctures =

$$\left(\frac{1}{9}+\frac{1}{6}\right) = \left(\frac{5}{18}\right)$$

So both punctures will make the type flat in

$$\left(\frac{18}{5}\right)mins = 3\frac{3}{5}mins$$

18. A completes 80% of a work in 20 days. Then B also joins and A and B together finish the remaining work in 3 days. How long does it need for B if he alone completes the work?

A. $35\frac{1}{2}$

B. $36\frac{1}{2}$

C. $37\frac{1}{2}$

D. $38\frac{1}{2}$

Answer :

Option **C**

Explanation:

Work done by A in 20 days = 80/100 = 8/10 = 4/5

Work done by A in 1 day = (4/5) / 20 = 4/100 = 1/25 --- (1)

Work done by A and B in 3 days = 20/100 = 1/5 (Because remaining 20% is done in 3 days by A and B)

Work done by A and B in 1 day = 1/15 ---(2)

Work done by B in 1 day = 1/15 – 1/25 = 2/75

=> B can complete the work in 75/2 days = 37 (1/2) days

19. A can do a job in 16 days, B can do same job in 12 days. With the help of C they did the job in 4 days. C alone can do the same job in how many days ?

A. $6\frac{1}{2}days$

B. $7\frac{1}{2}days$

C. $8\frac{3}{5}days$

D. $9\frac{3}{5}days$

Answer :

Option **D**

Explanation:

In this question we having, A's work, B's work and A+B+C work. We need to calculate C's work.
We can do it by,
(A+B+C)'s work - (A's work + B's work).

Let's solve it now:

C's 1 day work =

$$\frac{1}{4} - \left(\frac{1}{16} + \frac{1}{12}\right)$$
$$= \left(\frac{1}{4} - \frac{7}{48}\right)$$
$$= \frac{5}{48}$$

So C can alone finish the job in 48/5 days,
Which is =

$$9\frac{3}{5}days$$

20. A alone can complete a work in 16 days and B alone can do in 12 days. Starting with A, they work on alternate days. The total work will be completed in

A. $13\frac{1}{4}$

B. $13\frac{1}{2}$

C. $13\frac{3}{4}$

D. $13\frac{4}{4}$

Answer :

Option **C**

Explanation:

A's 1 day work = 1/16
B's 1 day work = 1/12

As they are working on alternate day's
So their 2 days work = (1/16)+(1/12)
= 7/48

[here is a small technique, Total work done will be 1, right, then multiply numerator till denominator, as 7*6 = 42, 7*7 = 49, as 7*7 is more than 48, so we will consider 7*6, means 6 pairs]

Work done in 6 pairs = 6*(7/48) = 7/8

Remaining work = 1-7/8 = 1/8

On 13th day it will A turn,
then remaining work = (1/8)-(1/16) = 1/16

On 14th day it is B turn,

1/12 work done by B in 1 day

1/16 work will be done in (12*1/16) = 3/4 day

So total days =

$$13\frac{3}{4}$$

It may be a bit typical question, but if are not getting it in first try then give it a second try. Even not, then comment for explanation for this. We will be happy to help you.

21. A can do a certain job in 25 days which B alone can do in 20 days. A started the work and was joined by B after 10 days. The number of days taken in completing the wotk were ?

A. $14\frac{2}{3}kmph$

B. $15\frac{2}{3}kmph$

C. $16\frac{2}{3}kmph$

D. $17\frac{2}{3}kmph$

Answer :

Option **C**

Explanation:

Work done by A in lo days = (1/25) *10 = 2/5
Remaining work = 1 - (2/5) = 3/5
(A+B)s 1 days work = (1/25) + (1/20) = 9/100

9/100 work is done by them in 1 day.
hence 3/5 work will be done by them in (3/5)*(100/9)
= 20/3days.

Total time taken = (10 + 20/3) = 16*(2/3) days

Speed, Time and Distance

Speed is a measure of how fast or slow an object moves or changes its position. Numerically, Speed = Distance / Time

One of the most important topics under SSC, this topic requires the understanding of the question and a logical approach to be answered correctly. This section has questions which make or break an exam as either they will get sorted easily if you find the right approach but if you go wrong you are bound to waste a minute or two on these questions which is a major loss keeping the time constraints in hand under SSC CGL. Regular practice of this topic is essential along with the hacks and examples provided below if SSC is your goal this time. Read these :

- 1 Kmph = (5/18) m/s
- 1 m/s = (18/5) Kmph
- Speed(S) = Distance(d)/Time(t)
- Average Speed = Total distance / Total Time = $(d_1+d_2)/(t_1+t_2)$
- When $d_1 = d_2$, Average speed = $2S_1S_2 / (S_1+S_2)$, where S_1 and S_2 are the speeds for covering d_1 and d_2 respectively.
- When $t_1 = t_2$, Average speed = $(S_1+S_2) / 2$, where S_1 and S_2 are the speeds during t_1 and t_2 respectively.
- Relative speed when moving in opposite direction is $S_1 + S_2$.
- Relative speed when moving in same direction is $S_1 - S_2$.

- A person goes certain distance (A to B) at a speed of S_1 kmph and returns back (B to A) at a speed of S_2 kmph. If he takes T hours in all, the distance between A and B is $T(S_1 S_2 / S_1 + S_2)$.

- When two trains of lengths l_1 and l_2 respectively travelling at the speeds of s_1 and s_2 respectively cross each other in time t, then the equation is given as,
$s_1 + s_2 = (l_1 + l_2) / t$.

- When a train of lengths l_1 travelling at a speed s_1 overtakes another train of length l_2 travelling at speed s_2 in time t, then the equation is given as $s_1 - s_2 = (l_1 + l_2) / t$.

- When a train of lengths l_1 travelling at a speed s_1 crosses a platform /bridge/ tunnel of length l_2 in time t, then the equation is given as $s_1 = (l_1+l_2) / t$

- When a train of lengths l travelling at a speed s crosses a pole/pillar/flag post in time t, then the equation is given as $s = l/t$.

- If two persons A and B start at the same time from two points P and Q towards each other and after crossing they take T_1 and T_2 hours in reaching Q and P respectively, then (A's speed) / (B's speed) = $\sqrt{T_2} / \sqrt{T_1}$.

These hacks when applied to the questions below, will enhance your understanding as well as approach towards this topic.

Solve the questions that follow:

1. A man is walking at the rate of 5 km/hr crosses a bridge in 15 minutes. The length of the bridge is

A. 1000 meters

B. 1050 meters

C. 1200 meters

D. 1250 meters

Answer :

Option **D**

Explanation:

We need to get the answer in meters. So we will first of change distance from km/hour to meter/sec by multiplying it with 5/18 and also change 15 minutes to seconds by multiplying it with 60.

$$Speed = 5 * \frac{5}{18} = \frac{25}{18} m/sec$$
$$Time = 15 * 60 seconds = 900 seconds$$
$$Distance = Time * Speed$$
$$Distance = \frac{25}{18} * 900 = 1250 meter$$

2. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. What is the duration of the flight ?

A. 3 hours

B. 2.4 hours

C. 1.4 hours

D. 1 hour

Answer :

Option **D**

Explanation:

Let the duration of the flight be x hours.
Then

$$\frac{600}{x} - \frac{600}{x + \frac{1}{2}} = 200$$
$$\frac{600}{x} - \frac{1200}{2x+1} = 200$$
$$x(2x+1) = 3$$
$$2x^2 + x - 3 = 0$$
$$=> (2x+3)(x-1) = 0$$

Neglecting the negative value for x we get x = 1

3. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr

A. $1\frac{1}{5} min$

B. $2\frac{1}{5} min$

C. $3\frac{1}{5} min$

D. $4\frac{1}{5} min$

Answer :

Option **A**

Explanation:

We know that,

$$Time = \frac{Distance}{Speed}$$

$$Speed = 20 \text{ km/hr} = 20 * \frac{5}{18} m/sec$$
$$= \frac{50}{9} m/sec$$
$$\text{Time} = \left(400 * \frac{9}{50}\right)$$
$$= 72 sec = 1\frac{1}{5} min$$

4. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minutes ?

A. 50 meter

B. 100 meter

C. 110 meter

D. None of above

Answer :

Option **B**

Explanation:

Relative speed of the thief and policeman = (11 – 10) km/hr = 1 km/hr

Distance covered in 6 minutes =

$$\frac{1}{60} * 6 = \frac{1}{10}$$
$$= 100 meters$$

So distance between them after 6 minutes = 200 - 100

5. A Man travelled a distance of 61 km in 9 hours. He travelled partly on foot at 4 km/hr and partly on bicycle at 9 km/hr. What is the distance travelled on foot?

A. 16 km

B. 14 km

C. 12 km

D. 10 km

Answer :

Option **A**

Explanation:

Let the time in which he travelled on foot = x hour
Time for travelling on bicycle = (9 - x) hr

Distance = Speed * Time, and Total distance = 61 km
So,
4x + 9(9-x) = 61
=> 5x = 20
=> x = 4

So distance traveled on foot = 4(4) = 16 km

6. 2 trains starting at the same time from 2 stations 200 km apart and going in opposite direction cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds ?

A. 11:9

B. 13:9

C. 17:9

D. 21:9

Answer :

Option **A**

Explanation:

We know total distance is 200 Km
If both trains crossed each other at a distance of 110 km then one train covered 110 km and other 90 km [110+90=200km]
So ratio of their speed = 110:90 = 11:9

7. A train covers a distance in 50 minutes, if it runs at a speed of 48kmph on an average. Find the speed at which the train must run to reduce the time of journey to 40 minutes.

A. 50 km/hr

B. 60 km/hr

C. 65 km/hr

D. 70 km/hr

Answer :

Option **B**

Explanation:

We are having time and speed given, so first we will calculate the distance. Then we can get new speed for given time and distance.
Lets solve it.

Time = 50/60 hr = 5/6 hr
Speed = 48 mph
Distance = S*T = 48 * 5/6 = 40 km

New time will be 40 minutes so,
Time = 40/60 hr = 2/3 hr
Now we know,
Speed = Distance/Time

8. A person crosses a 600 meter long street in 5 minutes. What is the speed in Km/hr

A. 6.2 km/hr

B. 7.2 km/hr

C. 8.2 km/hr

D. 9.2 km/hr

Answer :

Option **B**

Explanation:

Two things to give attention on this question.
First time is in minutes, we need to change it to seconds to get speed in m/sec, then we need to get the final answer in km/hr.
So lets solve this.

$$Speed = \frac{Distance}{Time}$$
$$Distance = 650meter$$
$$Time = 5 \text{ minutes} = 300sec$$
$$Speed = \frac{600}{300} = 2m/sec$$
$$=> 2 * \frac{18}{5} km/hr = 7.2km/hr$$

9. Robert is travelling on his cycle and has calculated to reach point A at 2 P.M. if he travels at 10 kmph, he will reach there at 12 noon if he travels at 15 kmph. At what speed must he travel to reach A at 1 P.M.?

A. 9 km/hour

B. 10 km/hour

C. 11 km/hour

D. 12 km/hour

Answer :

Option **D**

Explanation:

We need to calculate the distance, then we can calculate the time and finally our answer.
Lets solve this,

Let the distance travelled by x km
Time = Distance/Speed

$$\frac{x}{10} - \frac{x}{15} = 2$$
$$\text{[because, 2 pm - 12 noon = 2 hours]}$$
$$3x - 2x = 60$$
$$x = 60.Time = \frac{Distance}{Speed}$$
$$Time@10km/hr = \frac{60}{10} = 6hours$$

So 2 P.M. - 6 = 8 A.M
Robert starts at 8 A.M.

He have to reach at 1 P.M. i.e, in 5 hours

So, Speed = 60/5 = 12 km/hr

10. Two boys starting from the same place walk at a rate of 5kmph and 5.5kmph respectively. What time will they take to be 8.5km apart, if they walk in the same direction?

A. 15 hours

B. 16 hours

C. 17 hours

D. 18 hours

Answer :

Option **C**

Explanation:

In this type of questions we need to get the relative speed between them,
The relative speed of the boys = 5.5kmph – 5kmph
= 0.5 kmph

Distance between them is 8.5 km
Time = Distance/Speed
Time= 8.5km / 0.5 kmph = 17 hrs

11. The ratio between the speeds of two trains is 7: 8. If the second train runs 400 kms in 4 hours, then the speed of the first train is ?

A. 83.5 km/hr

B. 84.5 km/hr

C. 86.5 km/hr

D. 87.5 km/hr

Answer :

Option **D**

Explanation:

Let the speeds of two trains be 7X and 8X km/hr.

$$8X = \frac{400}{4}$$
$$=> X = 12.5 Km/hr$$

So speed of first train is 12.5*7 = 87.5 km/hr

Explanation:

$$Speed = \left(80 * \frac{5}{18}\right) m/sec$$
$$= \frac{200}{9} m/sec$$
$$= 22\frac{2}{9} m\,\text{sec}$$

12. A car moves at 80 km/hr. What is the speed of the car in meters per second ?

A. $20\frac{2}{9} m \sec$

B. $22\frac{2}{9} m \sec$

C. $24\frac{2}{9} m \sec$

D. $26\frac{2}{9} m \sec$

Answer :

Option **B**

13. An athlete runs 200 meters in 24 seconds. His speed is ?

A. 10 km/hr

B. 17 k/hr

C. 27 km/hr

D. 30 km/hr

Answer :

Option **D**

Explanation:

$$Speed = \frac{Distance}{Time}$$
$$= \frac{200}{24} m/sec = \frac{25}{3} m/sec$$
$$\frac{25}{3} * \frac{18}{5} km/hr = 30 km/hr$$

14. A man in a train notices that he can count 41 telephone posts in one minute. If they are known to be 50 metres apart, then at what speed is the train travelling?

A. 60 km/hr

B. 100 km/hr

C. 110 km/hr

D. 120 km/hr

Answer :

Option

Explanation:

Number of gaps between 41 poles = 40
So total distance between 41 poles = 40*50
= 2000 meter = 2 km
In 1 minute train is moving 2 km/minute.
Speed in hour = 2*60 = 120 km/hour

15. A person travels equal distances with speed of 3 km/hr, 4 km/hr and 5 km/hr and takes a total of 47 minutes. Find the total distane

A. 3 km

B. 4 km

C. 6 km

D. 9 km

Answer :

Option **A**

Explanation:

Let the distance be 3x km,
then,

$$\frac{x}{3}+\frac{x}{4}+\frac{x}{5}=\frac{47}{60}$$
$$\frac{47x}{60}=\frac{47}{60}$$
$$x=1$$

So total distance = 3*1 = 3 Km

16. The distance between two cities A and B is 330 Km. A train starts from A at 8 a.m. and travel towards B at 60 km/hr. Another train starts from B at 9 a.m and travels towards A at 75 Km/hr. At what time do they meet?

A. 10 am

B. 11 am

C. 12 pm

D. 1 pm

Answer :

Option **B**

Explanation:

Suppose they meet x hrs after 8 a.m.
then
(Distance moved by first in x hrs) + [Distance moved by second in (x-1) hrs]
= 330
So, 60x+75(x-1) = 330

x=3.

So,they meet at (8+3). i.e 11a.m.

17. A train traveled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point.

A. 6 hours 24 mins

B. 6 hours 21 mins

C. 6 hours 18 mins

D. 6 hours 15 mins

Answer :

Option **B**

Explanation:

Time taken to cover 600 km = 6 hours [because train speed is 100 km/hr]

Number of stops =
egin{aligned}
frac{600}{75} - 1 = 7 \
ext{Total time on stops =}7*3 = 21 mins

end{aligned}
Total time taken by train = 6 hours 21 mins

18. A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. Find the average speed for first 320 km of tour.

A. 70.11 km/hr

B. 71.11 km/hr

C. 72.11 km/hr

D. 73.11 km/hr

Answer :

Option **B**

Explanation:

We know Time = Distance/speed

So total time taken =

$$\left(\frac{160}{64}+\frac{160}{80}\right)$$
$$=\frac{9}{2}hours$$
$$\text{Time taken for 320 Km} =$$
$$320*\frac{2}{9}=71.11km/hr$$

19. A cyclist covers a distance of 750 meter in 2 minutes 30 seconds. What is the speed in km/hr of cyclist

A. 16 km/hr

B. 17 km/hr

C. 18 km/hr

D. 19 km/hr

Answer :

Option **C**

Explanation:

$$Speed=\frac{Distance}{Time}$$
$$Distance=750meter$$
$$Time=2\text{ min }30\text{ sec}=150sec$$
$$Speed=\frac{750}{150}=5m/sec$$
$$=>5*\frac{18}{5}km/hr=18km/hr$$

20. A man complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km/hr and second half at the rate of 24 km/hr. Find the total journey in km.

A. 200 Km

B. 222 Km

C. 224 Km

D. 248 Km

Answer :

Option **C**

Explanation:

Let time taken to travel the first half = x hr
Then time taken to travel the second half = (10 - x) hr

Distance covered in the the first half = Distance covered in the the second half
So,
21x = 24(10 - x)
=> 45x = 240
=> x = 16/3

Total Distance = 2*21(16/3) = 224 Km [multiplied by 2 as 21x was distance of half way]

21. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour ?

A. 8 minutes

B. 10 mintues

C. 12 minutes

D. 14 minutes

Answer :

Option **B**

Explanation:

Due to stoppages, it covers 9 km less.

Time taken to cover 9 km = (9/54) hour
= (1/6)*60 minutes
= 10 minutes

22. A person travels from P to Q at a speed of 40 km/hr and returns by increasing his speed by 50%. What is his average speed for both the trips ?

A. 44 km/hour

B. 46 km/hour

C. 48 km/hour

D. 50 km/hour

Answer :

Option **C**

Explanation:

Speed while going = 40 km/hr
Speed while returning = 150% of 40 = 60 km/hr

Average speed =

$$\frac{2xy}{x+y}$$
$$= \frac{2*40*60}{40+60} = \frac{4800}{100}$$
$$= 48Km/hr$$

23. A walks around a circular field at the rate of one round per hour while B runs around it at the rate of six rounds per hour. They start at same point at 7:30 am. They shall first cross each other at ?

A. 7:15 am

B. 7:30 am

C. 7: 42 am

D. 7:50 am

Answer :

Option **C**

Explanation:

Relative speed between two = 6-1 = 5 round per hour

They will cross when one round will complete with relative speed,
which is 1/5 hour = 12 mins.

So 7:30 + 12 mins = 7:42

1. A train running at the speed of 60 km/hr crosses a pole in 9 seconds. Find the length of the train.

A. 150 meter

B. 145 meter

C. 140 meter

D. 135 meter

Answer :

Option **A**

Explanation:

Speed = 60*(5/18) m/sec = 50/3 m/sec

Length of Train(Distance) = Speed * Time

$$= \frac{50}{3} * 9 = 150 meter$$

2. Two trains 140 metre and 160 metre long run the speed of 60 km/hr and 40 km/hr respectively opposite direction on parallel tracks. What time these will take to cross each other ?

A. 10.7 Seconds

B. 10.8 Seconds

C. 10.9 Seconds

D. 11.8 Seconds

Answer :

Option **B**

Explanation:

Relative Speed = 60+40 = 100 Kmph
= 100*(5/18) = 250/9 m/sec

Distance to be covered = 140 + 160 = 300 metres

Time = Distance/Speed

$$Time = 300 * \frac{9}{250}$$
$$= \frac{54}{5} = 10.8 \text{ seconds}$$

3. A train is 360 meter long is running at a speed of 45 km/hour. In what time will it pass a bridge of 140 meter length.

A. 20 seconds

B. 30 seconds

C. 40 seconds

D. 50 seconds

Answer :

Option **C**

Explanation:

Speed = 45 Km/hr = 45*(5/18) m/sec
= 25/2 m/sec
Total distance = 360+140 = 500 meter

Time = Distance/speed

$$= 500 * \frac{2}{25} = 40 seconds$$

4. A train speeds past a pole in 15 seconds and a platform 100 meter long in 25 seconds. What is length of the train ?

A. 140 meter

B. 145 meter

C. 150 meter

D. 155 meter

Answer :

Option **C**

Explanation:

Let the length of the train is x meter and Speed of the train is y meter/second
Then x/y = 15 [because distance/speed = time]
=> y = 15/x

$$=> \frac{x+100}{25} = \frac{x}{15}$$
$$x = 150 \text{ meters}$$

So length of the train is 150 meters

5. A train is moving at a speed of 132 km/hour. If the length of the train is 110 meters, how long wi take to cross a railway platform 165 meters long.

A. 7 second

B. 7.5 second

C. 8 second

D. 8.5 second

Answer :

Option **B**

Explanation:

As we need to calculate answer in seconds, so first convert speed into meter/sec.

we know 1 km/hr = 1*(5/18) m/sec
So, Speed = 132* (5/18) = 110/3 m/sec

Distance need to be covered in passing the platform = Length of train + length of platform = 110 + 165
= 275 meters

Time = Distance/Speed

$$=> Time = 275 * \frac{3}{110} = \frac{15}{2}$$
$$= 7.5 seconds$$

6. Speed of a goods train is 72 km/hr. This train crosses a 250 meter platform in 26 seconds. Then find the length of goods train.

A. 250 meters

B. 260 meters

C. 270 meters

D. 280 meters

Answer :

Option **C**

Explanation:

First convert speed from km/hr to m/sec

So, Speed = 72*(5/18) = 20 m/sec
Time = 26 seconds

Let the length of the train be x meters.
We know, Distance = Speed*Time.
[you can remember this formula as remembering DUST = D*ST... Distance=Speed*Time]

x+250 = 20*26
=> x = 270 meters

So length of the goods train is 270 meter.

7. Two trains are running in opposite directions in the same speed. The length of each train is 120 meter. If they cross each other in 12 seconds, the speed of each train (in km/hr) is

A. 30 Km/hr

B. 36 Km/hr

C. 80 Km/hr

D. 90 Km/hr

Answer :

Option **B**

Explanation:

Distance covered = 120+120 = 240 m
Time = 12 s

Let the speed of each train = x.
Then relative velocity = x+x = 2x

2x = distance/time = 240/12 = 20 m/s

Speed of each train = x = 20/2 = 10 m/s

= 10*18/5 km/hr = 36 km/hr

8. A train, 800 meter long is running with a speed of 78 km/hr. It crosses a tunnel in 1 minute. What is the length of the tunnel ?

A. 650 meter

B. 555 meter

C. 500 meter

D. 458 meter

Answer :

Option **C**

Explanation:

Let length of tunnel is x meter
Distance = 800+x meter
Time = 1 minute = 60 seconds

Speed = 78 km/hr = 78*5/18 m/s = 65/3 m/s

Distance = Speed*Time

$$=> 800 + x = \frac{65}{3} * 60$$
$$=> 800 + x = 20 * 65 = 1300$$
$$=> x = 1300 - 800 = 500$$

So the length of the tunnel is 500 meters.

[If you want to calculate it practically, then please have your copy and pen and take a ride of Kalka to Shimla toy train in north India. It has many tunnels on the way ☺]

9. A 300 meter long train crosses a platform in 39 seconds while it crosses a signal pole in 18 seconds. What is the length of the platform.

A. 310 meter

B. 335 meter

C. 345 meter

D. 350 meter

Answer :

Option **D**

Explanation:

Speed = Distance/time = 300/18 = 50/3 m/sec

Let the length of the platform be x meters then

$$Distance = Speed * Time$$
$$x + 300 = \frac{50}{3} * 39$$
$$=> 3(x + 300) = 1950$$
$$=> x = 350 \text{ meters}$$

10. How many seconds will a 500 meter long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr

A. 25 Seconds

B. 28 Seconds

C. 30 Seconds

D. 35 Seconds

Answer :

Option **C**

Explanation:

Relative Speed = 63-3 = 60 Km/hr
= 60 *(5/18) = 50/3 m/sec

Time taken to pass the man will ne

$$500 * \frac{3}{50} = 30 \text{ seconds}$$

1. A tank can be filled by a tap in 20 minutes and by another tap in 60 minutes. Both the taps are kept open for 10 minutes and then the first tap is shut off. After this, the tank will be completely filled in what time ?

A. 10 mins

B. 15 mins

C. 20 mins

D. 25 mins

Answer :

Option **C**

Explanation:

How we can solve this question ?
First we will calculate the work done for 10 mins, then we will get the remaining work, then we will find answer with one tap work, As

Part filled by Tap A in 1 min = 1/20
Part filled by Tap B in 1 min = 1/60

(A+B)'s 10 mins work =

$$10*\left(\frac{1}{20}+\frac{1}{60}\right)$$
$$=10*\frac{4}{60}=\frac{2}{3}$$
$$\text{Remaining work}=1-\frac{2}{3}$$
$$=\frac{1}{3}$$

METHOD 1

$$=>\frac{1}{60}:\frac{1}{3}=1:X$$
$$=>X=20$$

METHOD 2

$$1/60 \text{ part filled by B in} = 1min$$
$$1/3 \text{ part will be filled in}$$
$$=\frac{\frac{1}{3}}{\frac{1}{60}}$$
$$=\frac{60}{3}=20$$

2. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely ?

A. 6 min to empty

B. 7 min to full

C. 6 min to full

D. 7 min to empty

Answer :

Option **A**

Explanation:

There are two important points to learn in this type of question,
First, if both will open then tank will be empty first.

Second most important thing is,
If we are calculating filling of tank then we will subtract as (filling-empting)

If we are calculating empting of thank then we will subtract as (empting-filling)

So lets come on the question now,

Part to emptied 2/5

Part emptied in 1 minute =

$$\frac{1}{6} - \frac{1}{10}$$
$$= \frac{1}{15}$$
$$=> \frac{1}{15} : \frac{2}{5} :: 1 : x$$
$$=> \frac{2}{5} * 15 = 6mins$$

3. A cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time cistern will get filled ?

A. 7 hours

B. 7.1 hours

C. 7.2 hours

D. 7.3 hours

Answer :

Option **C**

Explanation:

When we have question like one is filling the tank and other is empting it, then we subtraction as,

Filled in 1 hour = 1/4
Empties in 1 hour = 1/9

Net filled in 1 hour = 1/4 - 1/9
= 5/36

So cistern will be filled in 36/5 hours i.e. 7.2 hours

4. Taps A and B can fill a bucket in 12 minutes and 15 minutes respectively. If both are opened and A is closed after 3 minutes, how much further time would it take for B to fill the bucket?

A. 8 min 15 sec

B. 7 min 15 sec

C. 6 min 15 sec

D. 5 min 15 sec

Answer :

Option **A**

Explanation:

Part filled in 3 minutes =

$$3 * \left(\frac{1}{12} + \frac{1}{15}\right)$$
$$= 3 * \frac{9}{60} = \frac{9}{20}$$
$$\text{Remaining part } = 1 - \frac{9}{20}$$
$$= \frac{11}{20}$$

$$=> \frac{1}{15} : \frac{11}{20} = 1 : X$$
$$=> X = \frac{11}{20} * \frac{15}{1}$$
$$=> X = 8.25 mins$$

So it will take further 8 mins 15 seconds to fill the bucket.

5. A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres a minute. When the tank is full, the inlet is opened and due to the leak the tank is empty in 8 hours. The capacity of the tank (in litres) is

A. 5780 litres

B. 5770 litres

C. 5760 litres

D. 5750 litres

Answer :

Option

Explanation:

Work done by the inlet in 1 hour =

$$\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$$

Work done by inlet in 1 min

$$= \frac{1}{24} * \frac{1}{60}$$

$$= \frac{1}{1440}$$

=> Volume of 1/1440 part = 4 liters

Volume of whole = (1440 * 4) litres = 5760 litres.

Basic Algebraic identities

In mathematics, an algebraic identity is an equality relation A = B, such that A and B contain some variables and A and B produce the same value as each other regardless of what values are substituted for the variables.
In other words, A = B is an identity if A and B define the same functions.

This topic is the best topic to score because you can verify your answer while attempting the question itself. The difficulty level is easy and you just have to remember the formulas and implement them accordingly. The formulas mentioned below are mostly the old ones which we have already studied in school. So take a quick look at these and move straight to the exercises to practice.

- $(x+y)^2 = x^2 + 2xy + y^2$
- $(x-y)^2 = x^2 - 2xy + y^2$
- $(x+y)(x-y) = x^2 - y^2$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$
- $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x+y)^3 = x^3 + y^3 + 3xy(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$
- $(x-y)^3 = x^3 - y^3 - 3xy(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$
- $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

The revision must not have lasted more than 5 minutes, we believe such is the topic. The implementation will be clear once you go through these handpicked examples and questions.

Q1. The value of, $\frac{1}{a^2+ax+x^2} - \frac{1}{a^2-ax+x^2} + \frac{2ax}{a^4+a^2x^2+x^4}$ is,

1. 2
2. 1
3. -1
4. 0

Solution

When you meet such balanced nearly whole square equations in the denominator with one middle term plus and the other minus, you can straightway pair the first and third terms a^2 and x^2 together so that when you combine the first two terms of the expression, the denominator turns to $(a^2+x^2)^2 - a^2x^2$.

Immediately your attention shifts to the third denominator and quickly you can transform it to the same expression mentally.

$$\begin{aligned}(x^4+a^2x^2+x^4) &= (x^4+2a^2x^2+x^4) - a^2x^2 \\ &= (a^2+x^2)^2 - a^2x^2\end{aligned}$$

Now only the task of evaluating the numerator remains.

First two terms in an instant produces $-2ax$ which cancels out with the third term numerator resulting in a zero in the numerator.

Answer: Option 4: 0.

Q2. If $x^3+y^3=9$ and $x+y=3$ then the value of x^4+y^4 is,

1. 81
2. 32
3. 27
4. 17

Solution

Whenever you meet $x^3+y^3=9$ and $x+y=3$ together straightway go for the expression $x^3+y^3=(x+y)\times(x^2-xy+y^2)$ which results in,

$$\begin{aligned}9&=3\times((x+y)^2-3xy)\\&=3\times(9-3xy)\\&=27-9xy\end{aligned}$$

or, $xy=2$.

Now easiest way to get x^4+y^4 is to multiply the first two given expressions together, giving,

$x^4+y^4+xy(x^2+y^2)=27$. So,

$$\begin{aligned}x^4+y^4&=27-2\times((x+y)^2-2xy)\\&=27-2\times(9-4)\\&=17\end{aligned}$$

Answer: Option 4 : 17.

Q3. For any real number x the maximum value of $4-6x-x^2$ is,

1. 4
2. 7
3. 9
4. 13

Solution

By analyzing the expression with the knowledge that x can be either positive or negative, we find in both cases of nature of x, the third term being square of x, will remain to be negative.

So to maximize the value of the expression, the second term must be positive, that is, x must be negative.

Next we see that the third term being a square it will increase faster than the second term. So combining this knowledge with the requirement of minimizing the third term (being negative), we conclude that absolute value of x must be as small as possible.

Starting with $x=-1$, we get target expression $E=4+6-1=9$.

With $x=-2$, expression $E=4+12-4=12$ - increasing.

Decreasing x still further for $x=-3$, we get, expression $E=4+18-9=13$.

We stop here **as we notice 13 to be the maximum value among the choices.**

Answer: Option 4: 13.

Q4. If $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$ then value of x is,

1. $\frac{25}{4}$
2. 4
3. 6
4. 9

Solution

From basic mathematical sense, we can take the first decision as $\sqrt{x}$ must be an integer. Derivation of the value of x and then the term values if $\sqrt{x}$ is a fraction would be too complicated in the expected time given for answer and also beyond the scope of syllabus.

Secondly, 5 and 12 having a large separation, if the power is large, increasing power for 13 cannot be compensated by same increasing power of 12. With this reasoning, we would start iteration with power as 1, and then at $\sqrt{x} = 2$ we get equality,

$5^2 + 12^2 = 13^2$.

So, $x = 4$.

Answer: Option 2: 4.

Q5. If $a + b + c = 0$ then the value of $\frac{a^2+b^2+c^2}{a^2-bc}$ is,

1. 0
2. 1
3. 2
4. 3

Solution

Sensing that the denominator must be a factor of the numerator by noticing the nature of the choice values, we resort straightaway to algebraic manipulation of isolating the factor $a^2 - bc$ in the numerator of the target expression.

Thus at the first step we get, denominator

$$\begin{aligned} D &= a^2 - bc + (b+c)^2 - bc \\ &= a^2 - bc + (-a)^2 - bc \\ &= a^2 - bc + a^2 - bc \end{aligned}$$

So expression, $E = 2$.

Answer: Option 3: 2.

Q6. If $x^2+2=2x$ then the value of $x^4-x^3+x^2+2$ will be,

1. 0
2. 1
3. -1
4. $\sqrt{2}$

Solution

The very first step that we will take is to **transpose the equation 1 to have a numerical value on the right of the equality sign.** We intend to factor the target expression with this transposed expression, $x^2-2x=-2$.

Factoring this way we get expression,

$$E=x^2(x^2-2x)+2x^3-x^3+2x=x^3-2x^2+2x.$$

We have straightaway simplified the tail by using $x^2+2=2x$ for substitution.

So, $E=x(x^2-2x)+2x=-2x+2x=0$

Answer: Option 1 : 0.

Q7. If $x=(\sqrt{2}+1)^{-\frac{1}{3}}$, then the value of $\left(x^3-\frac{1}{x^3}\right)$,

1. 0
2. -2
3. $-\sqrt{2}$
4. $\sqrt{2}$

Solution

We see x^3 in target expression and power $\frac{1}{3}$ in the given expression of x. So we straightway, cube the given expression and use the value in the target expression, giving target expression,

$$E=\frac{1}{\sqrt{2}+1}-(\sqrt{2}+1)=\frac{1-(\sqrt{2}+1)^2}{\sqrt{2}+1}$$

So, $E=\frac{1-(2+2\sqrt{2}+1)}{\sqrt{2}+1}=\frac{-2-2\sqrt{2}}{\sqrt{2}+1}=-2$

Answer: Option 2: -2.

Q8. If $4b^2 + \frac{1}{b^2} = 2$, then value of $8b^3 + \frac{1}{b^3}$ is,

1. 0
2. 2
3. 1
4. 5

Solution

If we forget the given expression for the moment and consider the target expression in the form of $x^3 + y^3$, we get,

$E = (2b + \frac{1}{b})(4b^2 - 2 + \frac{1}{b^2})$

Now if we use the given expression in the second factor, it turns out to be 0, resulting in the answer as 0.

Answer: Option 1: 0.

Q9. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} - z^{\frac{1}{3}} = 0$ then value of $(x + y - z)^3 + 27xyz$ is,

1. -1
2. 1
3. 0
4. 27

Solution

In this type of sums always use transposition to avoid dealing with cubing or squaring a three variable expression. Thus from the given expression we get,

$x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$

Now cubing both sides we get,

$x + 3x^{\frac{1}{3}}y^{\frac{1}{3}}(x^{\frac{1}{3}} + y^{\frac{1}{3}}) + y = z$

or, $(x + y - z) = -3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

Cubing again both sides, $(x + y - z)^3 = -27xyz$.

So answer is 0.

Answer: Option 3: 0.

Q10. If $x^{x\sqrt{x}} = (x\sqrt{x})^x$ then x is equal to,

1. $\frac{4}{9}$
2. $\frac{2}{3}$
3. $\frac{9}{4}$
4. $\frac{3}{2}$

Solution

In indices sums, first explore ***equalization of base technique***. The LHS is already in powers of x. Let's then bring the RHS in powers of x,

$x^{x\sqrt{x}} = (x\sqrt{x})^x = (x^{\frac{3}{2}})^x = x^{\frac{3x}{2}}$.

Now equating powers on both sides, we get,

$x\sqrt{x} = \frac{3x}{2}$, or, $\sqrt{x} = \frac{3}{2}$,

or $x = \frac{9}{4}$.

Answer: Option 3: $\frac{9}{4}$.

10 QUESTION PRACTICE SET

Q1. The number of possible values of x in the equation,
$\sqrt{x^2-x+1}+\dfrac{1}{\sqrt{x^2-x+1}}=2-x^2$ is,

a. 1
b. 2
c. 0
d. 4

$$LHS = E = \sqrt{x^2-x+1}+\frac{1}{\sqrt{x^2-x+1}}$$

$$= \left(p+\frac{1}{p}\right), \text{ where } p=\sqrt{x^2-x+1}$$

$$= \left(\frac{p^2+1}{p}\right).$$

As any real p^2 in the numerator will increase much faster than p in the denominator for values of $p \geq 1$, the expression E will reach its minimum value only when the $p=1$ and $E=2$.

To deal with the values of $p \leq 1$, we substitute p by $\dfrac{1}{q}$ transforming the expression to,

$$E = \left(p+\frac{1}{p}\right)$$

$$= \left(\frac{1}{q}+q\right),$$

$$= \left(q+\frac{1}{q}\right).$$

Now when $p \leq 1, q \geq 1$ and the value of expression E reaches its minima 2 also when $q=p=1$.

So for all real values of p, the sum of its inverses will have minimum value 2.

Answer: Option a: 1.

Q2. If $p+\dfrac{1}{p}=5$, then the value of $\dfrac{p^4+\dfrac{1}{p^2}}{p^2-3p+1}$ is,

a. 50
b. 55
c. 70
d. 110

$$p+\frac{1}{p}=5$$

Or, $p^2-1+\dfrac{1}{p^2}=25-3=22$.

So,

$$p^3+\frac{1}{p^3}=\left(p+\frac{1}{p}\right)\left(p^2-1+\frac{1}{p^2}\right)$$

$$=5\times 22$$

$$=110.$$

Thus numerator $=110p$.

Now we will use the given expression in a second way in expanded form. This is direct application of ***Multiple input use technique.***

Expanding the given expression and rearranging we get,

$$p^2-5p+1=0.$$

So denominator is,

$$p^2-3p+1=2p.$$

Finally then the desired value of the target expression is,

$$E=\frac{110p}{2p}=55.$$

Answer: Option b : 55.

Q3. If $\sqrt{4x-9}+\sqrt{4x+9}=5+\sqrt{7}$, find the value of x.

a. 3
b. 4
c. 5
d. 7

Raising the given equation to the power of 2,

$(\sqrt{4x-9}+\sqrt{4x+9})^2=(5+\sqrt{7})^2$,

Or, $8x+2\sqrt{16x^2-81}=32+10\sqrt{7}$.

Equating the non-square-root terms of LHS and RHS,

$8x=32$,

Or, $x=4$.

This can also verified by equating the square root parts of LHS and RHS or substituting value of x
.

Answer: Option b: 4.

Q4. If $\sqrt{2x}-\sqrt{3y}=0$ and $\sqrt{7x}+\sqrt{2y}=0$ then the value of $x+y$ is,

a. 1
b. 2
c. 3
d. 0

Though the two variables x and y are under square roots, essentially the two equations can be considered to two linear equations in two variables. The variables here turn out to be $\sqrt{x}$ and $\sqrt{y}$ instead of plain x and y. This is use of abstraction. Instead of $\sqrt{x}$ and $\sqrt{y}$ the two variables could have been x^2 and y^2 and in this form we could have then solved for x^2 and y^2. The form of the equation is important not the form of the variables. The form of the equations here is linear.

To solve two linear equations in two variables we take the simple approach of equalizing the coefficients of one variable in two equations by changing the coefficient of the chosen variable in one equation suitably. This we do by multiplying the whole equation by a suitable factor.

In this case $\sqrt{y}$ calls for elimination because of its opposite signs in two equations. We select the first equation and multiply it by $\sqrt{\frac{2}{3}}$ to transform the coefficient of $\sqrt{y}$ from $\sqrt{3}$ to $\sqrt{2}$ thus making it equal to the corresponding coefficient in the second equation.

$\sqrt{2x} - \sqrt{3y} = 0$

Or, $\sqrt{\frac{4x}{3}} - \sqrt{2y} = 0$.

Adding this equation with the second equation $\sqrt{7x} + \sqrt{2y} = 0$ we get,

$\sqrt{\frac{4x}{3}} + \sqrt{7x} = 0,$

Or, $\sqrt{x}\left(\sqrt{\frac{4}{3}} + \sqrt{7}\right) = 0.$

So,

$\sqrt{x} = x = 0$ and substituting it in any of the two equations we get $y = 0$ also.

Finally then, $x + y = 0$.

Answer: Option d: 0.

Q5. Find the remainder when $x^5 - 9x^2 + 12x - 14$ is divided by $(x - 3)$.

a. 56
b. 184
c. 0
d. 1

So we apply the continued factor extraction on the pair of expressions,

$x^5 - 9x^2 + 12x - 14$

$= x^4(x-3) + 3x^4 - 9x^2 + 12x - 14$

$\Rightarrow 3x^3(x-3) + 9x^3 - 9x^2 + 12x - 14$

$\Rightarrow 9x^2(x-3) + 27x^2 - 9x^2 + 12x - 14$

$\Rightarrow 18x^2 + 12x - 14$

$= 18x(x-3) + 54x + 12x - 14$

$\Rightarrow 66x - 14$

$= 66(x-3) + 198 - 14$

$\Rightarrow 184.$

Answer: Option b: 184.

Q6. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then value of $c + \frac{1}{a}$ is,

a. 1
b. 0
c. 2
d. $\frac{1}{2}$

Finding b in terms of a from the first equation,

$a+\frac{1}{b}=1$

Or, $\frac{1}{b}=1-a$,

Or, $b=\frac{1}{1-a}$.

Substituting this value in the second equation,

$b+\frac{1}{c}=1$,

Or, $\frac{1}{1-a}+\frac{1}{c}=1$,

Or, $\frac{1}{c}=1-\frac{1}{1-a}=-\frac{a}{1-a}$,

Or, $c=-\frac{1-a}{a}$,

Or, $c+\frac{1}{a}=1$

Answer: Option a : 1.

Q7. If $a:b=\frac{2}{9}:\frac{1}{3}, b:c=\frac{2}{7}:\frac{5}{14}$ and $d:c=\frac{7}{10}:\frac{3}{5}$ find the value of $a:b:c:d$.

a. $8:12:15:7$
b. $4:6:7:9$
c. $30:35:24:16$
d. $16:24:30:35$

Normalizing the first ratio values we have,

$$a : b = \frac{2}{9} : \frac{1}{3},$$

Or, multiplying the ratio values by 9,

$$a : b = 2 : 3.$$

Normalizing the second ratio values,

$$b : c = \frac{2}{7} : \frac{5}{14}$$

Or multiplying the values by 14,

$$b : c = 4 : 5.$$

Similarly normalizing the third ratio values by multiplying the values by 10,

$$d : c = \frac{7}{10} : \frac{3}{5} = 7 : 6.$$

To join $a : b = 2 : 3$ and $b : c = 4 : 5$ we identify b as the common quantity. The values corresponding to b in the two ratios being 3 and 4, the target equalization value of this quantity will be the LCM of the two values, that is, $3 \times 4 = 12$.

We now transform the two ratios to equalize the values corresponding to quantity b as 12,

$a : b = 2 : 3 = 8 : 12$ and

$b : c = 4 : 5 = 12 : 15$

Joining the two ratios we get,

$a : b : c = 8 : 12 : 15$

Now we have to join this ratio with the third ratio, $d : c = 7 : 6$.

To join the first step we have to take is to bring the common quantity c in this case to the numerator of the third ratio,

$d : c = 7 : 6,$

Or, $c : d = 6 : 7.$

The target value at which the two values of c are to be equalized now will be the LCM of 6 and 15 which is 30.

For base equalization, transforming the two ratios to be joined we get,

$a:b:c = 8:12:15 = 16:24:30$ and

$c:d = 6:7 = 30:35$

Finally then the four quantity ratio is,

$a:b:c:d = 16:24:30:35$

Answer: Option d: $16:24:30:35$.

Q8. If a and b are the roots of the equation $3x^2+2x+1=0$ which of the equations will have the roots, $\dfrac{1-a}{1+a}$ and $\dfrac{1-b}{1+b}$?

a. $y^2-2y+3=0$
b. $y^2+2y-3=0$
c. $y^2-2y-3=0$
d. $y^2+2y+3=0$

First we normalize the quadratic equation to remove the coefficient of x^2 and make it 1,

$3x^2+2x+1=0$

Or, $x^2+\dfrac{2}{3}x+\dfrac{1}{3}=0$

So sum and product of the roots a and b,

$a+b=-\dfrac{2}{3}$ and,

$ab=\dfrac{1}{3}$.

Mark here, we don't have the values of the roots a and b, but only have the values of their sum and product.

$p = \frac{1-a}{1+a}$ and,

$q = \frac{1-b}{1+b}$.

So,

$$p + q = \frac{(1-a)(1+b) + (1-b)(1+a)}{(1+a)(1+b)}$$

$$= \frac{1 - a + b - ab + 1 - b + a - ab}{1 + a + b + ab}$$

$$= \frac{2 - 2ab}{1 + a + b + ab}$$

$$= \frac{2 - \frac{2}{3}}{1 - \frac{2}{3} + \frac{1}{3}}$$

$$= \frac{\frac{4}{3}}{\frac{2}{3}}$$

$$= 2$$

Similarly,

$$pq = \frac{(1-a)(1-b)}{(1+a)(1+b)}$$

$$= \frac{1-(a+b)+ab}{\frac{2}{3}}$$

$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{\frac{2}{3}}$$

$$= \frac{2}{\frac{2}{3}}$$

$$= 3$$

So the desired equation will be,

$y^2 - 2y + 3 = 0.$

Answer: Option a: $y^2 - 2y + 3 = 0.$

Q9. If $2x^2 - 7xy + 3y^2 = 0$, then the value of $x : y$ is,

a. $3 : 2$
b. $5 : 6$
c. $2 : 3$
d. $3 : 1$ and $1 : 2$

Solution 9 - Problem analysis and execution

By mid-term splitting and observation we factorize the given quadratic equation in two variables as,

$2x^2 - 7xy + 3y^2 = (2x - y)(x - 3y) = 0$.

This gives us two relations between x and y satisfying the quadratic equation,

$2x = y$ and

$x = 3y$.

In the first case the desired ratio is,

$x : y = 1 : 2$ and in the second case,

$x : y = 3 : 1$.

Finally then the desired ratio have two values for two root value pairs,

$1 : 2$ and $3 : 1$.

Ans: Option d: $3 : 1$ and $1 : 2$.

Q10. Find the value of α when the expression $\frac{x^2}{y^2} + \alpha x + \frac{y^2}{4}$ is a perfect square.

a. ± 1
b. 0
c. ± 2
d. ± 7

Solution 10 - Problem analysis and execution

Examining the quadratic expression in two variables we detect that both the square terms are in proper square forms,

$a^2 = \left(\frac{x}{y}\right)^2$ and

$b^2 = \left(\frac{y}{2}\right)^2$.

For the quadratic equation to be a perfect square then the mid-term must be,

$2ab = 2 \times \frac{x}{y} \times \frac{y}{2} = x$.

Thus for the given equation to be a perfect square,

$\alpha x = x$,

Or, $\alpha = 1$.

But we have considered only the form $(a+b)^2$. For the form $(a-b)^2$, the mid-term will be $-2ab = -x$ and the desired value of α will be,

$\alpha = -1$.

Finally then for all possibilities the desired value of α will be,

$\alpha = \pm 1$.

Answer: Option a: ± 1.

10 QUESTION PRACTICE SET

Q1. If $4y - 3x = 13$ and $xy = 14$, then $64y^3 - 27x^3$ is,

a. 8479
b. 8400
c. 8740
d. 8749

$4y - 3x = 13$,

Or, $(p-q)^2 = p^2 - 2pq + q^2 = 169$,

Or, $p^2 + pq + q^2 = 169 + 3pq$, the term $3pq$ added to both sides,

Or, $p^2 + pq + q^2 = 169 + 504 = 673$.

So,

$64y^3 - 27x^3 = p^3 - q^3$

$= (p-q)(p^2 + pq + q^2)$

$= 13 \times 673$

$= 8749$.

Answer: Option d: 8749.

Q2. If $x = (0.09)^2$, $y = \dfrac{1}{(0.09)^2}$ and $z = (1 - 0.09)^2 - 1$, then which of the following relations is true?,

a. $y < x$ and $x = z$
b. $y < z < x$
c. $z < x < y$
d. $x < y$ and $x = z$

By substitution, $p = 0.09$, where $p < 1$ we have the transformed given equations as,

$x = p^2$,

$y = \frac{1}{p^2}$, and

$z = (1 - p)^2 - 1 = p^2 - 2p$.

When comparing x with y we can conclude that,

$y > x$, as $p < 1$ (dividing 1 by a value less than 1 makes y larger than 1, whereas x is less than 1).

Comparing x with z we can conclude that,

$x > z$, as p is positive.

These two conclusions are sufficient to finally form the desired comparative relation between the three variables as,

$y > x > z$,

Or, $z < x < y$.

Answer: Option c : $z < x < y$.

Q3. Minimum value of $x^4 + \dfrac{1}{x^4+1} - 3$ is,

a. 0
b. -1
c. -2
d. -3

The target expression is then expressed as,

$$E = x^4 + \frac{1}{x^4+1} - 3$$

$$= (x^4+1) + \frac{1}{(x^4+1)} - 4$$

$$= p + \frac{1}{p} - 4, \text{ where } p = x^4 + 1$$

$$= E_1 - 4, \text{ where } E_1 = p + \frac{1}{p}.$$

By applying minima of sum inverses concept we get the minimum value of E_1 as 2,

So the minimum value of the target expression E is,

$2 - 4 = -2.$

Answer: Option c: -2.

Q4. If $\frac{p}{3} = \frac{q}{2}$ then the value of $\frac{2p+3q}{3p-2q}$ is,

a. 1
b. $\frac{12}{5}$
c. $\frac{5}{12}$
d. $\frac{12}{7}$

Evaluating the target expression we get,

$$E = \frac{2p+3q}{3p-2q}$$

$$= \frac{3q}{2q} \times \frac{\frac{2p}{3q}+1}{\frac{3p}{2q}-1}$$

$$= \frac{3}{2} \times \frac{1+1}{\frac{9}{4}-1}$$

$$= \frac{3}{2} \times \frac{2}{\frac{5}{4}}$$

$$= \frac{3}{2} \times \frac{8}{5}$$

$$= \frac{12}{5}.$$

Answer: Option b: $\frac{12}{5}$.

Q5. If $(a-4)^2 + (b-9)^2 + (c-3)^2 = 0$, then the value of $\sqrt{a+b+c}$ is,

a. ± 4
b. 4
c. -4
d. ± 2

By the ***principle of zero sum of square terms***, when all the terms in an additive expression equated to zero are square terms, each of the terms must be zero for the equation to be valid. This happens because each of the squares, if non-zero, will be positive resulting in a non-zero positive result on summation which will violate its zero sum status.

So in our given problem we have,

$(a-4)=0$,

Or, $a=4$.

$(b-9)=0$,

Or, $b=9$, and

$(c-3)=0$,

Or, $c=3$.

Thus target expression is,

$E=\sqrt{a+b+c}=\sqrt{16}=4$.

Answer: Option b: 4.

Q6. If $\frac{1}{x+y}=\frac{1}{x}+\frac{1}{y}$, where, $x\neq 0$, $y\neq 0$ and $x\neq y$, the value of x^3-y^3 is,

a. 0
b. 2
c. 1
d. -1

The given equation is,

$$\frac{1}{x+y}=\frac{1}{x}+\frac{1}{y}$$

Multiplying both sides by xy,

$$\frac{xy}{x+y}=x+y$$

Or, $xy=(x+y)^2$,

Or, $x^2+xy+y^2=0$.

This is the second factor in target expression so that target expression evaluates finally to 0.

Answer: Option a : 0.

Q7. If $a^x=(x+y+z)^y$, $a^y=(x+y+z)^z$ and $a^z=(x+y+z)^x$ find the value of $a+b+c$ where $a\neq 0$.

a. 0
b. 1
c. a^3
d. a

On analyzing the three expressions we discover the key pattern of base equalization, and decide to multiply the three expressions together giving,

$a^x.a^y.a^z=(x+y+z)^{x+y+z}$,

Or, $a^{x+y+z}=(x+y+z)^{x+y+z}$.

Here the ***equalized base is the power*** and ***not the indice base***.

So the indice bases are equal, or,

$x+y+z=a$.

Answer: Option d: a.

Q8. If $\frac{x-a^2}{b^2+c^2}+\frac{x-b^2}{c^2+a^2}+\frac{x-c^2}{a^2+b^2}=3$ then the value of x is,

a. a^2+b^2
b. $a^2+b^2+c^2$
c. $a^2+b^2-c^2$
d. $a^2-b^2-c^2$

We have the given equation as,

$$\frac{x-a^2}{b^2+c^2}+\frac{x-b^2}{c^2+a^2}+\frac{x-c^2}{a^2+b^2}=3,$$

Or, $(x-a^2-b^2-c^2)\times\left(\frac{1}{b^2+c^2}+\frac{1}{c^2+a^2}+\frac{1}{a^2+b^2}\right)=0.$

So $x-a^2-b^2-c^2=0,$

Or, $x=a^2+b^2+c^2.$

Answer: Option b: $a^2+b^2+c^2$.

Q9. If $ax^2+bx+c=a(x-p)^2$, then the correct relation between a, b and c would be,

a. $2b=a+c$
b. $b^2=4ac$
c. $b^2=ac$
d. $abc=1$

Expanding the RHS of the given expression we have,

$ax^2+bx+c=a(x-p)^2$,

Or, $ax^2+bx+c=ax^2-2apx+ap^2$.

Equating the coefficients of like terms on the two sides of the equation we get,

$b=-2ap$, and

$c=ap^2$.

We will now raise the first equation to a square and divide the two equations with each other to eliminate p leaving only a, b and c.

$b=-2ap$,

Or, $b^2=4a^2p^2$.

Dividing this equation by the second equation, $c=ap^2$ we get,

$\frac{b^2}{c}=4a$,

Or, $b^2=4ac$.

Ans: Option b: $b^2=4ac$.

Q10. If $\frac{y}{x} - \frac{x}{y} = 3$, then find the value of $\frac{x^3}{y^3} + \frac{y^3}{x^3}$.

a. $10\sqrt{3}$
b. $10\sqrt{13}$
c. $13\sqrt{3}$
d. $13\sqrt{10}$

$$p^2 - 2 + \frac{1}{p^2} = 9,$$

Or, $p^2 + 2 + \frac{1}{p^2} = 9 + 4 = 13,$

Or, $\left(p + \frac{1}{p}\right)^2 = 13,$

Or, $\left(p + \frac{1}{p}\right) = \sqrt{13}.$

Again from the square of the given expression we have,

$$p^2 - 2 + \frac{1}{p^2} = 9,$$

Or, $p^2 - 1 + \frac{1}{p^2} = 10.$

This is the second factor of the target cubes expression expanded. So we have the target expression as,

$$E = p^3 + \frac{1}{p^3},$$

$$= \left(p + \frac{1}{p}\right)\left(p^2 - 1 + \frac{1}{p^2}\right)$$

$$= \sqrt{13} \times 10 = 10\sqrt{13}$$

Answer: Option b: $10\sqrt{13}$.

10 QUESTION PRACTICE SET

Q1. Two numbers a and b (where $a > b$) are such that their sum is three times their difference. The value of $\frac{3ab}{4(a^2 - b^2)}$ is then,

a. $\frac{1}{3}$
b. $\frac{1}{2}$
c. $1\frac{1}{4}$
d. $\frac{5}{6}$

So we have,

$a + b = 3(a - b)$,

Or, $2a = 4b$,

Or, $a = 2b$, a very simple relation.

Substituting value of a in the target expression we get,

$$E = \frac{3ab}{4(a^2 - b^2)}$$

$$= \frac{6b^2}{4(4b^2 - b^2)}$$

$$= \frac{3b^2}{2(3b^2)}$$

$$= \frac{1}{2}$$

Answer: Option b: $\frac{1}{2}$.

Q2. If $\dfrac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}}=2$, then x is,

a. $\dfrac{12}{5}$
b. $\dfrac{5}{12}$
c. $\dfrac{5}{7}$
d. $\dfrac{7}{5}$

$$\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}}=2,$$

Or applying componendo-dividendo technique on two sides of the equation we have,

$$\frac{\sqrt{3+x}}{\sqrt{3-x}}=\frac{2+1}{2-1}=3,$$

Or squaring both sides,

$$\frac{3+x}{3-x}=9$$

Again we find the LHS to be nicely suitable for applying componendo-dividendo technique. Thus we get,

$$\frac{3}{x}=\frac{10}{8}$$

Or, $x=\dfrac{24}{10}=\dfrac{12}{5}$.

Answer: Option a : $\dfrac{12}{5}$.

Q3. If $a+b=1$, $c+d=1$ and $a-b=\frac{d}{c}$ then the value of c^2-d^2 is,

a. 1
b. $\frac{a}{b}$
c. $\frac{b}{a}$
d. -1

Avoiding the large number of given expressions we first examine the target expression to find it very friendly and waiting to be split up into two complementary age-old factors,

$$E=c^2-d^2=(c+d)(c-d) .$$

Now as we examine the given expressions we immediately make use of $c+d=1$ to further simplify the target expression to,

$$E=(c+d)(c-d)=c-d$$

We can see that from the other two given expressions we can get $c-d$ by subtracting one from the other,

$$(a+b)-(a-b)=1-\frac{d}{c} ,$$

Or, $2b=\frac{c-d}{c}$.

Now we only have to eliminate c by applying our experience in manipulating algebraic expressions. We foresee that summing up $a+b$ and $a-b$ will result in $c+d$ in the numerator, that is 1, divided by c. Without any hesitation we go ahead in this path,

$$(a+b)+(a-b)=2a=\frac{c+d}{c}=\frac{1}{c} \text{ , as } c+d=1.$$

Now just divide the first result equation by the second result equation eliminating c,

$$E=c-d=\frac{b}{a} .$$

Answer: Option c: $\frac{b}{a}$.

Q4. If $x+\frac{1}{x}=\sqrt{3}$, then the value of $x^{18}+x^{12}+x^{6}+1$ is,

a. 1
b. 3
c. 0
d. 2

$x+\frac{1}{x}=\sqrt{3}$,

Squaring both sides,

$x^2+2+\frac{1}{x^2}=3$,

Or, $x^2+\frac{1}{x^2}=1$.

Usually we keep this expression in this form and try to use it. But having already taken the decision to ***use the sum inverse of cubes,*** we further examine the result and recognize its potential of using it in a more convenient form,

$x^2+\frac{1}{x^2}=1$,

Or, $x^2-1+\frac{1}{x^2}=0$.

The LHS being the ***second factor of expansion of sum of inverses of cubes,***

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)=0$$

This is the ***extreme simplification*** we wanted using the techniques we had elaboarated in our discussion on ***principle of inverses.***

Now focusing our attention on the target expression we identify that ***to have sum of inverse of cubes as a factor,*** the ***pair of terms should have a difference of power of 6.*** This is ***identification of the key pattern*** for reaching the solution in only one step. We have arrived at this stage by using our ***deductive reasoning.***

Thus we have target expression,

$$E=x^{18}+x^{12}+x^{6}+1$$

$$=x^{15}\left(x^{3}+\frac{1}{x^{3}}\right)+x^{3}\left(x^{3}+\frac{1}{x^{3}}\right)=0\ .$$

We have used here a simplified form of ***continued factor extraction technique*** where second term compensation is not required.

Answer: Option c: 0.

Q5. If a and b are two real numbers and the expression ax^3+3x^2-8x+b is exactly divisible by the expressions $(x+2)$ and $(x-2)$ then the values of a and b are,

a. $a=-2; b=12$
b. $a=2; b=12$
c. $a=12; b=2$
d. $a=2; b=-12$

As both $(x+2)$ and $(x-2)$ are factors, the product, (x^2-4) is also a factor of the given expression, and we can express the given expression as,

$ax^3+3x^2-8x+b=(x^2-4)(px+q)$, where p and q are two unknown constants.

As the highest power of the given expression is 3, the second factor can only be a linear expression in x of the general form $px+q$. We have made this conclusion using the basic algebraic concept of power formation rule applicable for the terms of an equation in a single variable.

To express this rule in general form,

> *In a general product of factors of expressions in single variable x, the highest power of the term in x in the fully expanded form of the products will be determined by the **product of the highest power terms in each of the product expressions.***

This is the ***basic power formation principle of a product in single variable.***

In our problem example, one of the factors already has its maximum power term with power 2 in (x^2-4), whereas the expanded expression has maximum power as 3. Thus the second factor must have maximum power term of x as 1, that is, it must be a linear expression in x.

So we have,

$$ax^3+3x^2-8x+b=(x^2-4)(px+q)=px^3+qx^2-4px-4q$$

Now we will apply the ***rich algebraic concept of coefficient comparison of like terms.***

For x^3, coefficients $a=p$.

For x^2, coefficients $3=q$, or, $q=3$.

For x, coefficients $-8=-4p$, or, $p=2$.

Lastly for x^0, $b=-4q=-12$, as $q=3$,

And from the first relation,

$a=p=2$.

Answer: Option d: $a=2; b=-12$.

Q6. If $(a^2+b^2)^3=(a^3+b^3)^2$ then $\frac{b}{a}+\frac{a}{b}$ will be equal to,

a. $\frac{2}{3}$
b. $-\frac{1}{3}$
c. $-\frac{2}{3}$
d. $\frac{1}{3}$

While expanding the cube we take care to use the compact form of expansion thus ***minimizing the number of terms.*** This is appilication of ***Term number minimization principle.*** In a simplification process generally, unless required, it is recommended to keep the number of terms as low as possible. Thus we have,

$(a^2+b^2)^3=(a^3+b^3)^2$,

Or, $a^6+b^6+3a^2b^2(a^2+b^2)=a^6+b^6+2a^3b^3$,

Or, $3(a^2+b^2)=2ab$,

Now we transform the target expression by joining the two terms so that its form is similar to the form we reached from the given expression,

$$E=\frac{b}{a}+\frac{a}{b}$$
$$=\frac{b^2+a^2}{ab}$$

Using the results of the transformed given equation $3(a^2+b^2)=2ab$ then,

$$E=\frac{2}{3}.$$

Answer: Option a : $\frac{2}{3}$.

Q7. $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$ is equal to,

a. $\left(x^6+\frac{1}{x^6}\right)$
b. $\left(x^6-\frac{1}{x^6}\right)$
c. $\left(x^8+\frac{1}{x^8}\right)$
d. $\left(x^8-\frac{1}{x^8}\right)$

On first examination we find the first two expressions in the product similar in so far as these are the in the form of the first factor $(a \pm b)$ in the expansion of $(a^3 \pm b^3)$. We are not surprised to find the last two terms also in the form of the second factor $(a^2 \pm ab + b^2)$ in the expansion of the same sum of cubes, $(a^3 \pm b^3)$.

It just remains to pair these two sets of factors properly. We pair factor 1 with 3 and 2 with 4 to get the target expression transformed to,

$$E=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$$

$$=\left(x^6-\frac{1}{x^6}\right).$$

We reach the solution in just one step.

Answer: Option b: $\left(x^6-\frac{1}{x^6}\right)$.

Q8. If $x+y=a$ and $xy=b^2$ then the value of $x^3-x^2y-xy^2+y^3$ in terms of a and b is,

a. a^3-4b^2a
b. a^3+3b^2
c. $(a^2+4b^2)a$
d. a^3-3b^2

We have the target expression as,

$$E = x^3 - x^2y - xy^2 + y^3$$

$$= (x+y)^3 - 3xy(x+y) - xy(x+y)$$

$$= (x+y)^3 - 4xy(x+y)$$

$$= a^3 - 4b^2a.$$

Answer: Option a: $a^3 - 4b^2a$..

Q9. If $x = 99999$, the value of $\frac{4x^3 - x}{(2x+1)(6x-3)}$ is,

a. 11111
b. 66666
c. 33333
d. 22222

The value of x invites us to seacrch for $x+1$ in the target expression as $x+1 = 100000$. But in a quick analysis we reject this path and resort to the only alternative path available to us, that is, extreme simplification of the target expression.

Solution - Simplying actions

We have the target expression,

$$E = \frac{4x^3 - x}{(2x+1)(6x-3)}$$

$$= \frac{x(4x^2-1)}{3(2x+1)(2x-1)}$$

$$= \frac{x(4x^2-1)}{3(4x^2-1)}$$

$$= \frac{x}{3}$$

$$= 33333$$

Answer: Option c: 33333.

Q10. The length of the intercept of the graph of the equation $9x - 12y = 108$ between x and y axes is,

a. 12 units
b. 18 units
c. 9 units
d. 15 units

By putting $x = 0$ for intersection of y-axis and $y = 0$ for intersection of x-axis with the straight line represented by the linear equation $9x - 12y = 108$ in x and y we get respectively,

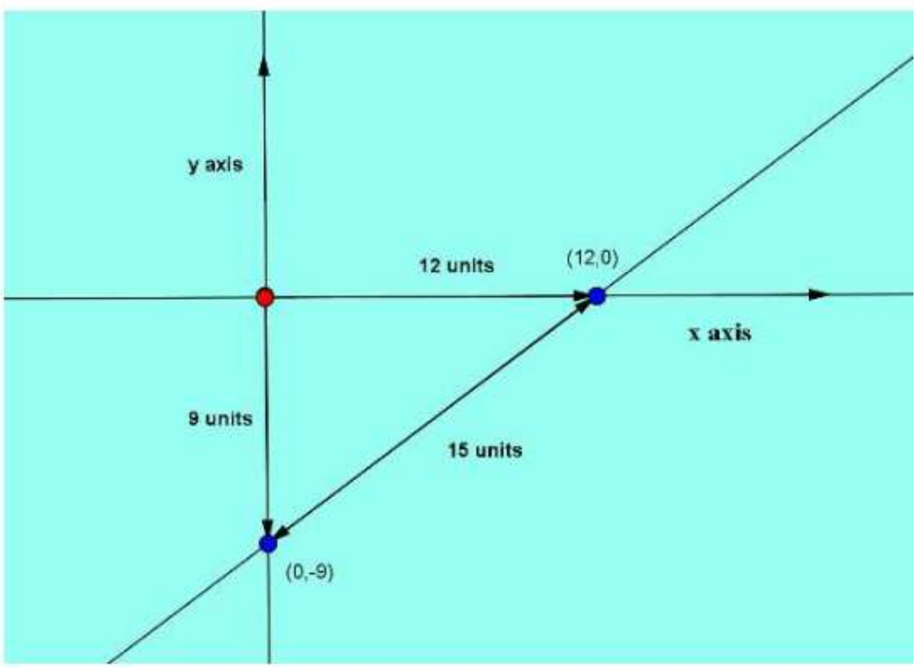
$-12y = 108$, for $x = 0$,

or, $y = -9$, and

$9x = 108$, for $y = 0$,

Or, $x = 12$.

So the straight line cuts y-axis 9 units below the x-axis in the negative y zone and similarly cuts the x-axis 12 units on the right of y-axis in positive x zone. The intercept forms the hypotenuse of a right angled triangle with the other two side lengths 12 units (on x axis from origin) and 9 units (on y-axis from origin). The following figure depicts the situation.



Applying Pythagoras theorem the length of the intercept is then,

$\sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$.

Answer: Option d: 15 units.

Triangles

A triangle is a polygon with three edges and three vertices. It is one of the basic shapes in geometry. A triangle with vertices A, B, and C is denoted as Δ ABC.

For SSC, it is an important and easy topic, a combination rarely found. The hacks and theorems below would be followed by questions related to this topic pertaining to the exact demands of the exam leaving you with knowledge as well as an appropriate approach. The important hacks and concepts are :

- **The sum of all the angles of a triangle is 180°.**

  - When base and corresponding height is given $= \frac{1}{2} \times$ base × height

  - Heron's formula (when all sides are given) = $\{s (s - a) (s - b) (s - c)\}^{1/2}$ where, s = (a+b+c)/2

  - When two sides and corresponding angle is given = $\frac{1}{2} \times a \times b \times \sin\theta$

  - When all the median are given : $\frac{4}{3} \times \{s(s - m_1) (s - m_2) (s - m_3\}^{1/2}$

  - where, s = $\frac{(m1 + m2 + m3)}{2}$ and m_1, m_2,m_3 are three medians of the triangle.
  - Note : Median is a line joining the vertex to the opposite side at midpoint.

  - When all the heights are given: $\frac{1}{\text{Area of triangle}} = 4\ \{G\ (G - 1/h_1)\ (G - 1/h_2)\ (G - 1/h_3)$,

- Note: Where, $G = \frac{1}{2}(1/h_1 + 1/h_2 + 1/h_3)$

- **Sine formulae of triangle :**

$\frac{a}{\text{Sin A}} = \frac{b}{\text{sin B}} = \frac{c}{\text{Sin C}} = 2R$ Where, R is the circumradius.

- **Cosine Formulae of triangle :**

Cos A = $(b^2 + c^2 - a^2)/ 2bc$

Cos B= $(a^2 + c^2 - b^2)/ 2ac$

Cos C = $(b^2 + a^2 - c^2)/ 2ba$

- **A result based on cosine formulae :**

If in a triangle, Cos A = $(b^2 + c^2 - a^2)/ 2bc$, Where b & c are the smaller sides then,

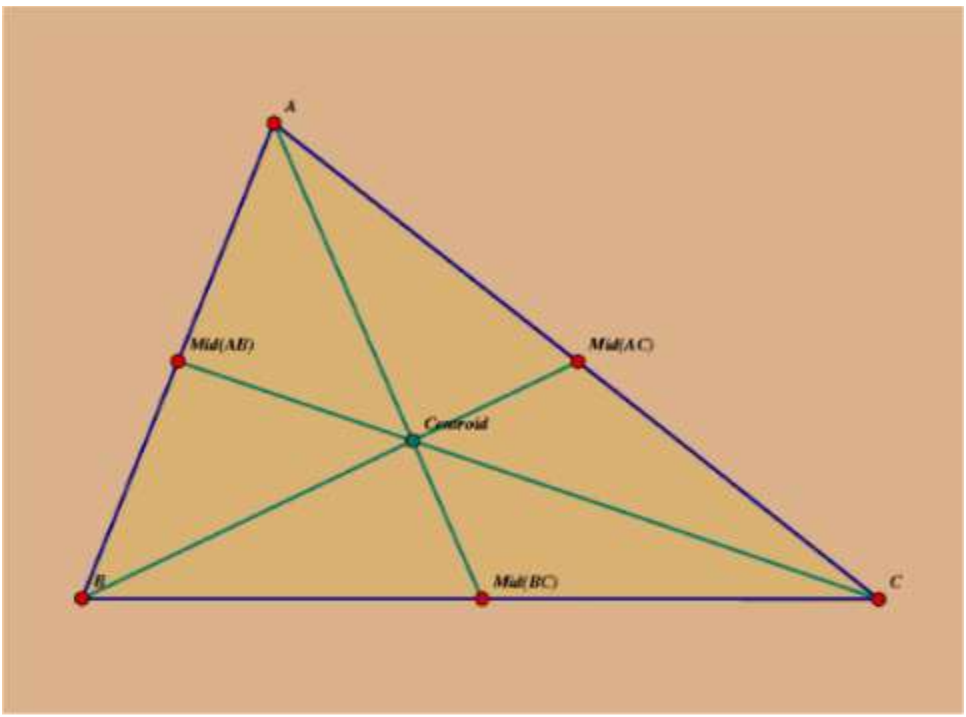
If $b^2 + c^2$ is greater than a^2, then angle A is acute.

If $b^2 + c^2$ is smaller than a^2, then angle A is obtuse.

If $b^2 + c^2$ is equals to a^2, then angle A is Right Angle.

➢ Medians of a Triangle

A median of a triangle is a line segment joining a vertex to the midpoint of the opposing side. Every triangle has exactly three medians, one from each vertex, and they all intersect each other at a point called centroid.



In the adjoining figure the midpoints of the sides of the triangle BC, AC and AB. Line segments in green are the medians joining at the midpoints of the side of the triangle. They are meeting at a common point which is the centroid.

➢ Properties of Centroid :

Centroid divides the median in the ratio 2:1.

Apollonius Theoram : To find the length of median when all sides are given,use the formula :

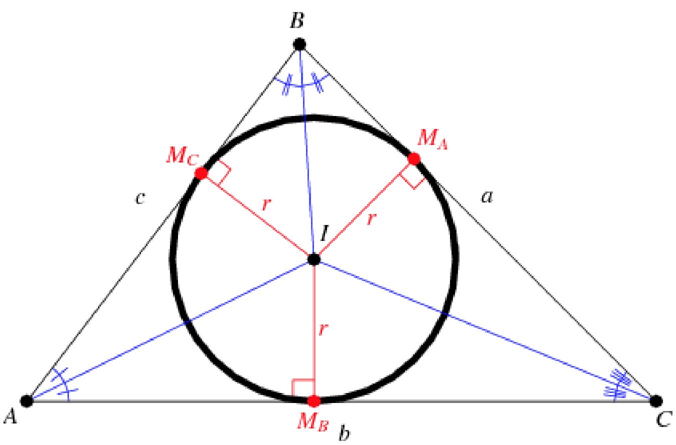
$4 \times (\text{median})^2 = 2(AC^2 + AB^2) - BC^2$

All 3 medians of a triangle divides the triangle in 6 equal parts.

Note: For isosceles and equilateral triangles, a median bisects any angle at a vertex whose two adjacent sides are equal in length.

- **Angle Bisectors**

An angle bisector divides the angle into two angles with equal measures. An angle only has one bisector. Each point of an angle bisector is equidistant from the sides of the angle.



In the adjoining figure line segments IA, IB & IC are angle bisectors and point I is Incentre of the triangle.

Incentre is the only point from which we can draw a circle inside the triangle which will touch all the sides of the triangle at exactly one point & this circle is called Incircle. The radius of this circle is known as Inradius. It is calculated by using the formula,

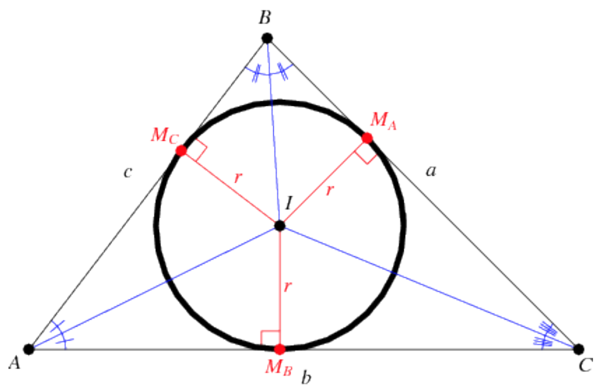
Inradius = Area of Δ ABC / Semi perimeter of Δ ABC

Angle formed at the incentre by any two angle bisector = Angle AIC = (Angle ABC / 2) + 90°.

➢ Perpendicular Bisectors of Sides of the Triangle :

A perpendicular bisector of a line segment is a line segment perpendicular to and passing through the midpoint. The perpendicular bisectors of the sides of the triangle intersect at a common point known as **circumcentre**. We can draw a circle which will enclose the triangle in a way such that all the vertices of the triangle lie on the circle. The radius of this circle is known as **circumradius**.

➢ Properties of Perpendicular Bisectors :



In the adjoining figure IM_A, IM_B and IM_C are perpendicular bisector of sides of the triangle. Point I is the circumcentre.

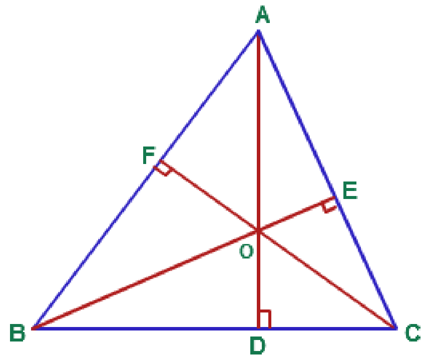
Circumradius = $\dfrac{(AC \times CB \times BA)}{4 \times \text{Area of triangle}}$

Angles formed at the circumcentre by any two line segments joining circumcentre to the vertex has this relation

Angle AIC = 2 × Angle ABC

➢ Altitudes of a Triangle :

An altitude of a triangle is a line segment through a vertex and perpendicular to the baseor the opposite side of the triange. The orthocenter of a triangle is the point where these three altitudes intersect. This point may be inside, outside or on the triangle.



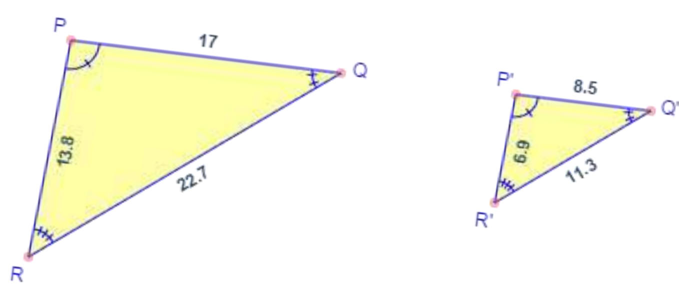
In the adjoining figure AD, BE, & CF are three altitudes of the Δ ABC and O is the orthocenter.

Angle BOC + Angle BAC = 180°

SIMILAR TRIANGLES :

This is a very easy topic under triangles and around 2-3 questions are there. Basically you get these marks for knowing just a few propeties and the section is yours. If two triangles are similar, then the ≈ symbol is used to define the relation and certain properties are to be agreed upon.

Properties of Similar Triangles:



In the adjoining figure, the ratio of sides of triangle is proportional to each other, such that

$$\frac{PQ}{PQ} = \frac{PR}{PR} = \frac{QR}{QR} \rightarrow \frac{8.5}{17} = \frac{6.9}{13.8} = \frac{11.3}{22.7} = \frac{1}{2}$$

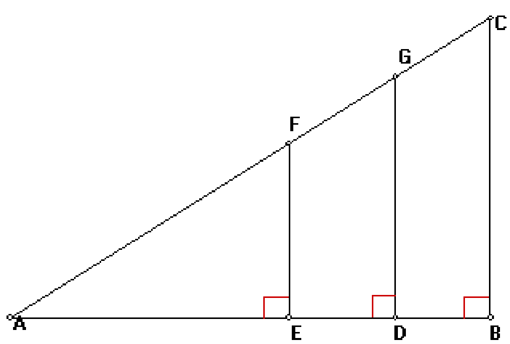
The height, angle bisector, inradius & circumradius are proportional to the sides of triangle.

$$\frac{\text{Median triangle ABC}}{\text{Median triangle DEF}} = \frac{\text{Height triangle ABC}}{\text{Height triangle DEF}} = \frac{AB}{DE}$$

The areas of the triangles are proportional to the square of the sides of the corresponding triangle.

$$\frac{\text{Area triangle ABC}}{\text{Area triangle DEF}} = \left[\frac{AB}{DE}\right]^2$$

In a right angled triangle, the triangles on each sides of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to each other and to the parent triangle too.



In the adjoining figure,

Δ ABC ≈ Δ AED

Δ ABC ≈ Δ ADG

Δ DBC ≈ Δ AEF

Proof: In Δ ABC & Δ AED. Angle B and Angle D is 90°. and Angle A is common in both. Hence by AA rule Both triangles are similar.

These hacks along with the additional knowledge of similarity must be enough to move on to the questions and practice.

Solve the questions below:

1. **In a right angled triangle the product of its two sides equals half of the square of the third side which is the hypotenuse. One of the acute angles must then be,**

A. 150°

B. 300°

C. 450°

D. 600°

Answer : C

2. **In ΔABC, two points DD and EE are taken on the lines AB and BC respectively in such a way that AC is parallel to DE. ΔABC and ΔDBE are then,**

A. always similar

B. always congruent

C. similar only if D lies outside the line segment AB

D. congruent only if D lies outside the line segment AB

Answer : A

3. The angle formed by the internal and external bisectors of an angle between them is,

A. 70°

B. 60°

C. 90°

D. 80°

Answer : D

4. O and C are the Orthocenter and Circumcenter of an acute angled triangle △PQR respectively. The points P and O are joined and extended to meet QR at S. If ∠QCR=130° and ∠PQS=60°, then ∠RPS is,

A. 100°

B. 35°

C. 30°

D. 60°

Answer : A

5. If I is the incenter of △ABC, ∠ABC=65° and ∠ACB=55°, then ∠BIC is,

A. 100°

B. 110°

C. 120°

D. 130°

Answer : B

6. If the median drawn on the base of a triangle is half its base, the triangle will be,

A. acute-angled triangle

B. obtuse-angled triangle

C. right-angled triangle

D. Equilateral triangle

Answer : B

7. △ABC is an isosceles triangle with AB=AC and AD is the median to base BC. If ∠ABC=35°, then ∠BAD is,

A. 70°

B. 50°

C. 55°

D. 110°

Answer : C

8. In isosceles triangle △FGH, FG < 3 cm and GH = 8 cm. The correct relation between FG and GH is,

A. GH = FH

B. GH < FH

C. GF = GH

D. FH > GH

Answer : C

9. The sum of three altitudes of a triangle is,

A. equal to the sum of three sides

B. twice the sum of sides

C. greater than the sum of sides

D. less than the sum of sides

Answer : A

10. The length of the sides of a triangle are 6cm, 8cm and 10cm respectively. The length of the median to the greatest side will be,

A. 5cm

B. 8cm

C. 4.8cm

D. 6cm

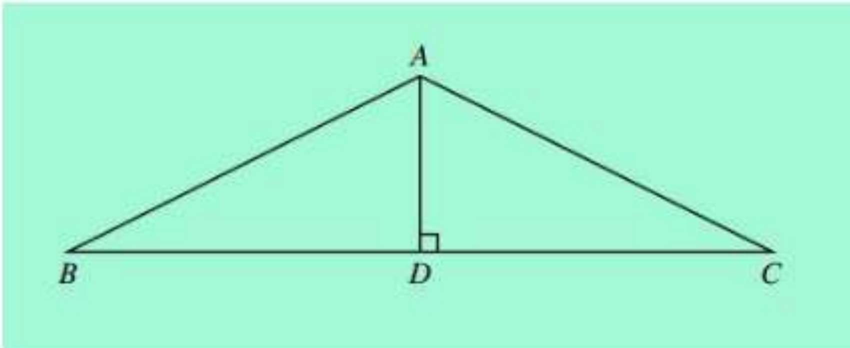
Answer : C

PRACTICE QUESTIONS

Q1. $\triangle ABC$ is an isosceles triangle with $AB = AC$ and AD as the median to base BC. If $\angle ABC = 35^0$, the $\angle BAD$ is

a. 70^0
b. 35^0
c. 55^0
d. 110^0

The following isosceles triangle has its $\angle ABC = 35^0$ and AB = AC.



As AD is median to the base BC, it bisects the side BC so that, $BD = DC$. In two triangles, $\triangle ABD$ and $\triangle ACD$ with common side AD all three pairs of corresponding sides are equal to each other and so the triangles are congruent. This gives us, $\angle ADB = \angle ADC = 90^0$. And so the $\angle BAD = 180^0 - 35^0 - 90^0 = 55^0$.

Answer: Option c: 55^0.

Q2.

In isosceles triangle $\triangle FGH$, $FG < 3$cm and $GH = 8$cm. Then the correct relation is,

a. $GH = FH$
b. $GH < FH$
c. $GF = GH$
d. $FH > GH$

If $FG < 3$cm, it is less than half of the second side $GH = 8$cm. It means if in the isosceles triangle, the third side equals the smaller side, the two of them will be smaller than the other side GH which violates the basic condition of formation of a triangle. The basic law of formation of a triangle states,

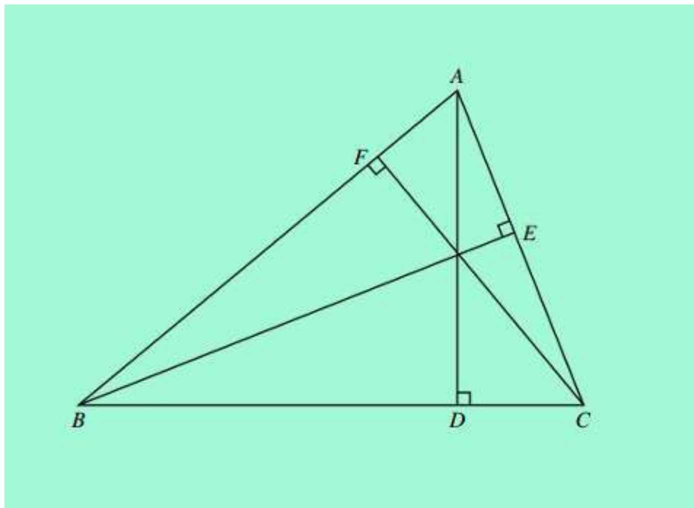
> *Sum of lengths of any two sides of a triangle must be larger than the third side.*

Thus, the equal sides are the larger sides, that is, $GH = FH = 8$cm.

Answer: Option a: $GH = FH$.

Q3. The sum of three altitudes of a triangle is,

a. equal to the sum of three sides
b. twice the sum of sides
c. greater than the sum of sides
d. less than the sum of sides



The altitudes of the triangle $\triangle ABC$ are AD, BE and CF. As these are the heights, these are the shortest distances to the opposite sides, that is, a height is lesser in length than both its adjacent sides. To be specific, the length of AD will be less than both the adjacent sides, AB and AC. This will be true for the other two altitudes BE and CF. So if you add up **One adjacent side corresponding to each height,** the sum of heights will always be less than the sum of three sides.

For example we may add up, BA for AD, AC for CF and CB for BE.

Or, $(AD + CF + BE) < (BA + AC + CB)$

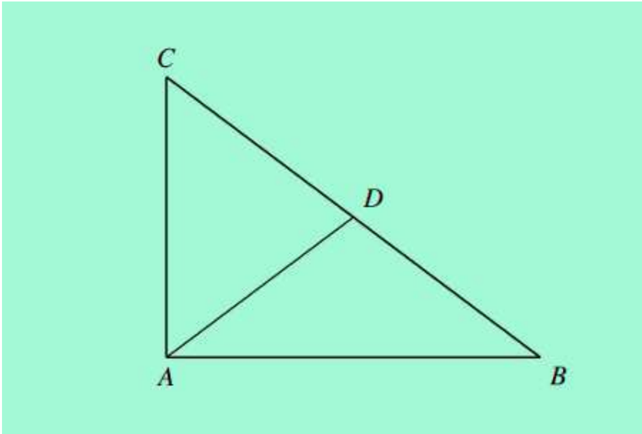
as, $AD < BA$, $CF < AC$ and $BE < CB$.

Answer: Option d: less than the sum of sides.

Q4. The length of 3 sides of a triangle are, 6cm, 8cm and 10cm. The length of the median to the greatest side is then,

a. 5cm
b. 8cm
c. 4.8cm
d. 6cm

Q5.



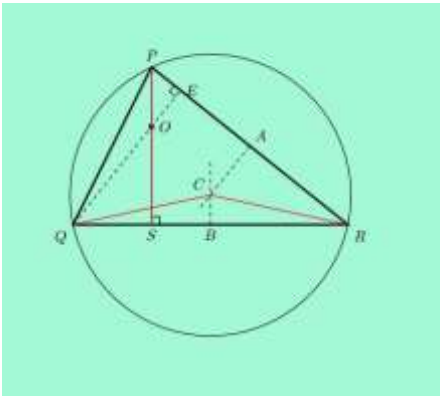
As $6^2 + 8^2 = 10^2$ in the given triangle, it is a right triangle $\triangle ABC$ with base side $AB = 8$cm, height side $AC = 6$cm and hypotenuse $BC = 10$cm. AD is the median drawn from A to center point D of largest side BC.

We know that the diameter of a circle subtends an angle of 90^0 at its periphery and so we may consider the three vertices of the triangle to lie on the periphery of the circumscribing circle with diameter as BC and center at D. So, AD will be another radius and will be equal to $BD = 5$cm.

Answer: a: 5cm.

O and C are the Orthocenter and the Circumcenter of an acute angled triangle $\triangle PQR$ respectively. The points P and O are joined and produced to meet the side QR at S. If $\angle QCR = 130^0$ and $\angle PQS = 60^0$ then $\angle RPS$ is,

a. 100^0
b. 35^0
c. 30^0
d. 60^0



The chord QR subtends an angle $\angle QCR = 130^0$ at the center that is double the $\angle RPQ$ subtended at the periphery. So, $\angle RPQ = 65^0$.

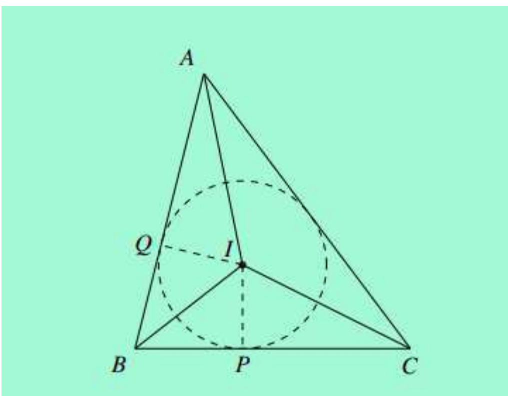
Again in right-angled $\triangle PQS$, $\angle PQS = 60^0$. So, the other angle in the $\triangle PQS$, $\angle QPS = 30^0$.

Finally then, desired $\angle RPS = \angle RPQ - \angle QPS = 65^0 - 30^0 = 35^0$.

Answer: b: 35^0.

If I is the incenter of $\triangle ABC$, $\angle ABC = 65^0$ and $\angle ACB = 55^0$, the $\angle BIC$ is,

a. 110^0
b. 120^0
c. 130^0
d. 140^0



In $\triangle BIC$ as BI and CI are angle bisectors,

$$\angle BIC = 180^0 - \tfrac{1}{2}(\angle ABC + \angle ACB)$$
$$= 180^0 - \tfrac{1}{2}(65^0 + 55^0)$$
$$= 120^0.$$

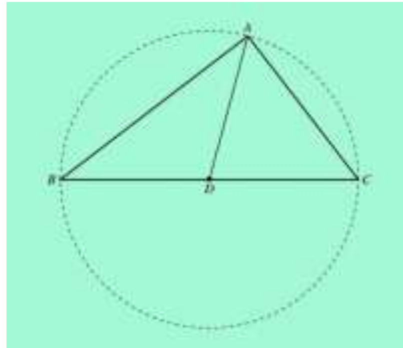
Answer: b: 120^0.

Q6.

Q7. If the median drawn on the base of a triangle is half its base, the triangle will be,

1. acute-angled
2. obtuse-angled
3. right-angled
4. equilateral

Q8.

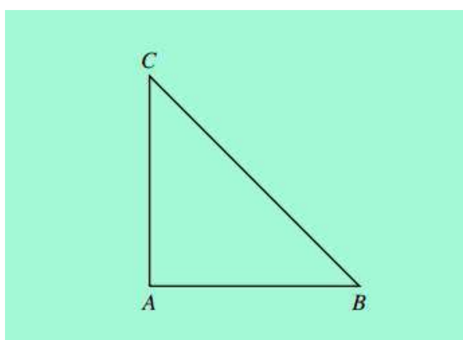


The median AD drawn on the base BC of the triangle $\triangle ABC$, is equal to half of BC, that is, $AD = BD = DC$. This is the situation where we can consider the point D as the center of a circle with the three points A, B and C lying on the periphery of the circle and $AD = BD = CD$ as the radii and BC as the diameter. As the diameter subtends an angle of 90^0, the triangle is a right angled triangle.

Answer: c: right-angled.

In a right angled triangle the product of its two sides equals half of the square of the third side which is the hypotenuse. One of the acute angles must then be,

a. 15^0
b. 30^0
c. 45^0
d. 60^0



By the definition of the problem we have,

$AC \times AB = \frac{1}{2}BC^2$,

Or, $2AC \times AB = BC^2$.

Again by Pythagoras theorem,

$AC^2 + AB^2 = BC^2$.

Subtracting the first equation from the second we have,

$(AC - AB)^2 = 0$,

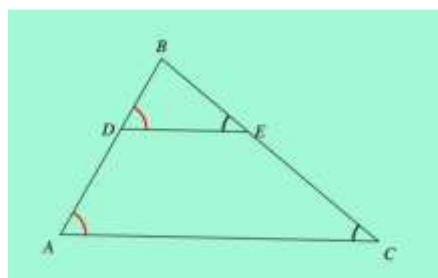
Or, $AC = AB$.

It means the right-angled triangle is also an isosceles triangle with angles = 45^0.

Answer: c: 45^0

In $\triangle ABC$, two points D and E are taken on the lines AB and BC respectively in such a way that AC is parallel to DE. The $\triangle ABC$ and $\triangle DBE$ are then,

a. always similar
b. always congruent
c. similar only if D lies outside the line segment AB
d. congruent only if D lies outside the line segment AB



Q9.

As $AC||DE$, in two triangles $\triangle ABC$ and $\triangle DBE$, $\angle CAB = \angle EDB$, and $\angle ACB = \angle DEB$. Apex angle being common, all three angles of $\triangle ABC$ equal the corresponding angles in $\triangle DBE$. So the two triangles are **similar** to each other.

As D and E are moved down the sides BA and BC respectively keeping the paralellism of $AC||DE$ undisturbed, the two triangles $\triangle ABC$ and $\triangle DBE$ remain similar to each other even when point D lies outside the line segment AB.

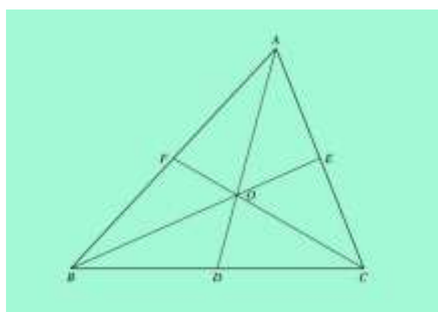
Only when D and E coincide with A and C respectively, the two triangles become congruent to each other. But even in this case, the *similarity characteristic holds.*

So similarity holds always, whereas congruency hold only on one occasion.

Answer: a: always similar.

Q10. AD is a median of $\triangle ABC$ and O is the centroid such that $AO = 10cm$. Length of OD (in cm) is,

1. 7
2. 5
3. 4
4. 2

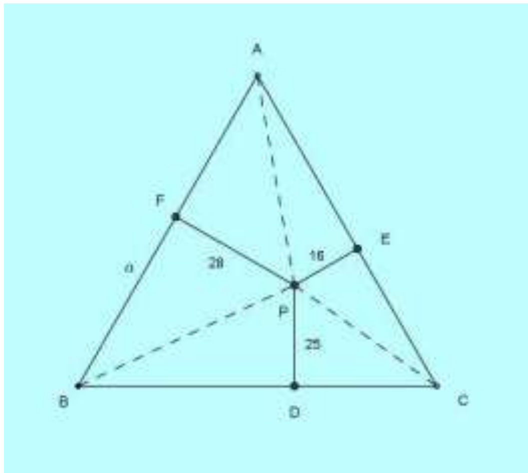


O is the centroid of the triangle $\triangle ABC$ and is the point of intersection of its three medians. As the centroid divides a median in the ratio of 2 : 1 from the vertice, in this case we have, $AO : OD = 2 : 1$ As $AO = 10$cm, OD is half of it, that is, 5cm.

Answer: b: 5.

Q11 Lengths of three perpendiculars from a point inside an equilateral triangle on the three sides being 16 cm, 25 cm and 28 cm, the area of the triangle (in cm^2) will be.

a. $1587\sqrt{3}$
b. 1587
c. 2116
d. $2116\sqrt{3}$



Proceeding in this way,

$$A_{ABC} = A_{PAB} + A_{PBC} + A_{PCA},$$

Or, $A_{ABC} = \frac{1}{2}a(28 + 25 + 16) = \frac{69}{2}a.$

Again $\triangle ABC$ being an equilateral triangle of side length a its area is,

$$A_{ABC} = \frac{\sqrt{3}}{4}a^2 = \frac{69}{2}a,$$

Or, $a = \frac{4}{\sqrt{3}} \times \frac{69}{2} = 46\sqrt{3}.$

So the desired area of the triangle is,

$$A_{ABC} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (46\sqrt{3})^2 = 1587\sqrt{3}\text{ cm}^2.$$

Answer: a: $1587\sqrt{3}$.

Q12 If the ratio of sines of angles of a triangle is $1:1:\sqrt{2}$, the ratio of square of the greatest side to sum of the squares of the other two sides is,

a. 1:2
b. 1:1
c. 2:1
d. 3:4

The problem definition clearly urges us to use the important concept of laws of Sines in a triangle,

$$\frac{a}{sin\ A} = \frac{b}{sin\ B} = \frac{c}{sin\ C},$$

where the sides are, a, b and c and the corresponding opposite angles are $\angle A$, $\angle B$ and $\angle C$.

Assuming the angles corresponding to the three ratio values as, $\angle A$, $\angle B$, and $\angle C$ and using the first pair of ratio variables and laws of sines we get,

$$\frac{a}{sin\ A} = \frac{b}{sin\ B},$$

Or, $\frac{a}{b} = \frac{sin\ A}{sin\ B} = 1.$

So,

$a = b$, and

$\angle A = \angle B.$

Using the second pair of ratio value with laws of sines,

$$\frac{b}{c} = \frac{sin\ B}{sin\ C} = \frac{1}{\sqrt{2}},$$

Or, $c = \sqrt{2}b.$

So, c is the largest side.

Ratio of the three sides is,

$a : b : c = b : b : \sqrt{2}b.$

Thus ratio of square of the largest side to the sum of squares of the other two sides is,

$$\frac{2b^2}{b^2 + b^2} = 1 : 1$$

Answer: b: 1:1.

Q13 The perimeters of similar triangles $\triangle ABC$ and $\triangle PQR$ are 24 cm and 36 cm respectively. If $AB = 10$ cm, then PQ (in cm) is,

a. 18
b. 12
c. 15
d. 16

As the $\triangle ABC$ and $\triangle PQR$ are similar, the ratio of corresponding sides are equal,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = p, \text{ say.}$$

So perimeters of the two triangles are related as,

$$AB + BC + CA = p(PQ + QR + RP),$$

Or, $24 = 36p$,

Or, $p = \frac{24}{36} = \frac{2}{3}$.

Again,

$$AB = 10 = p \times PQ = \frac{2}{3}PQ.$$

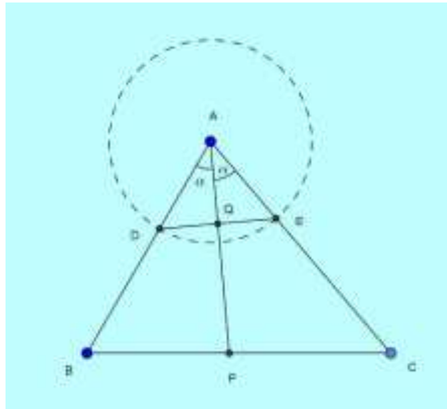
Finally then,

$$PQ = \frac{3}{2} \times 10 = 15 \text{ cm}$$

Answer: c: 15.

Q14 In $\triangle ABC$, the bisector of $\angle A$ is AP and it meets BC at P. If a line DE intersects AB, AP and AC at D, Q and E respectively and is perpendicular to AP, then which of the following is true?

a. AQ=QP
b. AD=AE
c. BP=PC
d. QP=EC



Let us analyze the angles in the two triangles, $\triangle AQD$ and $\triangle AQE$.

The triangles are similar as $\angle DAQ = \angle EAQ$ and a second pair of angles are 90^0. So the third pair of angles are also equal, and the triangles are similar to each other.

As we know, in similar triangles ratio of corresponding sides are equal.

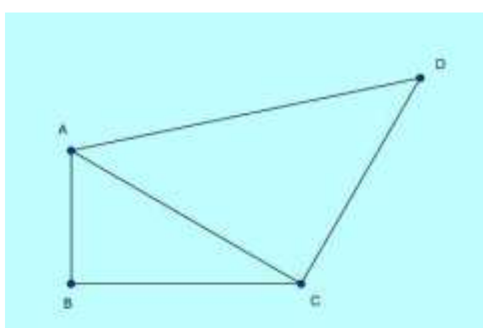
But as, the two triangles have a common side AQ which represents a pair of corresponding sides in the two adjoining triangles, the ratio of corresponding sides are $1:1$. In other words the corresponding sides are equal.

This results in, $AD = AE$.

Answer: b: AD=AE.

Q15 In a quadrilateral ABCD, $\angle B = 90^0$, and $AD^2 = AB^2 + BC^2 + CD^2$. Then the $\angle ACD$ is,

a. 45^0
b. 60^0
c. 50^0
d. 90^0



Given $\angle ABD = 90^0$ is the only angle information. the other relationship is on sides, that too of squares. This immediately urges us to use the Pythagoras theorem that converts the property of a 90^0 in a triangle to a relationship of squares of sides.

In $\triangle ABC$ formed by the diagonal AC and the two sides AB and BC,

$$AC^2 = AB^2 + BC^2.$$

Using this result with the given relation of squares of sides,

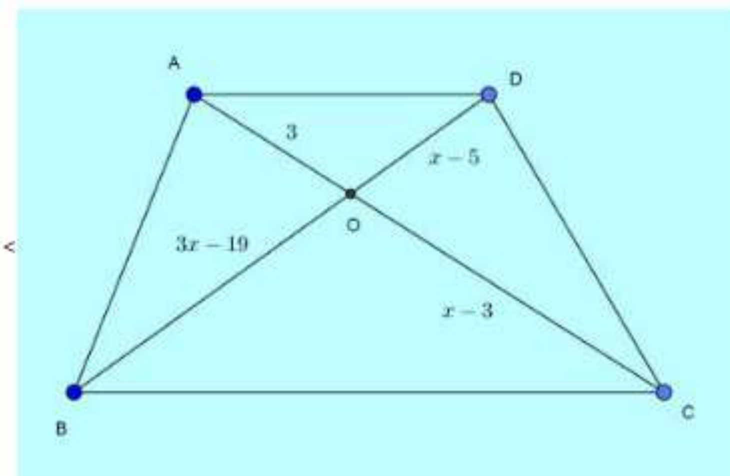
$$AD^2 = AB^2 + BC^2 + CD^2 = AC^2 + CD^2.$$

It means, in the $\triangle ACD$, AD must be the hypotenuse. In other words, $\angle ACD = 90^0$.

Answer: d: 90^0.

Q16

In figure below the lengths of the four sections of the two diagonals of a trapezium ABCD, namely, AO, BO, CO and DO are given.



Find the possible values of x.

a. 6, 8
b. 8, 9
c. 9, 7
d. 7, 8

The most important characteristic of a trapezium that we recognize is, two sides of it AD and BC parallel to each other. This results in the two triangles $\triangle AOD$ and BOC as similar because, at the diagonal intersecting point O opposite angle are equal as well as opposite internal angles of intersection of AC and BD with parallel lines AD and BC are equal.

This results in equality of ratios of corresponding sides of the two triangles. We are particularly interested in the ratios involving the four diagonal sections as,

$$\frac{3x-19}{x-5}=\frac{x-3}{3}.$$

To simplify the numerator of the LHS we multiply both denominators by 3 and subtract 1 from both sides,

$$\frac{3x-19}{3x-15}-1=\frac{x-3}{9}-1,$$

Or, $\frac{-4}{x-5}=\frac{x-12}{3}$,

Or, $-12=x^2-17x+60$,

Or, $x^2-17x+72=0$,

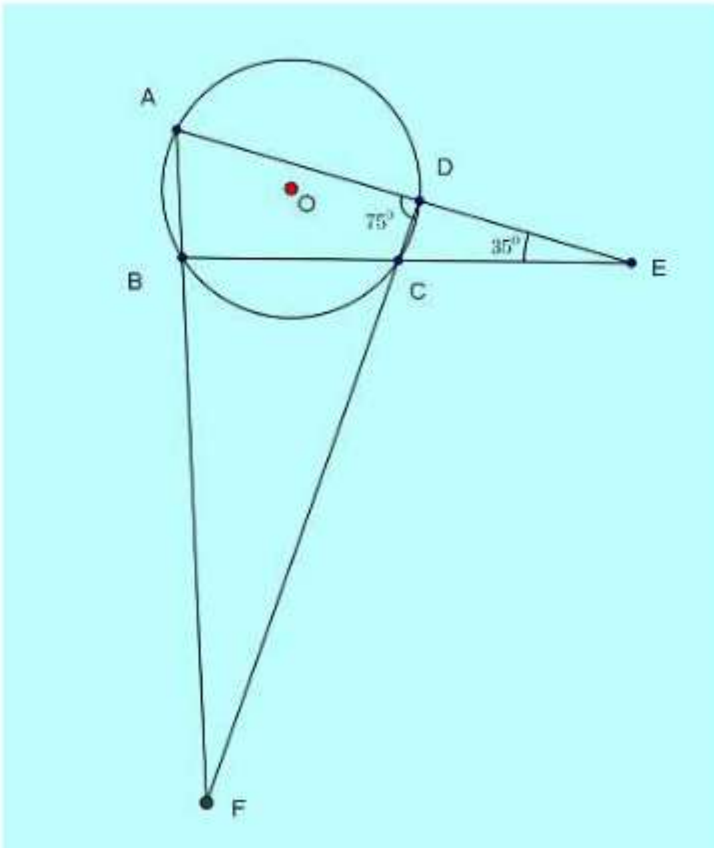
Or, $(x-8)(x-9)=0$.

So the possible values of x are 8 and 9.

Answer: b: 8, 9.

Q17

In figure below $\angle AEB = 35^0$ and $\angle ADF = 75^0$.



The $\angle AFD$ is,

a. 65^0
b. 45^0
c. 35^0
d. 55^0

In $\triangle CDE$, $\angle CDE = 180^0 - 75^0 = 105^0$, being an angle at the point of incidence of FD on AE forming two angles with $\angle CDA = 75^0$.

So,

$\angle DCE = \angle BCF = 180^0 - (105^0 + 35^0) = 40^0$, $\angle BCF$ being the opposite angle to $\angle DCE$ at the point of intersection of two lines.

Now in cyclic quadrilateral ABCD as sum of two opposite angles is 180^0,

$\angle ABC = 180^0 - 75^0 = 105^0$.

Lastly, $\angle ABC = 105^0$ is the external angle to the $\triangle BFC$, and so it equals sum of two opposite internal angles,

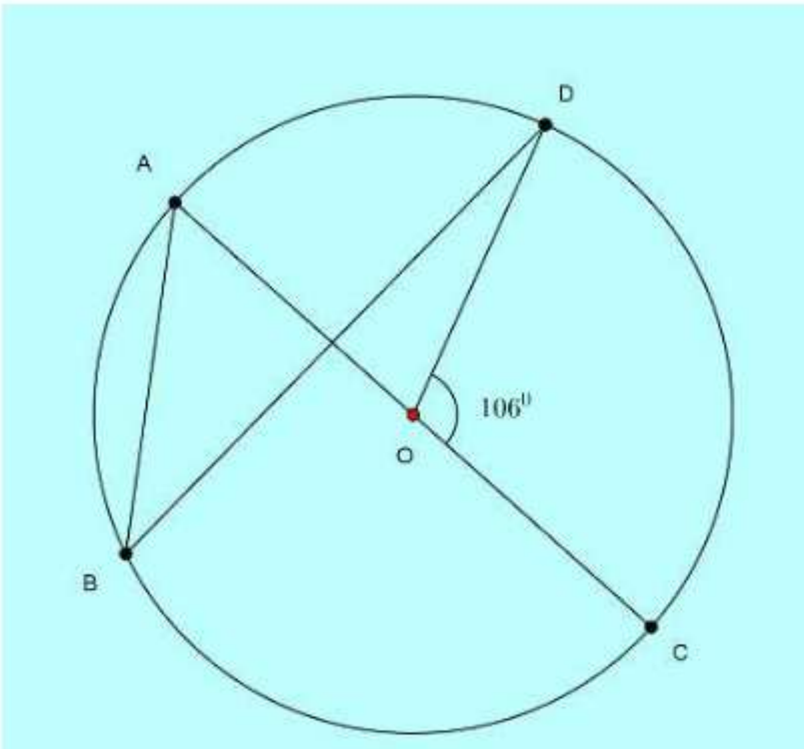
$\angle ABC = 105^0 = \angle BFC + \angle BCF$,

Or, $\angle AFD = \angle BFC = 105^0 - 40^0 = 65^0$.

Answer: a: 65^0.

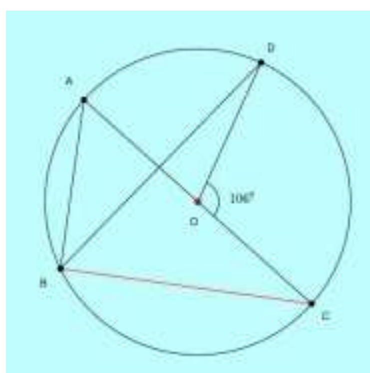
Q18.

In the figure below, O is the centre of a circle with $\angle COD = 106^0$ and AC as the d

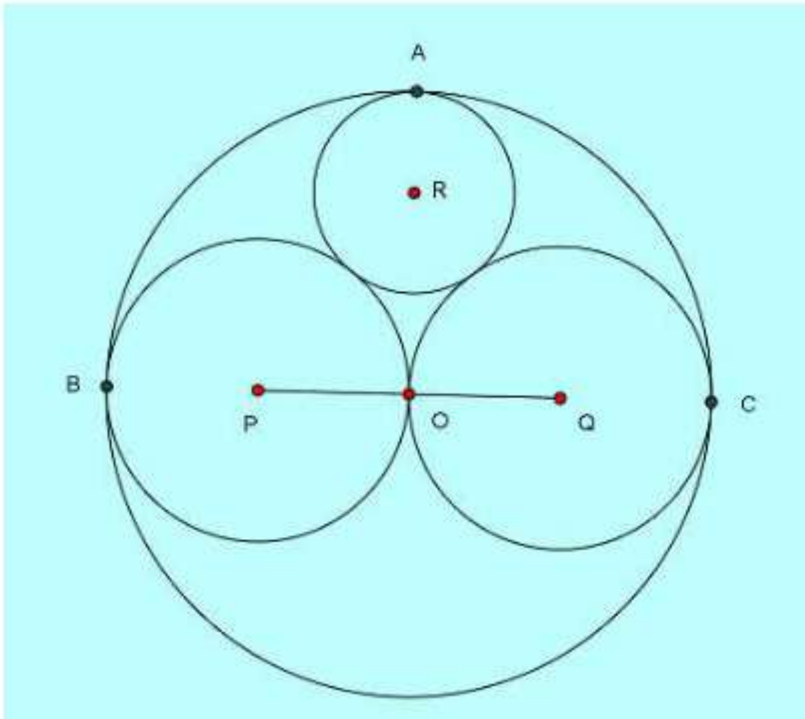


Find $\angle ABD$.

a. 55^0
b. 40^0
c. 37^0
d. 43^0

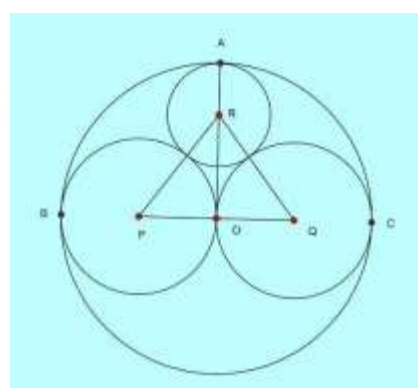


Q19. In the following figure, two circles with centres at P and Q and each of radius 1 cm touch each other at O which is the centre of a third circle of radius 2 cm. The fourth circle with centre at R touches all the three circles.



The radius of the smallest fourth circle (in cm) is,

a. 5
b. $\frac{2}{3}$
c. $\frac{3}{5}$
d. $\frac{3}{7}$



We introduce a new geometric element, that is, line BC, as a key to the solution.

By arc angle subtending concept $\angle CBD$ will be half of $\angle COD = 106^0$, that is, $\angle CBD = \frac{1}{2}$ of $(106^0) = 53^0$.

As AC is the diameter, $\angle ABC = 90^0$.

So,

$\angle ABD = 90^0 - 53^0 = 37^0$.

Answer: c: 37^0.

Let the radius of the smallest fourth circle be r.

As PR and QR both connect the centre of 1 cm radius circle to centre of the smallest circle passing through the common tangent points, $PR = QR = 1 + r$, and angle bisector of $\angle PRQ$ which is PO is perpendicular to PQ and also bisects it.

Additionally, O being the tangent point of the two 1 cm radius circles, as well as the centre of the larger 2 cm radius circle, OR extended to tangent point A, that is, OA is the radius of the larger circle.

With this background in place let us relate the sides of the $\triangle PRO$ applying Pythagoras theorem.

In $\triangle PRO$,

$PR^2 = OP^2 + OR^2$,

Or, $(1+r)^2 = 1^2 + (OA - AR)^2$,

Or, $1 + 2r + r^2 = 1 + (2^2 - 4r + r^2)$,

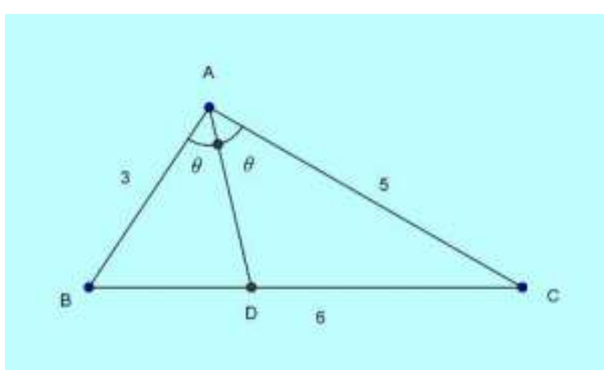
Or, $6r = 4$,

Or, $r = \frac{2}{3}$ cm.

Answer: b: $\frac{2}{3}$.

Q20. In a $\triangle ABC$, $AB = 3$ cm, $BC = 6$ cm and $CA = 5$ cm. Bisector of $\angle A$ meets BC at D. The length of BD is,

a. 2.25 cm
b. 2 cm
c. 2.5 cm
d. 3 cm



By laws of sines, in $\triangle ABD$,

$$\frac{BD}{sin\theta} = \frac{AD}{sin\ B},$$

Or, $AD = \dfrac{sin\ B}{sin\theta} BD.$

Similarly applying laws of sines in $\triangle ACD$ we get,

$$AD = \frac{sin\ C}{sin\theta} CD.$$

Equating the two relations and eliminating common denominator of $sin\theta$ we get,

$$BD \times\ sin\ B = CD \times\ sin\ C,$$

Or, $\dfrac{CD}{BD} = \dfrac{sin\ B}{sin\ C}.$

We have used the common element AD in the two triangles to link these two parts of $\triangle ABC$. This is application of ***link element use technique.***

Now is the time to use the basic trigonometric concept of sine-cosine complementary function relation,

Circles

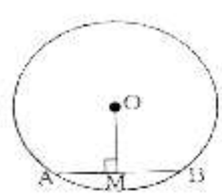
A circle is the set or locus of all points in a plane that are at an equivalent distance from a given point, the centre. The distance between any of the points and the centre is called the radius.

Geometry is easy if you can visualise and circles constitute a large chunk of the questions that are asked in SSC. The best part about the questions in circles is that you just need to find a relation of the unknown and apply theorems to equate this relation. There are not many different varieties of questions under this and we have covered all types in the questions carefully. A few important terms that you need to know before we begin with the theorems and questions are :

- **Secant**: A line which touches the circle at two distinct points.
- **Chord** : A line Segment which lies inside the circle and its end points always lie on the circumference of the circle.
- **Diameter** : The chord which passes through the centre of the circle.
  It is double of the radius .i.e. D = 2R.
- **Tangent**: A line which touches the circle at only one point at its circumference.
  This is the most important and we will discuss this in detail after we complete circles.

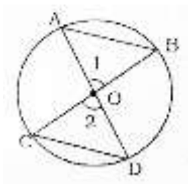
The important hacks and theorems for Circles follow :

- If two chords of a circle are given, the one which is greater is nearer to the centre.

- The perpendicular from the centre of the circle bisects the chord implying that the radius always bisects the chord if perpendicular. 

- Any line segment joining the centre of the circle to the mid point of the chord is perpendicular to the chord.

- Equal chords of the circle always subtends equal angles at the centre of the circle.

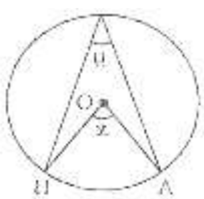
- 

- If the angle subtended by two chords at the centre are equal then the chords are always equal.

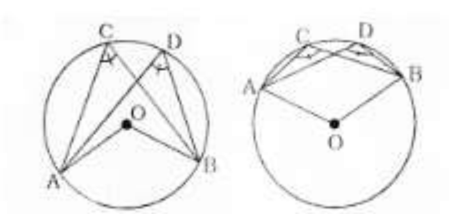
- Equal chords of the circle are at equal distance from the centre.

- Chords which are at equidistant from the centre of the circle are always equal.

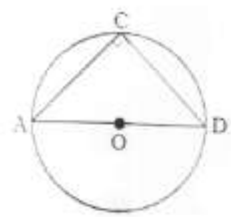
- The angle subtended by any arc at the centre of the circle is double the angle subtended by it at any point on the remaining part of the circle. $\angle x = 2\angle y$



- The angles subtended by an arc in the same segment of the circle are equal. $\angle ACB = \angle ADB$



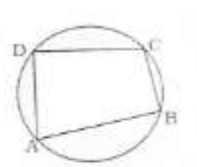
- The angle in a semi circle is always a right angle meaning the angle subtended by the



diameter is always a right angle.

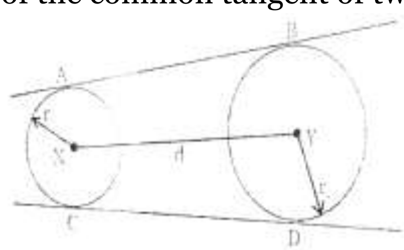
- The circle drawn with the hypotenuse of a right angle triangle as diameter, passes through its opposite vertex.

- The sum of the opposite angles of a cyclic quadrilateral is always 180°.
(Cyclic quadrilateral is a quadrilateral, all the vertices of which lie on a single circle.)



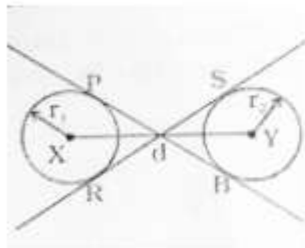
- If two circles touch each other internally or externally, the point of contact P lies on the line joining their centres.
Distance between the centres, when they touch internally =AP - BP
Distance between the centres, when they touch externally, distance =AP + BP

- If R_1 and R_2 are the radius of two circles and d is the distance between the centres of the circle then the length of the common tangent of two circles,



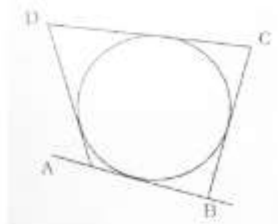
= $= \sqrt{d^2 - (r_1 - r_2)^2}$

- If R_1 and R_2 are the two radius of a circle and d is the distance between them then the length of the transverse common tangent,



= $= \sqrt{d^2 - (r_1 + r_2)^2}$

- If a circle touches all the four sides of a quadrilateral then sum of the opposite pair of



sides are equal.

- If two chords of a circle are equal then their angle bisector of the angle formed by these passes through the centre of the cirlcle.

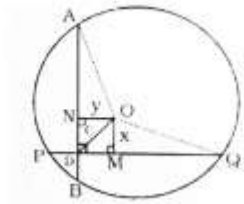
- The equilateral formed by the angle bisectors of a cyclic quadrilateral is also cyclic.

- If a cyclic trapezium is isosceles, its diagonals are equal.

- An angle in the major segment of a circle is acute while an angle in the minor segment is obtuse.

- If two circles of same radius r are such that the centre of one lies on the circumference of the other then,
Length of the common chord, $l = \sqrt{3} \times r$

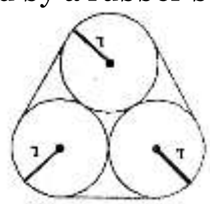
- If 2a and 2b are length of two chords which intersect at right angle and if the distance between the centre of the circle and the intersecting point of the chords is C then,



Radius of circle = $\frac{\sqrt{a^2+b^2+c^2}}{2}$

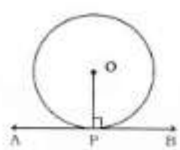
- If three circles of radius r are bound by a rubber band then,

Length of rubber band = $6r + 2\pi r$



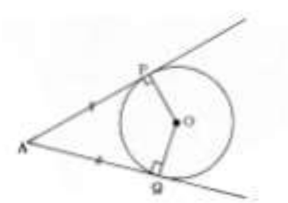
Now let's move to Tangents :

- Tangent and radius always make a right angle at the point where the tangent touches the circle.

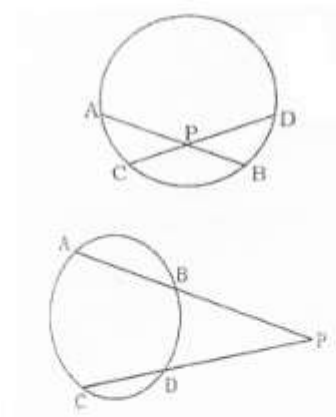


- Length of two tangents drawn from the same external point to a circle is always equal.

AP=AQ



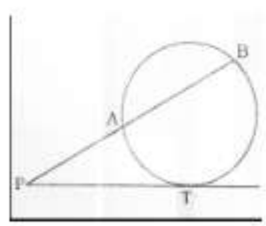
- If two chords AB and CD intersect internally or externally at point P then,



$PA \times PB = PC \times PD$

- If PAB is a secant which intersects the circle at A and B and PT is a tangent at T, then

$PT^2=PA*PB$

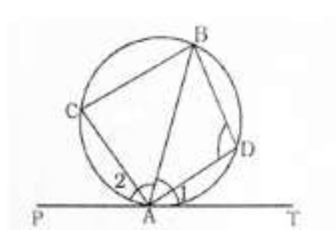


$PT^2 = PA \times PB$

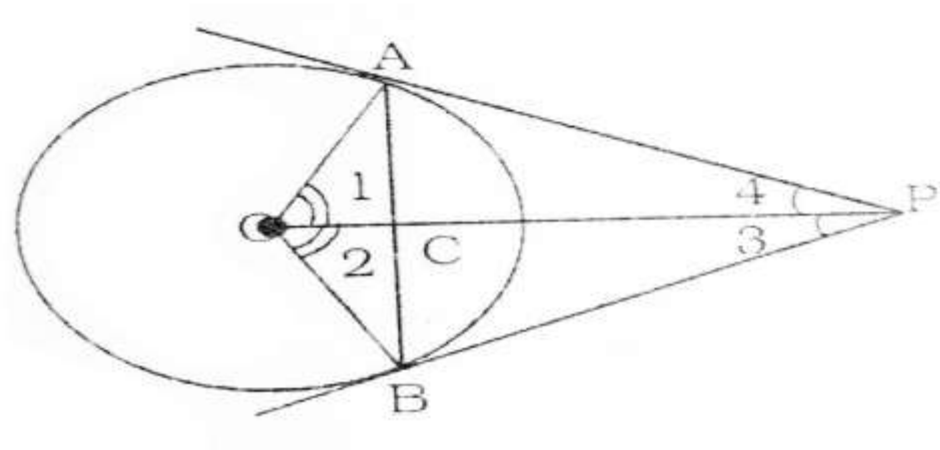
- If from the point of contact of tangent with the circle, a chord is drawn ,then the angles which the chord makes with the tangent line are equal to the angles formed in the corresponding alternate segment respectively.

$\angle BAT=\angle BCA=\angle 1$

$\angle BAP=\angle BDA=\angle 2$



- If two tangents PA and PB are drawn from the external point P, then according to this figure :



$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

- OP is perpendicular to AB and AC=BC.

Solve the questions that follow:

Q1. **AC and BC are two equal chords of a circle. BA is produced to any point P and CP when joined cuts circle at T. Then**

A) CT : TP = AB : CA
B) CT : TP = CA : AB
C) CT : CB = CA : CP
D) CT : CB = CP : CA

Draw the figure according to the question. Now concentrate on the two triangles CPA & CAT. We have to choose these two triangles only as the ratio of the sides of only these triangles is given in the answer options. CB too is there but it's equal to CA only.

Well, now see if the angles of these triangles could be proved equal. As the quadrilateral CTAB is cyclic, angles CTA+CBA=180; and angles CTA+ATP also equal to 180 (angles on straight line), so ∠ATP = ∠CBA.

Now you can easily show that ∠CAP= ∠CTA. ∠PCA= ∠ACT (same angle). So ∠CAP= ∠CAT (remaining angles}. From above you can see that these two triangles are similar. Now think over a bit, you'll find option 'C' as correct.

Q2. **P & Q are mid-points of two chords AB and AC of a circle. Center is O, OP and OQ are extended to meet the circle at R & S respectively. T is a point anywhere on the major arc RS of the circle. ∠BAC = 32°. Find ∠RTS.**

A) 32
B) 74
C) 106
D) 64

Concentrate on the quadrilateral OPAQ. In this ∠OPA = ∠OQA = 90° (both P & Q are mid points of two chords and they join the centre O.

So their sum equals 180°. Means the sum of the two remaining angles of the quadrilateral OPAQ i.e. ∠PAQ and ∠POQ is also 180°. But ∠PAQ=32° (∠PAQ and ∠BAC are the same). So ∠POQ=180 - 32 = 148.

But angles POQ and ROS are same; therefore ∠ROS = 148. Therefore ∠RTS = 148/2 = 74 (angle subtended by the arc RS on the remaining part of the circle) [option 'B']

THEOREM: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Q3. **In clock between 5 and 6 when hour and minute hands will stick together?**

A) 5 hour 25 min
B) 5 hours 300/11 min
C) 5 hr 289/11 min

AT 5.00 Angle = 150°. Now with the every 1′, the difference is reduced by (11/2)°. Hence to stick together difference must be 0°.

NOW, (11/2)° in 1′, Hence 150° in 150*2/11 = 300/11 min. So required time = 5 hrs + 300/11 min (option 'B')

Q4. **O is the center of a circle and OX and OY are two radii; ∠XOY = 90 and area of ΔXOY is 32 cm². Find the Area of the circle.**

A) 60
B) 64π
C) 32π
D) 64

The ΔXOC is a right angle triangle which is isosceles also where OX = OC (each a radius of the same circle). We know area of an isosceles right triangle is side^2 × 1/2; where side is the

But area is 32 cm^2

Therefore radius^2 × ½ = 32

=> radius=8

Now the area of the circle=$\pi 8^2$

=64π (option 'B')

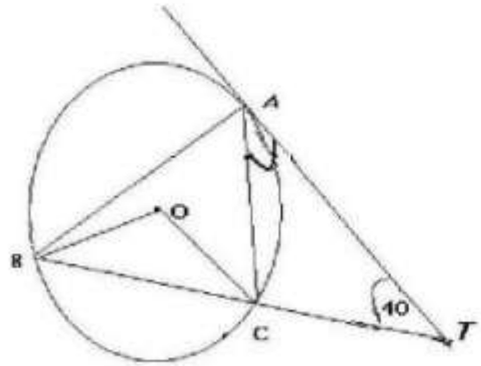
Q5. **A, B, C are three points on a circle. The tangent at A meets BC produced at T, ∠BTA=40, ∠CAT=44. The angle subtended by BC at the centre of the circle is ?**

A) 84

B) 92

C) 96

D) 104



Given ∠CAT = 44° and ∠CTA = 40°

We have ∠CAT = ∠ABC = 44° (alternate segment of the circle)

Again ∠BCA = ∠CAT + ∠CTA = 44 + 40 = 84° (exterior angle of a triangle)

Now ∠ABC + ∠BCA + ∠CAB = 180°
=> 44° + 84° + ∠CAB = 180°
=> ∠CAB = 52°

Again ∠BOC = 2∠CAB = 2 × 52 = 104° (angle subtended by a chord on the center is double to the angle subtended on the circle) [option 'D']

Q6. **A, B, C are 3 points on the circumference of a circle. Chord AB forms 90° at the centre and chord AC forms 110°. Find the ∠BAC.**

A) 160°
B) 180°
C) 80°
D) 100°

Let the center of the circle be ‘O’; then ∠AOB + ∠AOC + ∠BOC = 360

=> 90 + 110 + ∠BOC = 360
=> BOC = 160

But ∠BAC is subtended on the other part of the arc BOC.
So ∠BOC = 2∠BAC (The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

=> ∠BAC = 160/2 = 80° (option ‘C)

Q7. **The area of circle whose radius is 8 cm is trisected by two concentric circles. The ratio of radii of the concentric circles in ascending order is ?**

A) 1, 2, 3
B) 2, 3, 5
C) 1, $\sqrt{2}$, $\sqrt{3}$
D) $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

Let the radii be a, b, c in ascending order
Since the area is trisected, therefore $\pi a\^2 = \pi(b^2 - a^2) = \pi(c^2 - b^2)$
From 1 & 2
b = $\sqrt{2}$a

From 2 & 3
c = $\sqrt{3}$a

=> a : b : c = a : $\sqrt{2}$a : $\sqrt{3}$a
= 1 : $\sqrt{2}$: $\sqrt{3}$ (dividing all by ‘a’) Option ‘C’

Q8. **If the center of a circle passing through points A, B, C makes ∠BAO = 30° and ∠BCO = 40°; find ∠AOC.**

A) 240
B) 210
C) 220
D) 120

As the circle passes through A, B, C, so according to the question BA and BC are two chords of the circle with B as the common point.
∠BAO = 30 & ∠BOC = 40 (given)
Therefore ∠ABC = ∠BAO + ∠BOC = 30+40 = 70 (Angle at the meeting point of two chords of the same circle is the sum of angles made by their end points with the center)

Again ABCO is a quadrilateral with ∠ABC & ∠AOC, and ∠BAO & ∠BCO as two opposite pairs.
Therefore ∠ABC + ∠AOC = 360 – 70 = 290
=> ∠AOC = 290 – ∠ABC
= 290 – 70 = 220 (option 'C).

NOTE: If 220 is not in the answer options the desired angle will be 360 – 220 = 140

Q9. **Two circle of 15 cm and 20 cm intersect. Distance between the two centers is 25 cm. The length of common chord is ?**

A) 20
B) 24
C) 15
D) 12

Let the centres of circles be A and B; D being the intersecting point of the radii and the two circles meet the circles at P and Q; therefore PQ is the common chord.
In this way we'll have a triangle with sides 20, 15 and 25
Now the area of the triangle =$\sqrt{s(s-a)(s-b)(s-c)}$; where 's' is half of the sum of the sides of the triangle
=> $\sqrt{[30(30 - 20)(30 - 15)(30 - 25)]}$
= 150

But the common chord intersects the line joining the centres of the circles; therefore PD is the height of this ∠APB
Thus the area of the ∠APB is also equal to 1/2 (25*h) [Area of a triangle also = ½base *height]

Therefore 25/2h = 150
h = 12
But this height is half of the chord
Hence length of the chord = 2*12 = 24 (option 'B')

Q10 **The lengths of two chords of a circle intersecting each other perpendicularly are 6 cm and 8 cm respectively. If the point of intersection is at a distance of 5 cm from the center; find the radius of the circle ?**

A) 7 cm
B) 8 cm
C) 10 cm
D) 5 cm

Draw perpendicular 'm' from the center of the circle on 8 cm chord and 'n' on 6 cm chord and let 'r' be the radius of the center. Join center of the circle to one of the end point of 8 cm chord and to one of the end point of 6 cm chord.

In this way we'll have two right triangles; one with sides 'm' (1/2 of 8) = 4 cm and radius as diagonal and the other with sides 'n' (1/2 of 6) = 3cm and radius as diagonal.

Now using pythagoras
$r^2 = m^2 + 4^2$ i
$r^2 = n^2 + 3^2$ ii
Adding (i) and (ii)
$2r^2 = 4^2 + 3^2 + m^2 + n^2$ iii
=> $2r^2 = 5^2 + (m^2 + n^2)$

If we see carefully we'll also have a rectangle whose length and breadth will be 'n' and 'm' respectively and the line joining the intersecting point of the chords to the center as one of its diagonals.
Therefore $m^2 + n^2 = 5^2$

Now putting this in equation (iii)
$2r^2 = 5^2 + 5^2$
=> $2r^2 = 2*5^2$^2
=> r = 5 (option 'D')

Q11 **Radius of a circle is 10. Two chords of 16 cm and 17 cm cut each other perpendicularly; then distance between center of the circle and the intersecting point of two chords is ?**

A) 6.5
B) 7.6
C) 8
D) 7.2

The line joining the centre of the cirlce to the intersecting point of two chords is actually the diagonal of the rectangle formed by the altitudes drawn from centre to the two chords respectively.

We know that diagonal of a rectangle = $\sqrt{(\text{length}^2 + \text{breadth}^2)}$
We see that the length and breadth of this rectangle actually are the altitudes drawn from centre to the two chords respectively.

So letting the altitude on the chord with 16 cm of length as H and the altitude on the chord with 17 cm of length as h we can write the above equation like this: diagonal = $\sqrt{(H^2 + h^2)}$

But, $H^2 = 10^2 – (16/2)^2 = 100 – 64 = 36$
and $h^2 = 10^2 – (17/2)^2 = 100 – 289/4 = 27.25$

So according to the Pythagoras
the required difference = $\sqrt{(36 + 27.25)} = \sqrt{63.25} = 8$ (appox)————option 'C'

Q12 **AB is the diameter of a circle with centre O and P is a point on it. If ∠POA=120, then the value of ∠PBO=?**

A) 60
B) 70
C) 55
D) 80

In the circle we get two triangles POA and POB;

In ΔPOA, ∠POA=120 (given)
Therefore in ΔPOB, ∠POB=180 – 120=60 (∠POB being the linear pair with ∠POA)

Therefore ∠PBO+∠OPB=180 – 60 = 120 (according to the angle sum property of a triangle)

Now ∠PBO = OPB (OB and PB being the radii of the same circle and hence equal; and both of these angles opposite to these sides)
Therefore ∠PBO and ∠OPB each equal to 60

So ∠PBO=60 (option 'A')

Q13. **The distance between the centers of two equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is?**

A) 4 cm
B) 5 cm
C) 6 cm
D) 8 cm

Remember you can call TRANSVERSE TANGENT as TRANSVERSE COMMON TANGENT also.

Let A & B be the centers of the two circles respectively and PQ the transverse tangent intersecting the circle with center A at P & intersecting the circle with center B at Q and the line joining the centers at X.

Now X will be the bisector point of AB as a transverse tangent divides the line joining the centers internally in the ratio of their radii

Hence in triangle APX
AX = 10/2 = 5 cm
AP = 3 cm (radius)
$\angle APX = 90^\circ$ (P being the point of contact of radius and tangent)

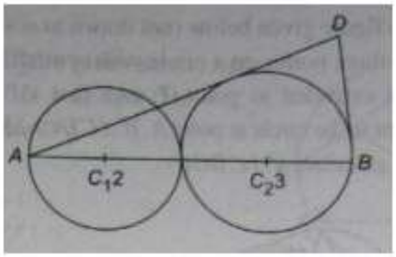
Hence by Pythagoras
$5^2 - 3^2 = PX^2$
=> PX = 4
Hence PQ = 2*4 = 8 cm (PQ =2*PX as it's the transverse tangent intersecting the line joining the centers of two equal circles.) [option 'D']

Q14 **A circle and square have the equal perimeter, then**

A) area of circle is greater
B) area of square is greater
C) both have equal areas
D) cannot be compared

The perimeters of a circle and a square are $2\pi r$ and 4a respectively, so we have to choose a number divisible by 22 and 4 both so that calculations are easy. Now let the perimeter is 44, therefore a = 11 and r = 7
Now area of the circle = $(22/7)*7^2 = 154$
And area of the square = $11^2 = 121$
So area of circle is greater (option 'A')

Q15 **Two circles C1 and C2 of radii 2 cm and 3 cm respectively touch each other as shown in the figure. If AD and BD are tangents then the length of BD is?**



A) $15\sqrt{10}$ cm

B) $1.5\sqrt{10}$ cm

C) $10\sqrt{10}$ cm

D) $\sqrt{10}$ cm

Let the point of intersection of the tangent AD to the circle C2 be T and the center of C2 be O and join O and T to form OT. Thus ΔOTA is a right angle triangle, where ∠OTA = 90°, AO = diameter of C1+radius of C2 = 4+3 = 7 and OT = radius of C2 = 3, and AO being the hypotenuse.

Therefore AT = $\sqrt{(7^2 - 3^2)} = \sqrt{40} = 2\sqrt{10}$ ....by Pythagoras

Now, as BD is another tangent to C2, B being the point of tangent, and it's joined with the center of the circle, so ΔABD too is a right triangle, where angle ABD = 90°, AB = sum of the diameters of the two circles given = 4+6 = 10 cm and AD being the hypotenuse.

Note that BT and TD are equal as they both are tangents to the same circle joined together (here joined at D)
Now let BD = x cm, therefore AD = $(2\sqrt{10} + x)$ cm

Hence by Pythagoras $BD^2 = (2\sqrt{10} + x)^2 - 10^2$

=> $x^2 = 40 + x^2 + 4x\sqrt{10} - 100$

=> $4x\sqrt{10} - 60 = 0$

=> $x\sqrt{10} = 15$

=> $x = 15/\sqrt{10} = (15/10)\sqrt{10}$ ....by rationalization of $\sqrt{10}$

=> $x = 1.5\sqrt{10}$ (option 'B')

Q16 **In a circle of radius 5 cm, AB and AC are two equal chords of lengths 6 cm each. Find the length of BC.**

A) 10 cm
B) 8 cm
C) 9.6 cm
D) 10.8 cm

In a circle as O its center draw the figure according to the question. Join AO and OB. Obviously both are radii of the circle, means each is of length 5 cm.

Now you'll see that ΔAOB is isosceles with AO and OB 5 cm each and AB 6 cm.

Now area of an isosceles triangle = $(b/4)\sqrt{(4a^2 - b^2)}$, where a is the length of one of the equal sides and b the third side.
Therefore area of ΔAOB = $(6/4)\sqrt{(4*5^2 - 6^2)}$
= 12

We know that line going through the center and the intersecting point of the two equal chords is the perpendicular bisector of the line segment joining the two chords (here BC).

Now, Let this perpendicular bisecting point be M, so BM is the height of the ΔAOB corresponding to the base AO

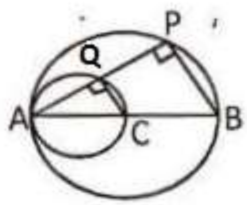
Hence BM = (area/AO)*2 …………….(From area of the triangle = (base*height)/2)

=> BM = (12/5)*2 = 24/5 = 4.8 cm

But BM = 1/2 of BC…………….told above
Hence BC = 2*4.8 = 9.6 cm (option 'C')

Q17. **Let C be a point on a straight line AB. Circles are drawn with diameters AC and AB. Let P be any point on the circumference of the circle with diameter AB. If AP meets the other circle at Q, then?**

A) QC || PB
B) QC is never parallel to PB
C) QC = 1/2 PB
D) QC || PB and QC = 1/2 PB



∠APB = ∠AQC = 90 (both are angles of semi- circles)
Now in ΔAPB and ΔAQC
∠A = ∠PAB = ∠QAC ....same angles
∠APB = ∠AQC ....proved above

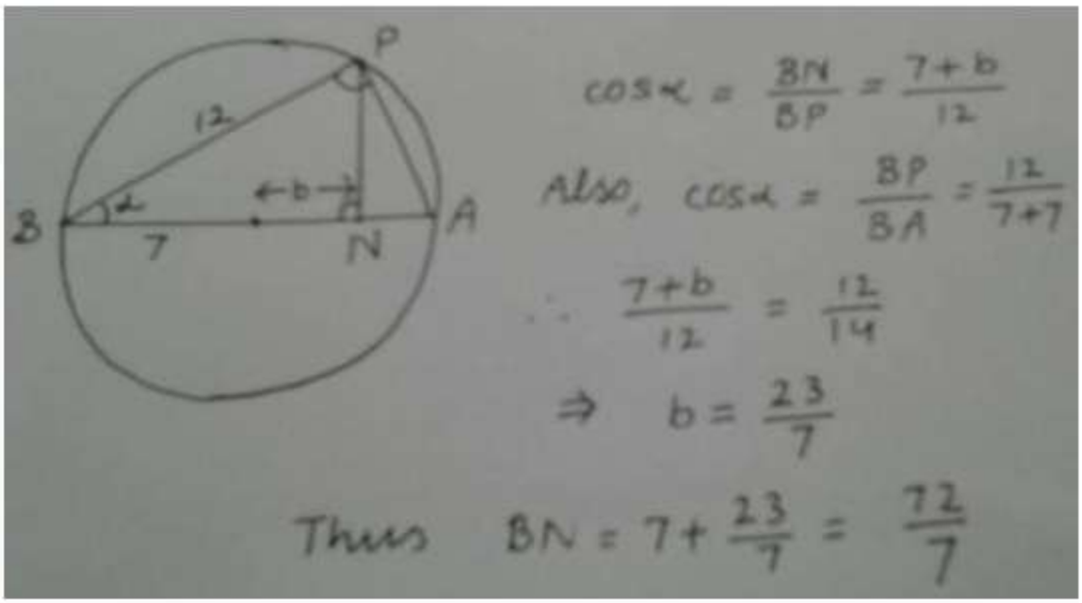
Therefore both triangles are similar with ∠APB and ∠AQC being corresponding.
Hence QC II PB

QC cannot necessarily be 1/2 PB as 'C' must be half of AB then, which cannot be proved

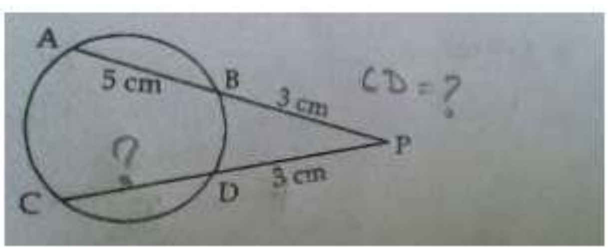
Hence QC II PB (option 'A')

Q18. N is the perpendicular from a point P of a circle with radius 7 cm on a diameter AB of the circle. If the length of the chord PB is 12 cm, the distance of the point N from the point B is?

A) 26/7
B) 72/7
C) 47/7
D) 86/7



Q19



A) 4 cm
B) 3 cm
C) 2.5 cm
D) 5 cm

RULE: If two chords of a circle intersect inside, or outside the circle when produced the rectangle formed by the two segments of a chord is equal to the area to the rectangle formed by the two segments of the other chord.

Therefore AB*BP = CD*DP

=> CD = (5*3)/3 = 5 cm (option 'D')

Q20. PQ is a chord of length 8 cm of a circle with center O and of radius 5 cm. The tangents at P and Q intersect at a point T. The length of TP is?

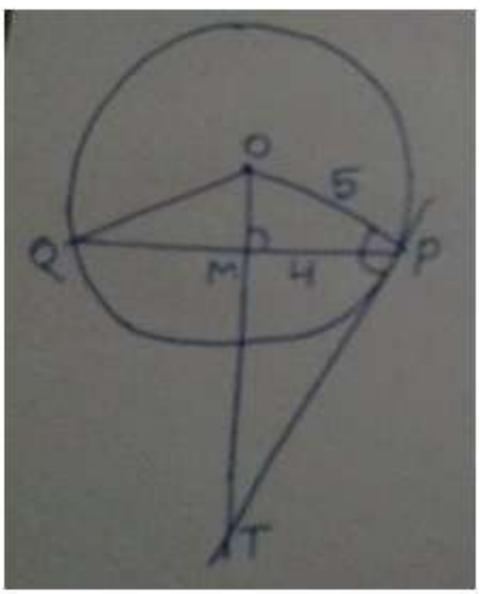
A) 15/4 cm

B) 20/3 cm

C) 21/4 cm

D) 10/3 cm

In the following figure T is the point of intersection of the two tangents at P and Q. One of the tangents has not been shown as it was not needed in the solution.



Since the line joining the center of the circle to the intersecting points of the tangents at a chord cuts the chord on right angle; ∠OMP = 90°s and hence ΔOMP is a right angle triangle. Now,

Since $PQ = 8$; $MP = \frac{8}{2} = 4$

Thus $OM = \sqrt{5^2 - 4^2} = 3$

It can easily be made out that triangle OMP is similar to triangle OPT hence

Now $\frac{OM}{MP} = \frac{OP}{PT}$

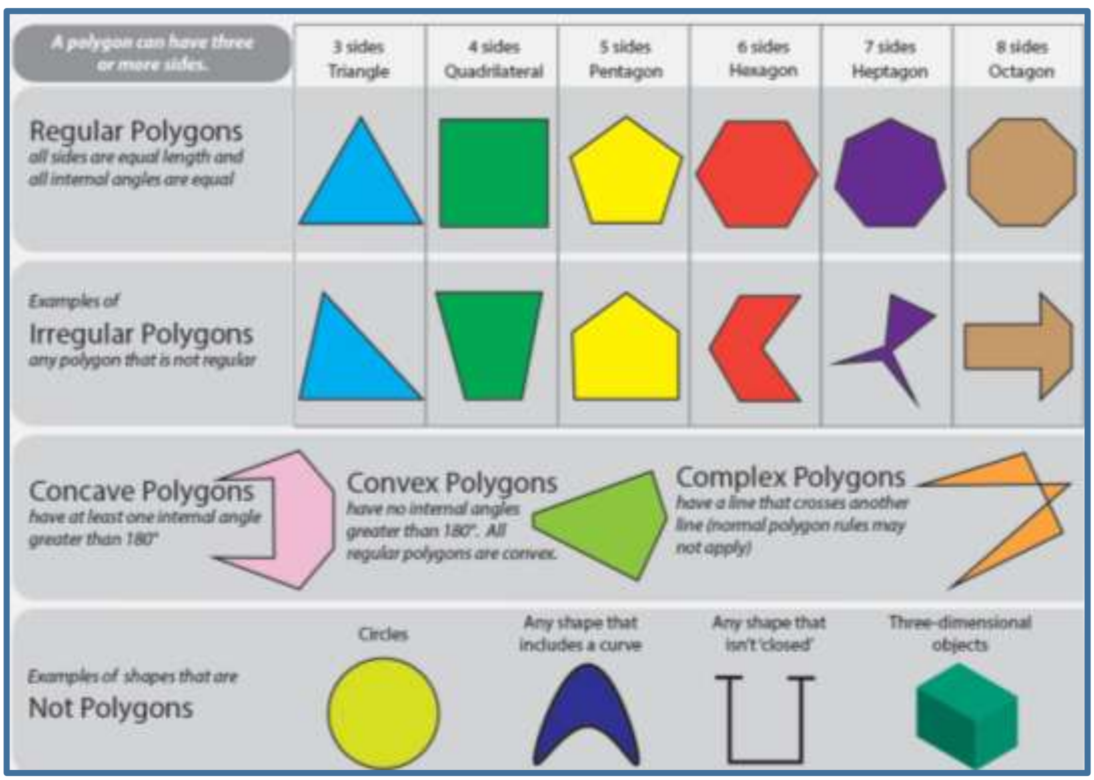
$\Rightarrow \frac{3}{4} = \frac{5}{PT}$

$\Rightarrow PT = \frac{20}{3}$

Quadrilaterals and Polygons

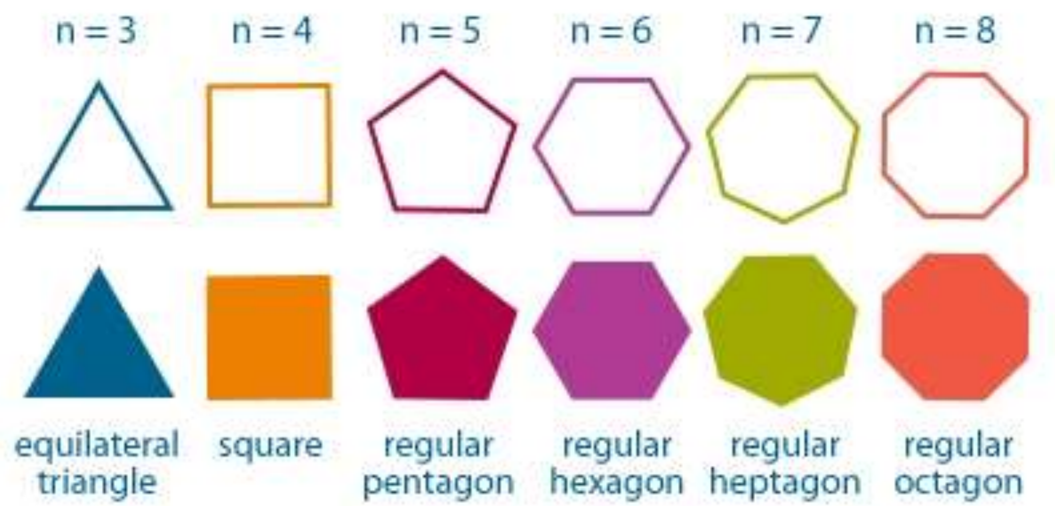
In geometry, a polygon is a plane figure that is bounded by a finite chain of straight line segments closing in a loop to form a closed chain or circuit. These line segments must be more than 3 and are called its edges or sides, and the points where two edges meet are the vertices or corners. An n-gon is a polygon with n sides implying that a triangle is a 3-gon.

Polygons can be anything, as can be seen in this picture below:



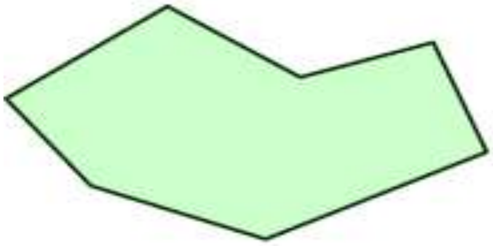
Although everything is not a polygon, as has been pointed in the last section of the figure above. So, this topic has many subtopics within it and we have ensured to compile them all here with a vision of scoring the maximum marks possible. Under this section, we will leave only those topics that are covered seperately.

This topic is the first topic you should revise before the exam day because it is easy and covers a lot. The amount of quuestions is good relatively and the amount of work involved is lesser. The best thing is that this topic is a geometrical topic, so all you need is practice to visualise and theorems to create the answer. First, let's see the types of polygons based on the number of sides.

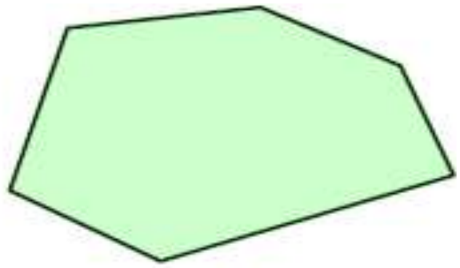


A polygon is either called with these special names or simply as an n-gon where n is the number of its edges or segments. A regular polygon is a convex polygon with all sides and angles congruent.

Examples of convex pentagon and concave pentagon are in the figure below :



Concave Polygon



Convex Polygon

Formula :

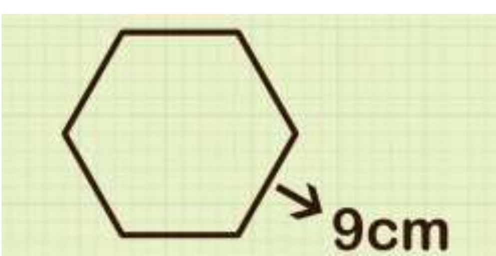
A. Area of a regular polygon = $\frac{1}{2}$ × number of sides × radius of the inscribed circle

B. Area of a Hexagon = $\frac{3\sqrt{3}}{2}$ × (side)²

C. Area of an Octagon = 2(√2 + 1) × (side)²

Examples:

- *Find the area of a regular hexagon whose side measures 9 cm.*



Area of a Hexagon = $\frac{3\sqrt{3}}{2} \times (\text{side})^2$

Here, a = 9 cm

$\frac{3\sqrt{3}}{2} \times (9)^2 = 210.4$ sq. cm approx

- *Find the length of the sides of a regular octagon whose area is 1 hectare.*

Area = 1000 m^2

$2(\sqrt{2} + 1) \times (\text{side})^2 = 10000$

On solving the equation, side = 46 meters approx.

The sum of the angles of a polygon can be calculated by the formula:

Sum of angles = 180 (n – 2)

Where, n is the number of sides of the respective polygon.

Sum of the angles of a triangle = 180°

Sum of the angles of a pentagon = 540°

Sum of the angles of a of a nonagon(9-sides) = 1260°

Inversely,

If a pentagon is regular, the measure of each interior angle by the same formula would be 108°.

This is all we have for Polygons and it's time to face the questions now.

Solve the questions that follow:

1) **If the ratio of an external angle and an internal angle of a regular polygon is 1 : 17, then the number of sides of the regular polygon is**

A. 20

B. 18

C. 36

D. 12

Answer : C

Hint : Suppose external angle as a and interior angle as 17a.

So, a + 17a = 180°

Hence, a = 10°

Formula : The sum of the measures of all the exterior angles of a polygon is always 360°.

$10 \times n = 180°$, n = 18 sides.

2) **A Polygon has 90 diagonals .The number of sides of the polygon are?**

A. 10

B. 20

C. 15

D. 25

Answer : C

Formula : Number of diagonals = $\frac{n(n-3)}{2}$, where n is the number of sides.

So, $\frac{n(n-3)}{2}$ = 90 , when calculated n comes to 15.

3) **How many sides does a polygon have if the sum of its interior angles is 180° ?**

A. 2

B. 3

C. 4

D. 5

Answer : B

Formula : Sum of interior angles = $180 \times (n - 2)$ where, n is the number of sides.

So, $180 \times (n - 2) = 180°$

$n - 2 \quad = 1$

$n = 3$

4) **What is the sum of the exterior angles of an octagon?**

A. 180°

B. 270°

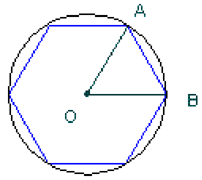
C. 360°

D. 1080°

Answer : C

Hint : The sum of the exterior angles of a polygon is always 360°.

5. A regular hexagon is inscribed in a circle of radius 10 cm, find the length of one side of the hexagon.



A. 5

B. 10

C. 15

D. 20

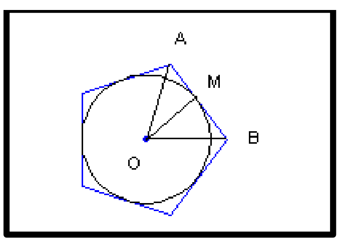
Answer : B

Hint : Angle (AOB) = 360° / 6 = 60°

OA = OB

Angle OAB = Angle OBA = 60° = Angle AOB

6. A circle of radius 6 cm is inscribed in a regular pentagon, find the length of one side of the pentagon.



Hint : Angle AOB = $360^o / 5 = 72^o$

The pentagon is regular and OA = OB.

Let M be the midpoint of AB so that OM is perpendicular to AB. OM is the radius of the inscribed circle and is equal to 6 cm. Now, $\tan\left(\frac{t}{2}\right) = \frac{MB}{OM}$

The side of the pentagon is equal to 2MB = $2 \times \tan\left(\frac{t}{2}\right) \times$ OM = 8.7 cm approx.

Mensuration

Mensuration is a branch of mathematics that deals with measurement of various parameters (area, perimeter, volume etc.) of geometric figures like circles, triangles, spheres, cones etc.

This topic is similar to algebraic identities in its concepts wherin you have a set of formulas to work around to calculate the answer. The complexity, however, increases many folds. The questions asked by SSC under this topic are trickier than most of the other topics but involve just understanding and a formula or two to solve. Hence, we should understand that the important tactic here would be to practice a lot more with quality questions and formulas mentioned below so that on the exam day you come out smiling after the test. The formulas and questions follow :

IMPORTANT FORMULAS FOR MENSURATION:

Circle :

1. Diameter, $D = 2r$

2. Area = πr^2 sq. units

3. Circumference = $2\pi r$ units

Square :

4. Area = a^2 sq. units

5. Perimeter = $4a$ units

6. Diagonal, $d = \sqrt{2}\, a$ units

Rectangle :

7. Area = $l \times b$ sq. units

8. Perimeter = $2(l+b)$ units

9. Diagonal, $d = \sqrt{(l^2 + b^2)}$ units

Scalene Triangle :

10.Area = $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

where $s = (a+b+c)/2$

11.Perimeter = $(a+b+c)$ units

Isosceles Triangle:

12.Area = $\frac{b}{4}\sqrt{(4a^2 - b^2)}$ sq units

13.Perimeter = $2a + b$ units

where b = base length; a = equal side length

Equilateral Triangle:

14.Area = $\frac{\sqrt{3}}{4}a^2$ sq. units

15.Perimeter = $3a$ units

where a = side of the triangle

Right-angled triangle :

16. Area = $\frac{1}{2}(b \times h)$ sq. units

17.Perimeter = b + h + hypotenuse

18.Hypotenuse = $\sqrt{(b^2 + h^2)}$ units

Cuboid:

19. Volume = (Cross section area × height) = $l \times b \times h$ cubic units

20. Lateral Surface Area (LSA) = $2[(l+b)h]$ sq. units

21. Total surface area (TSA) = $2(lb+bh+hl)$ sq. units

22. Length of the diagonals = $\sqrt{l^2 + b^2 + h^2}$ units

CUBE:

23.Volume = a^3 cubic units

24.Lateral Surface Area (LSA) = $4a^2$ sq. units

25.Total surface area (TSA) = $6a^2$ sq. units

26.Length of diagonal = $a\sqrt{3}$ units

SPHERE:

27.Volume = $\frac{4}{3}\pi r^3$ cubic units

28. Surface Area = $4\pi r^2$ sq. units

29. If R and r are the external and internal radii of a spherical shell, then

Volume = $\frac{4}{3}[R^3 - r^3]$ cubic units

HEMISPHERE:

30. Volume = $\frac{2}{3}\pi r^3$ cubic units

31. Total Surface Area(TSA) = $3\pi r^2$ sq. units

CYLINDER:

32. Volume = $\pi r^2 h$ cubic units

33. Curved surface Area (CSA) (excluding the areas of the top and bottom circular regions) = $2\pi rh$ sq. units

34. Total Surface Area(TSA) = Curved Surface Area + Areas of the top and bottom circular regions = $2\pi rh + 2\pi r^2 = 2\pi r\,[r+h]$ sq. units

CONE:

35.Volume = $\frac{1}{3}\pi r^2 h$ cubic units

36.Slant Height of cone, $l = \sqrt{(r^2 + h^2)}$ units

37.Curved surface Area (CSA) = πrl sq. units

38.Total Surface Area(TSA) = $\pi r\,(r + l)$ sq. Units

These are the formulas.

Now solve the practice questions that follow:

1. The curved surface of a right circular cone of height 15 cm and base diameter 16 cm is:

A. $116\pi cm^2$

B. $122\pi cm^2$

C. $124\pi cm^2$

D. $136\pi cm^2$

Answer :

Option **D**

Explanation:

$$\text{Curved surface area of cone} = \pi rl$$
$$l = \sqrt{r^2 + h^2}$$
$$l = \sqrt{8^2 + 15^2} = 17cm$$
$$\text{Curved surface area} = \pi rl$$
$$= \pi * 8 * 17 = 136\pi cm^2$$

2. Find the surface area of a 10cm*4cm*3cm brick.

A. 154 cm square

B. 156 cm square

C. 160 cm square

D. 164 cm square

Answer :

Option **D**

Explanation:

Surface area of a cuboid = 2(lb+bh+hl) cm square
So,
Surface area of a brick = 2(10*4+4*3+3*10) cm square
= 2(82) cm square = 164 cm square

3. A hemisphere and a cone have equal bases. If their heights are also equal, then the ratio of their curved surface will be :

A. $2:1$

B. $1:\sqrt{2}$

C. $\sqrt{2}:1$

D. $\sqrt{3}:1$

Answer :

Option **C**

Explanation:

Let the radius of hemisphere and cone be R,
Height of hemisphere H = R.
So the height of the cone = height of the hemisphere = R
Slant height of the cone

$$= \sqrt{R^2 + R^2}$$
$$= \sqrt{2}R$$
$$\frac{\text{Hemisphere Curved surface area}}{\text{Cone Curved surface area}} =$$
$$\frac{2\pi R^2}{\pi * R * \sqrt{2}R}$$
$$= \sqrt{2}:1$$

4. A hollow iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weights 8g/cm cube, then find the weight of the pipe.

A. 3.696 kg

B. 3.686 kg

C. 2.696 kg

D. 2.686 kg

Answer :

Option **A**

Explanation:

In this type of question, we need to subtract external radius and internal radius to get the answer using the volume formula as the pipe is hollow. Oh! line become a bit complicated, sorry for that, lets solve it.

External radius = 4 cm
Internal radius = 3 cm [because thickness of pipe is 1 cm]

$$\text{Volume of iron} = \pi r^2 h$$
$$= \frac{22}{7} * [4^2 - 3^2] * 21cm^3$$
$$= \frac{22}{7} * 1 * 21cm^3$$
$$= 462cm^3$$

Weight of iron = 462*8 = 3696 gm
= 3.696 kg

5. A cistern 6 m long and 4 m wide contains water up to a breadth of 1 m 25 cm. Find the total area of the wet surface.

A. 42 m sqaure

B. 49 m sqaure

C. 52 m sqaure

D. 64 m sqaure

Answer :

Option **B**

Explanation:

Area of the wet surface =
2[lb+bh+hl] - lb = 2 [bh+hl] + lb
= 2[(4*1.25+6*1.25)]+6*4 = 49 m square

6. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is:

A. 4 cm

B. 6 cm

C. 8 cm

D. 10 cm

Answer :

Option **B**

Explanation:

$$\text{Curved surface area of sphere} = \frac{4}{\pi}r^2$$
$$\text{Surface area of cylinder} = 2\pi rh$$
$$=> \frac{4}{\pi}r^2 = 2\pi rh$$
$$=> r^2 = \frac{6*12}{2}$$
$$=> r^2 = 36$$
$$=> r = 6$$

Note: Diameter of cylinder is 12 so radius is taken as 6.

7. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm is:

A. $79.8cm^3$

B. $79.4cm^3$

C. $89.8cm^3$

D. $89.4cm^3$

Answer :
Option **C**

Explanation:

Volume of the largest cone = Volume of the cone with diameter of base 7 and height 7 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} * \frac{22}{7} * 3.5 * 3.5 * 7$$
$$= \frac{269.5}{3} cm^3$$
$$= 89.8cm^3$$

Note: radius is taken as 3.5, as diameter is 7 cm

8. A hollow spherical metallic ball has an external diameter 6 cm and is 1/2 cm thick. The volume of metal used in the metal is:

A. $47\frac{1}{5}cm^3$

B. $47\frac{3}{5}cm^3$

C. $47\frac{7}{5}cm^3$

D. $47\frac{9}{5}cm^3$

Answer :

Option **B**

Explanation:

Please note we are talking about "Hollow" ball. Do not ignore this word in this type of question in a hurry to solve this question.

If we are given with external radius and thickness, we can get the internal radius by subtracting them. Then the volume of metal can be obtained by its formula as,

External radius = 3 cm,

Internal radius = (3-0.5) cm = 2.5 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} * \frac{22}{7} * [3^2 - 2.5^2]cm^3$$
$$= \frac{4}{3} * \frac{22}{7} * \frac{91}{8} cm^3$$
$$= \frac{143}{3} cm^3$$
$$= 47\frac{2}{3} cm^3$$

9. The radii of two cones are in ratio 2:1, their volumes are equal. Find the ratio of their heights.

A. 1:4

B. 1:3

C. 1:2

D. 1:5

Answer :

Option **A**

Explanation:

Let their radii be 2x, x and their heights be h and H resp.
Then,

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$\frac{\frac{1}{3} * \pi * (2x)^2 * h}{\frac{1}{3} * \pi * x^2 * H}$$
$$=> \frac{h}{H} = \frac{1}{4}$$
$$=> h : H = 1 : 4$$

10. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge.

A. 10000 cubes

B. 1000 cubes

C. 100 cubes

D. 50 cubes

Answer :

Option **B**

Explanation:

$$\text{Number of cubes} = \frac{100 * 100 * 100}{10 * 10 * 10} = 1000$$

Note: 1 m = 100 cm

11. If the volume of two cubes are in the ratio 27:1, the ratio of their edges is:

A. 3:1

B. 3:2

C. 3:5

D. 3:7

Answer :

Option **A**

Explanation:

Let the edges be a and b of two cubes, then

$$\frac{a^3}{b^3} = \frac{27}{1}$$
$$=> \left(\frac{a}{b}\right)^3 = \left(\frac{3}{1}\right)^3$$
$$\frac{a}{b} = \frac{3}{1}$$
$$=> a : b = 3 : 1$$

12. If a right circular cone of height 24 cm has a volume of 1232 cm cube, then the area of its curved surface is :

A. $450cm^2$

B. $550cm^2$

C. $650cm^2$

D. $750cm^2$

Answer :

Option **B**

Explanation:

Volume is given, we can calculate the radius from it, then by calculating slant height, we can get curved surface area.

$$\frac{1}{3} * \pi * r^2 * h = 1232$$

$$\frac{1}{3} * \frac{22}{7} * r^2 * 24 = 1232$$

$$r^2 = \frac{1232 * 7 * 3}{22 * 24} = 49$$

$$r = 7$$

Now, r = 7cm and h = 24 cm

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{7^2 + 24^2} = 25cm$$

Curved surface area $= \pi rl$

$$= \frac{22}{7} * 7 * 25 = 550cm^2$$

13. 66 cubic centimetres of silver is drawn into a wire 1 mm in diameter. The length if the wire in meters will be:

A. 76 m

B. 80 m

C. 84 m

D. 88 m

Answer :

Option **C**

Explanation:

Let the length of the wire be h

$$Radius = \frac{1}{2}mm = \frac{1}{20}cm$$
$$\pi r^2 h = 66$$
$$\frac{22}{7} * \frac{1}{20} * \frac{1}{20} * h = 66$$
$$=> h = \frac{66 * 20 * 20 * 7}{22}$$
$$= 8400cm$$
$$= 84m$$

14. A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden solid sphere of radius 9 cm. The percentage of the wood wasted is :

A. 45%

B. 56%

C. 67%

D. 75%

Answer :

Option **D**

Explanation:

We will first subtract the cone volume from wood volume to get the wood wasted.
Then we can calculate its percentage.

$$\text{Sphere Volume} = \frac{4}{3}\pi r^3$$

$$\text{Cone Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of wood wasted} = \left(\frac{4}{3}\pi * 9 * 9 * 9\right) - \left(\frac{1}{3}\pi * 9 * 9 * 9\right)$$

$$= \pi * 9 * 9 * 9 cm^3$$

$$\text{Required Percentage} = \frac{\pi * 9 * 9 * 9}{\frac{4}{3}\pi * 9 * 9 * 9} * 100\%$$

$$= \frac{3}{4} * 100\%$$

$$= 75\%$$

15. Two right circular cylinders of equal volumes have their heights in the ratio 1:2. Find the ratio of their radii.

A. $\sqrt{3} : 1$

B. $\sqrt{7} : 1$

C. $\sqrt{2} : 1$

D. $2 : 1$

Answer :

Option **C**

Explanation:

Let their heights be h and 2h and radii be r and R respectively then.

$$\begin{aligned} \pi r^2 h &= \pi R^2 (2h) \\ => \frac{r^2}{R^2} &= \frac{2h}{h} \\ &= \frac{2}{1} \\ => \frac{r}{R} &= \frac{\sqrt{2}}{1} \\ => r : R &= \sqrt{2} : 1 \end{aligned}$$

16. A boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets into it. The mass of the man is :

A. 50 kg

B. 60 kg

C. 70 kg

D. 80 kg

Answer :

Option B

Explanation:

In this type of question, first we will calculate the volume of water displaces then will multiply with the density of water.
Volume of water displaced = 3*2*0.01 = 0.06 m cube

Mass of Man = Volume of water displaced * Density of water

= 0.06 * 1000 = 60 kg

17. 12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. Find the diameter of each sphere.

A. 4 cm

B. 6 cm

C. 8 cm

D. 10 cm

Answer :

Option **A**

Explanation:

In this type of question, just equate the two volumes to get the answer as,

$$\text{Volume of cylinder} = \pi r^2 h$$
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
$$=> 12 * \frac{4}{3}\pi r^3 = \pi r^2 h$$
$$=> 12 * \frac{4}{3}\pi r^3 = \pi * 8 * 8 * 2$$
$$=> r^3 = \frac{8 * 8 * 2 * 3}{12 * 4}$$
$$=> r^3 = 8$$
$$=> r = 2cm$$
$$=> \text{Diameter} = 2 * 2 = 4cm$$

18. A swimming pool 9 m wide and 12 m long is 1 m deep on the shallow side and 4 m deep on the deeper side. Its volume is:

A. 260

B. 262

C. 270

D. 272

Answer :

Option **C**

Explanation:

Volume will be length * breadth * height, but in this case two heights are given so we will take average,

$$Volume = \left(12 * 9 * \left(\frac{1+4}{2}\right)\right) m^3$$
$$12 * 9 * 2.5m^3 = 270m^3$$

19. The maximum length of a pencil that can he kept is a rectangular box of dimensions 8 cm x 6 cm x 2 cm, is

A. $2\sqrt{17}$

B. $2\sqrt{16}$

C. $2\sqrt{26}$

D. $2\sqrt{24}$

Answer :

Option **C**

Explanation:

In this question we need to calculate the diagonal of cuboid,
which is =

$$\begin{aligned} &\sqrt{l^2+b^2+h^2} \\ &= \sqrt{8^2+6^2+2^2} \\ &= \sqrt{104} \\ &= 2\sqrt{26} \end{aligned}$$

20. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km square. The height of mountain is :

A. 2.3 km

B. 2.4 km

C. 2.5 km

D. 2.6 km

Answer :
Option **B**

Explanation:

Let the radius of the base be r km. Then,

$$\begin{aligned} \pi r^2 &= 1.54 \\ r^2 = \frac{1.54 * 7}{22} &= 0.49 \\ &= 0.7km \\ \text{Now l=2.5 km, r} &= \text{0.7 km} \\ h = \sqrt{2.5^2 - 0.7^2}\, km \\ = \sqrt{6.25 - 0.49} \\ = \sqrt{5.76}km \\ = 2.4km \end{aligned}$$

21. The perimeter of one face of a cube is 20 cm. Its volume will be:

A. $125cm^3$

B. $400cm^3$

C. $250cm^3$

D. $625cm^3$

Answer :

Option **A**

Explanation:

Edge of cude = 20/4 = 5 cm

Volume = a*a*a = 5*5*5 = 125 cm cube

22. A metallic sheet is of rectangular shape with dimensions 48 m x 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8 m, the volume of the box (in m cube) is:

A. 4120 m cube

B. 4140 m cube

C. 5140 m cube

D. 5120 m cube

Answer :

Option **D**

Explanation:

l = (48 - 16)m = 32 m, [because 8+8 = 16]
b = (36 -16)m = 20 m,
h = 8 m.

Volume of the box = (32 x 20 x 8) m cube
= 5120 m cube.

23. The cost of the paint is Rs. 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side.

A. Rs. 850

B. Rs. 860

C. Rs. 876

D. Rs. 886

Answer :

Option **C**

Explanation:

We will first calculate the Surface area of cube, then we will calculate the quantity of paint required to get answer.
Here we go,

$$\begin{aligned}\text{Surface area} &= 6a^2 \\ &= 6 * 8^2 = 384\text{sq feet} \\ \text{Quantity required} &= \frac{384}{16} \\ &= 24kg \\ \text{Cost of painting} &= 36.50 * 24 \\ &= Rs. 876\end{aligned}$$

24. How many bricks, each measuring 25cm*11.25cm*6cm, will be needed to build a wall 8m*6m*22.5m

A. 6100

B. 6200

C. 6300

D. 6400

Answer :

Option **D**

Explanation:

To solve this type of question, simply divide the volume of wall with the volume of brick to get the numbers of required bricks
So lets solve this

Number of bricks =

$$\frac{\text{Volume of wall}}{\text{Volume of 1 brick}}$$
$$= \frac{800 * 600 * 22.5}{25 * 11.25 * 6}$$
$$= 6400$$

25. A circular well with a diameter of 2 meters, is dug to a depth of 14 meters. What is the volume of the earth dug out.

A. $40m^3$

B. $42m^3$

C. $44m^3$

D. $46m^3$

Answer :

Option **C**

Explanation:

$$Volume = \pi r^2 h$$

$$Volume = \left(\frac{22}{7} * 1 * 1 * 14\right) m^3$$

$$= 44m^3$$

26. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by:

A. $11\frac{3}{7}cm$

B. $11\frac{2}{7}cm$

C. $11\frac{1}{7}cm$

D. $11cm$

Answer :

Option **A**

Explanation:

Let the drop in the water level be h cm, then,

$$\begin{aligned} &\text{Volume of cylinder} = \pi r^2 h \\ => &\frac{22}{7} * \frac{35}{2} * \frac{35}{2} * h = 11000 \\ => &h = \frac{11000 * 7 * 4}{22 * 35 * 35} cm \\ &= \frac{80}{7} cm \\ &= 11\frac{3}{7} cm \end{aligned}$$

TRIGONOMETRY

Trigonometry is a branch of mathematics that studies relationships involving lengths and Angles of triangles. This topic consists of formulas too and each one f us is already familiar with most of the formulas used. We begin with a quick revision of the formulas along with hacks and the questions follow after that:

PYTHAGOREAN IDENTITIES:

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \text{cosec}^2\theta$

NEGATIVE OF A FUNCTION :

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\text{cosec}(-x) = -\text{cosec}\, x$

$\sec(-x) = \sec x$

$\cot(-x) = -\cot x$

If Angle A + Angle B = 90° then,

Sin A = Cos B

$\text{Sin}^2A + \text{Sin}^2B = \text{Cos}^2A + \text{Cos}^2B = 1$

Tan A = Cot B

Sec A = cosec B

Example:

If tan (x + y) tan (x-y) = 1, then find tan (2x/3)?

Solution:

tan A = cot B » tan A × tan B = 1

So, Angle A + Angle B = 90^o

(x + y) + (x -y) = 90^o, 2x = 90^o , x = 45^o

tan $\frac{2x}{3}$ = tan 30^o = $\frac{1}{\sqrt{3}}$

If Angle A – Angle B = 90° , (if Angle A > Angle B) then,

Sin A = Cos B

Cos A = – Sin B

Tan A = – Cot B

If Angle A ± Angle B = 180^o then,

Sin A = Sin B

Cos A = – Cos B

If Angle A + Angle B = 180^o then,

tan A = – tan B

If Angle A – Angle B = 180^o then,

tan A = tan B

Example:

Find the Value of tan 60^o + tan 120^o ?

Solution:

Since $60° + 120° = 180°$

Therefore, $\tan 60^o + \tan 120^o = 1$

If Angle A + Angle B + Angle C = 180^o then,

$\text{Tan } A + \text{Tan } B + \text{Tan } C = \text{Tan } A \times \text{Tan } B \times \text{Tan } C$

$\sin\theta \times \sin 2\theta \times \sin 4\theta = \frac{1}{4}\sin 3\theta$

$\cos\theta \times \cos 2\theta \times \cos 4\theta = \frac{1}{4}\cos 3\theta$

Ex: What is the value of cos 20^o cos 40^o cos 60^o cos 80^o?

Solution: We know $\cos\theta \times \cos 2\theta \times \cos 4\theta = \frac{1}{4}\cos 3\theta$

Now, $(\cos 20^o \cos 40^o \cos 80^o)\cos 60^o$

$= \frac{1}{4}(\text{Cos } 3 \times 20°) \times \cos 60^o = \frac{1}{4}(\text{Cos } 60^o)^2 = \frac{1}{4} \times \left[\frac{1}{2}\right]^2$

$= \frac{1}{16}$

If a sin θ + b cos θ = m and a cos θ – b sin θ = n, then

$a^2 + b^2 = m^2 + n^2$

Example:

If $\sin\theta + \cos\theta = p$ and $\mathrm{cosec}\,\theta - \sec\theta = q$ then,

$$p - \frac{1}{p} = \frac{2}{q}$$

If $a\cot\theta + b\,\mathrm{cosec}\,\theta = m$ and $a\,\mathrm{cosec}\,\theta + b\cot\theta = n$ then,

$b^2 - a^2 \quad = m^2 - n^2$

If $\cot\theta + \cos\theta = x$ and $\cot\theta - \cos\theta = y$ then,

$$x^2 - y^2 = 4\sqrt{xy}$$

If $\tan\theta + \sin\theta = x$ and $\tan\theta - \sin\theta = y$ then,

$$x^2 - y^2 = 4\sqrt{xy}$$

If $y = a^2 \sin 2x + b^2\,\mathrm{cosec}\,2x + c$

$y = a^2 \cos 2x + b^2 \sec 2x + c$

$y = a^2 \tan 2x + b^2 \cot 2x + c$ then,

$y_{min} = 2ab + c$

y_{max} = not defined

Example:

If $y = 9 \sin^2 x + 16 \operatorname{cosec}^2 x + 4$ then, y_{min} will be :

Solution:

For, $y_{min} = 2\times \sqrt{9} \times \sqrt{16} + 4 = 2 \times 3 \times 4 + 20 = 24 + 4 = 28$

If $y = a \sin x + b \cos x + c$;

$y = a \tan x + b \cot x + c$

$y = a \sec x + b \operatorname{cosec} x + c$ then,

$y_{min} = + [\sqrt{(a^2+b^2)}] + c$

$y_{max} = - [\sqrt{(a^2+b^2)}] + c$

Example:

If $y = \dfrac{1}{12\sin x + 5\cos x + 20}$ then, y_{max} is:

Solution:

For, $y_{max} = \dfrac{1}{x}\min$

$= 1/-\sqrt{(12^2 + 5^2)} + 20 = \dfrac{1}{-13+20} = \dfrac{1}{7}$

For $\sin^2 \theta$, maxima value = 1, minima value = 0

For $\cos^2 \theta$, maxima value = 1, minima value = 0

Hopefully you are ready to practice now.

Quwstions for Practice:

Q1. $If (a^2 - b^2)sin\theta + 2ab\ cos\theta = a^2 + b^2, then\ value\ of\ tan\theta\ is$

Solution:

$(a^2 - b^2)sin\theta + 2ab\ cos\theta = a^2 + b^2$
$Dividing\ by\ cos\theta\ we\ get$
$(a^2 - b^2)tan\theta + 2ab = (a^2 - b^2)sec\theta$
$on\ squaring\ both\ sides$
$(a^2 - b^2)^2tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)tan\theta = (a^2 - b^2)^2sec^2\theta$
$(a^2 - b^2)^2tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)tan\theta = (a^2 - b^2)^2(1 + tan^2\theta)$
$(a^2 + b^2)^2tan^2\theta - (a^2 - b^2)^2tan^2\theta + [(a^2 + b^2)^2 - 4a^2b^2] - 4ab(a^2 - b^2)tan\theta = 0$
$4a^2b^2\ tan^2\theta + (a^2 - b^2)^2 - 4ab(a^2 - b^2)tan\theta = 0$
$((a^2 - b^2) - 2ab\ tan\theta)^2 = 0$
$a^2 - b^2 - 2ab\ tan\theta = 0$
$2ab\ tan\theta = a^2 - b^2$
$tan\theta = \frac{a^2 - b^2}{2ab}$

Q2. $If\ \theta\ is\ a\ positive\ acute\ angle\ and\ 2sin\theta + 15cos^2\theta = 7$

$then\ value\ of\ cot\theta\ is$

Solution:

$2sin\theta + 15cos^2\theta = 7$
$2sin\theta + 15(1 - sin^2\theta) = 7$
$2sin\theta + 15 - 15sin^2\theta = 7$
$15sin^2\theta - 2sin^2\theta - 8 = 0$
$15sin^2\theta - 12sin\theta + 10sin\theta - 8 = 0$
$3sin\theta(5sin\theta - 4) + 2(5sin\theta - 4) = 0$
$(3sin\theta + 2)(5sin\theta - 4) = 0$
$5sin\theta - 4 = 0$
$sin\theta = \frac{4}{5}$
$cosec\theta = \frac{5}{4}$
$cot\theta = \sqrt{cosec^2\theta - 1} = \sqrt{\frac{25}{16} - 1}$
$= \sqrt{\frac{9}{16}} = \frac{3}{4}$

Q3. $The\ maximum\ value\ of\ 24\ sin\theta + 7\ cos\theta\ is$

Solution:

$Maximum\ value\ of\ a\ sin\theta + b\ cos\theta = \sqrt{a^2 + b^2}$

$So\ maximum\ value = \sqrt{24^2 + 7^2} = 25$

Q4. $if\ tan2\theta.tan4\theta = 1 then\ value\ of\ tan3\theta\ is$

Solution:

$tan2\theta.tan4\theta = 1$

$tan2\theta = \frac{1}{tan4\theta}$

$tan2\theta = cot4\theta$

$tan2\theta = tan(90 - 4\theta)$

$2\theta = 90 - 4\theta$

$6\theta = 90$

$3\theta = 45$

$tan3\theta = tan45 = 1$

Q.5

If $x = rsin\alpha cos\beta, y = rsin\alpha sin\beta$ and $z = rcos\alpha$, then,

a. $x^2 - y^2 + z^2 = r^2$

b. $x^2 + y^2 + z^2 = r^2$

c. $x^2 + y^2 - z^2 = r^2$

d. $y^2 + z^2 - x^2 = r^2$

$x = ycot\beta,$

Or, $cot^2\beta = \frac{x^2}{y^2}$

Or, $cosec^2\beta = \frac{x^2+y^2}{y^2}$

Or, $sin^2\beta = \frac{y^2}{x^2+y^2}$

The second equation is,

$y = rsin\alpha sin\beta,$

Or, $y^2 = r^2 sin^2\alpha sin^2\beta = \frac{r^2y^2sin^2\alpha}{x^2+y^2}$,

Or, $sin^2\alpha = \frac{x^2+y^2}{r^2}$.

Now is the time to turn our attention to the third equation,

$z = rcos\alpha,$

Or, $z^2 = r^2\cos^2\alpha = r^2(1 - sin^2\alpha)$,

Or, $x^2 + y^2 + z^2 = r^2$.

Answer: b: $x^2 + y^2 + z^2 = r^2$.

Q.6 With $\angle\theta$ acute, the value of the expression, $\left(\frac{5\ cos\theta - 4}{3 - 5\ sin\theta} - \frac{3 + 5\ sin\theta}{4 + 5\ cos\theta}\right)$ is,

a. 1
b. 0
c. $\frac{1}{2}$
d. $\frac{1}{4}$

The given target expression,

$$E = \left(\frac{5\ cos\theta - 4}{3 - 5\ sin\theta} - \frac{3 + 5\ sin\theta}{4 + 5\ cos\theta}\right),$$

$$= \frac{((5cos\theta)^2 - 4^2) - (3^2 - (5sin\theta)^2)}{(3 - 5\ sin\theta)(4 + 5\ cos\theta)},$$

$$= \frac{25(sin^2\theta + cos^2\theta) - 25}{(3 - 5\ sin\theta)(4 + 5\ cos\theta)},$$

$$= 0.$$

Answer: b: 0.

Q.7

If $4 + 3\ tan\alpha = 0$, where $\frac{\pi}{2} < \alpha < \pi$, the value of $2\ cot\alpha - 5\ cos\alpha + \sin\alpha$ is,

a. $\frac{23}{10}$
b. $-\frac{53}{10}$
c. $\frac{37}{10}$
d. $\frac{7}{10}$

We have,

$tan\alpha = -\frac{4}{3}$,

Or, $1 + tan^2\alpha = sec^2\alpha = 1 + \frac{16}{9} = \frac{25}{9}$,

Or, $sec\alpha = -\frac{5}{3}$,

Or, $cos\alpha = -\frac{3}{5}$,

Or, $sin\alpha = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$.

So the given expression is,

$E = 2cot\alpha - 5cos\alpha + sin\alpha$

$= -\frac{3}{2} + 3 + \frac{4}{5}$

$= \frac{23}{10}$.

Answer: a: $\frac{23}{10}$.

Q.8

If $sin\theta + sin^2\theta = 1$, then which of the following is true?

a. $cos\theta + cos^2\theta = 1$
b. $cos^2\theta + cos^3\theta = 1$
c. $cos^2\theta + cos^4\theta = 1$
d. $cos\theta - cos^2\theta = 1$

The nature of the given expression immediately urges us to transform it to,

$sin\theta + sin^2\theta = 1$,

Or, $sin\theta = 1 - sin^2\theta = cos^2\theta$.

As all the options consist of only cos functions, as a next step we felt the need to convert the $sin\theta$ to $cos\theta$,

$sin\theta = cos^2\theta$,

Or, $sin^2\theta = 1 - cos^2\theta = cos^4\theta$,

Or, $cos^2\theta + cos^4\theta = 1$.

By target driven approach and end state analysis we find that we have reached the solution.

Answer: c: $cos^2\theta + cos^4\theta = 1$

Q.9

If $a = \sec\theta + \tan\theta$, then $\dfrac{a^2 - 1}{a^2 + 1}$ is,

a. $\sec\theta$
b. $\cos\theta$
c. $\tan\theta$
d. $\sin\theta$

Recognizing the presence of a friendly pair in the given expression, we express it as,

$$a = sec\theta + tan\theta = \frac{1}{sec\theta - tan\theta}.$$

Multiplying the two equations,

$$a^2 = \frac{sec\theta + tan\theta}{sec\theta - tan\theta},$$

Applying the componendo dividendo technique on the equation (subtracting 1 from both sides, again adding 1 to both sides of the original equation and taking the ratio of the two),

$$\frac{a^2 - 1}{a^2 + 1} = \frac{tan\theta}{sec\theta} = sin\theta.$$

We have reached the solution in only a few steps by applying rich trigonometry ***concept of friendly trigonometric function pair*** as well as ***rich algebraic technique of componendo dividendo.***

This is an example of achieving what we call ***elegant solution*** through the process of ***efficient problem solving.***

Answer: d: $sin\theta$.

Q.10

The value of $\frac{cot\theta + cosec\theta - 1}{cot\theta - cosec\theta + 1}$ is,

a. $cosec\theta - cot\theta$
b. $cosec\theta + cot\theta$
c. $sec\theta + cot\theta$
d. $cosec\theta + tan\theta$

Applying the inverse relationship of,

$cosec\theta + cot\theta = \dfrac{1}{cosec\theta - cot\theta}$, we transform the target expression to,

$$E = \frac{cot\theta + cosec\theta - 1}{cot\theta - cosec\theta + 1}$$

$$= \frac{\dfrac{1}{cosec\theta - cot\theta} - 1}{cot\theta - cosec\theta + 1}$$

$$= \frac{1}{cosec\theta - cot\theta} \times \frac{cot\theta - cosec\theta + 1}{cot\theta - cosec\theta + 1}$$

$$= \frac{1}{cosec\theta - cot\theta}$$

$$= cosec\theta + cot\theta.$$

Again detecting the very useful pattern of friendly trigonometric function pair and using the pattern judiciously we reach the solution elegantly in a few simple steps.

Answer: b: $cosec\theta + cot\theta$.

Q.11 If $\dfrac{sin\theta + cos\theta}{sin\theta - cos\theta} = 3$, then the value of $sin^4\theta - cos^4\theta$ is,

a. $\dfrac{2}{5}$
b. $\dfrac{1}{5}$
c. $\dfrac{4}{5}$
d. $\dfrac{3}{5}$

Applying componendo dividendo on the given equation we get,

$$\frac{sin\theta}{cos\theta} = \frac{3+1}{3-1} = 2,$$

Or, $sin\theta = 2cos\theta,$

Or, $tan\theta = 2,$

Or, $1 + tan^2\theta = sec^2\theta = 5,$

Or, $cos^2\theta = \frac{1}{5}.$

The target expression is,

$$E = sin^4\theta - cos^4\theta = (sin^2\theta + cos^2\theta)(sin^2\theta - cos^2\theta)$$

$$= 3cos^2\theta = \frac{3}{5}.$$

Answer: d: 1.

Q.12 If $asec\theta + btan\theta + c = 0$, and $psec\theta + qtan\theta + r = 0$, then the value of $(br - qc)^2 - (pc - ar)^2$ is,

a. $(aq + bp)^3$
b. $(aq - bp)^3$
c. $(aq + bp)^2$
d. $(aq - bp)^2$

We have

$asec\theta + btan\theta + c = 0$,

Or, $aqsec\theta + bqtan\theta + cq = 0$.

Similarly multiplying the second equation by b,

$bpsec\theta + bqtan\theta + br = 0$.

Subtracting the two,

$(aq - bp)sec\theta = (br - cq)$,

Or $sec\theta = \frac{br - cq}{aq - bp}$.

We can deduce similarly,

$tan\theta = \frac{pc - ar}{aq - bp}$.

So from $sec^2\theta - tan^2\theta = 1$,

$$\frac{(br - cq)^2 - (pc - ar)^2}{(aq - bp)^2} = 1,$$

Or, $(br - qc)^2 - (pc - ar)^2 = (aq - bp)^2$.

Though the problem looked awkward the methods followed being simple came in a few steps.

Answer: d: $(aq - bp)^2$.

Q.13

If $\alpha + \beta + \gamma = \pi$, then the value of $(sin^2\alpha + sin^2\beta - sin^2\gamma)$ is,

a. $2sin\alpha sin\beta cos\gamma$
b. $2sin\alpha$
c. $2sin\alpha cos\beta sin\gamma$
d. $2sin\alpha sin\beta sin\gamma$

$\alpha+\beta=\pi-\gamma$,

Or, $sin(\alpha+\beta)=sin(\pi-\gamma)=sin\gamma$,

Or, $(sin\alpha cos\beta+cos\alpha sin\beta)^2=sin^2\gamma$,

Or, $sin^2\alpha cos^2\beta+2sin\alpha cos\beta cos\alpha sin\beta+cos^2\alpha sin^2\beta$

$=sin^2\gamma$,

Or, $sin^2\alpha(1-sin^2\beta)+sin^2\beta(1-sin^2\alpha)-sin^2\gamma$

$=-2sin\alpha sin\beta cos\alpha cos\beta$,

Or, $sin^2\alpha+sin^2\beta-sin^2\gamma$

$=2sin^2\alpha sin^2\beta-2sin\alpha sin\beta cos\alpha cos\beta$,

Or, $sin^2\alpha+sin^2\beta-sin^2\gamma$

$=-2sin\alpha sin\beta(cos\alpha cos\beta-sin\alpha sin\beta)$,

Or, $E=-2sin\alpha sin\beta cos(\alpha+\beta)$, where E is the target expression,

Or, $E=-2sin\alpha sin\beta cos(\pi-\gamma)$,

Or, $E=2sin\alpha sin\beta cos\gamma$, as $cos(\pi-\theta)=-cos\theta$.

Answer: a: $2sin\alpha sin\beta cos\gamma$.

Q.14 If $sin\alpha sin\beta-cos\alpha cos\beta+1=0$, then the value of $cot\alpha tan\beta$ is,

a. -1
b. 1
c. 0
d. None of these

Detecting the compound angle pattern in the target expression we transform it,

$sin\alpha sin\beta - cos\alpha cos\beta + 1 = 0$,

Or, $cos(\alpha + \beta) = 1$,

Or, $\alpha + \beta = 0^0$.

So,

$sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta = 0$,

Or, $sin\alpha cos\beta = -cos\alpha sin\beta$,

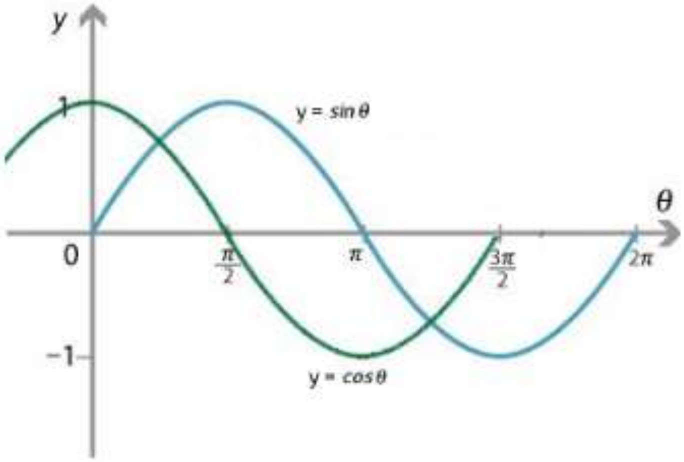
Or, $cot\alpha tan\beta = -1$.

Answer: a: -1.

Q.15

If $0^0 < \theta < 90^0$ and $2sin^2\theta + 3cos\theta = 3$ then the value of θ is,

a. 30^0
b. 60^0
c. 45^0
d. 75^0



From the figure it can never be forgotten that $cos\theta = 1$ when $\theta = 0^0$ and its value reduces to 0 when value of θ goes on increasing to 90^0. In between there are two significant values of $cos\theta$, namely $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ at θ values either 30^0 or 60^0. As $\frac{1}{2}$ is smaller than $\frac{\sqrt{3}}{2}$, and also as value of $cos\theta$ reduces from 1 to 0, it follows that $cos60^0 = \frac{1}{2}$.

This is direct application of the Principle of Less facts and more procedure.

Answer: Option b: 60^0.

Q.16

If $sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, then the value of $cot\theta$ will be,

a. $\frac{b}{a}$
b. $\frac{a}{b}$
c. $\frac{a}{b}+1$
d. $\frac{b}{a}+1$

Solution: We reason, to find the value of $cot\theta$, we need values of $sin\theta$ and $cos\theta$, one of which is given. Using the identity, $sin^2\theta + cos^2\theta = 1$ we can mentally arrive at the result,

$cos^2\theta = 1 - sin^2\theta = \frac{b^2}{a^2+b^2}$

Or, $cos\theta = \frac{b}{\sqrt{a^2+b^2}}$

So, $cot\theta = \frac{b}{a}$.

Answer: Option a : $\frac{b}{a}$.

Q.17 . If $tan\theta = \frac{3}{4}$ and $0 < \theta < \frac{\pi}{2}$ and $25xsin^2\theta cos\theta = tan^2\theta$, then the value of x

a. $\frac{7}{64}$
b. $\frac{9}{64}$
c. $\frac{3}{64}$
d. $\frac{5}{64}$

In the second expression, we cancel out the $sin^2\theta$ on both sides and take the $cos\theta$ on the RHS to get,

$25x = sec\theta sec^2\theta$

Now we use the simpler expression $tan\theta = \frac{3}{4}$ to derive,

$sec^2\theta = 1 + tan^2\theta = \frac{25}{16}$.

Substituting we get,

$25x = \frac{25}{16} sec\theta,$

Or, $x = \frac{sec\theta}{16}$

Now is the time to use the second condition in the expression, $sec\theta = \pm\frac{5}{4} = \frac{5}{4}$.

Finally we then get $x = \frac{5}{64}$.

Answer: Option d: $\frac{5}{64}$.

Q.18 If $xsin\theta - ycos\theta = \sqrt{x^2 + y^2}$ and $\frac{cos^2\theta}{a^2} + \frac{sin^2\theta}{b^2} = \frac{1}{x^2+y^2}$ then,

a. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
b. $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
c. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
d. $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$

$$x^2 sin^2\theta + y^2 cos^2\theta$$
$$-2xysin\theta cos\theta = x^2 + y^2$$

Transposing and collecting $sin^2\theta$ and $cos^2\theta$ it further simplifies to,

$$x^2 cos^2\theta + y^2 sin^2\theta + 2xysin\theta cos\theta = 0$$

Or, $(xcos\theta + ysin\theta)^2 = 0$

Or, $xcos\theta = -ysin\theta$

Or, $cot^2\theta = \frac{y^2}{x^2}$

Or, $cot^2\theta + 1 = cosec^2\theta = \frac{y^2+x^2}{x^2}$

Or, $sin^2\theta = \frac{x^2}{x^2+y^2}$

Or, $cos^2\theta = 1 - sin^2\theta = \frac{y^2}{x^2+y^2}$

Now your target is to eliminate $cos\theta$ and $sin\theta$, by substituting values of $cos^2\theta = \frac{y^2}{x^2+y^2}$, and $sin^2\theta = \frac{x^2}{x^2+y^2}$ in the second expression, giving,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Answer: Option b: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Q.19 . The value of $sin^2 1^0 + sin^2 3^0 + sin^2 5^0 + \ldots$
$+ sin^2 87^0 + sin^2 89^0$ is,

a. 22
b. $22\frac{1}{2}$
c. 23
d. $22\frac{1}{4}$

$sin\theta = cos(90^0 - \theta)$ for positive θ less than or equal to 90^0. Using this knowledge we can form pairs sum of each of which will be 1 as below,

$$sin^2 1^0 + sin^2 89^0 = cos^2 89^0 + sin^2 89^0$$
$$= 1$$

Question now is how many terms does the expression have? Without using any formula, just test for a similar expression ending with $sin^2 7^0$ and another with the next one $sin^2 9^0$. We know the first to have 4 terms and the second 5 terms. Knowing this we try the conjecture that number of terms for the first = $\frac{7+1}{2} = 4$ and for the second, $\frac{9+1}{2} = 5$ both tallying with our direct knowledge. Thus confirming our conjecture, we use the often used Principle of induction to get the number of terms in the given expression = $\frac{89+1}{2} = 45$.

With 45 terms, we get 22 pairs giving sum as 22 and a single term in the middle, the 23rd term. The second term has angle $2 \times (2-1) + 1 = 3^0$, the third, $2 \times (3-1) + 1 = 5^0$. It tallies. Using induction again, we get the angle of the middle term = $2 \times (23-1) + 1 = 45^0$.

As $sin45^0 = \frac{1}{\sqrt{2}}$, we get the given sum = $22\frac{1}{2}$.

Answer: Option b: $22\frac{1}{2}$.

Q.20 The minimum value of $cos^2\theta + sec^2\theta$ is,

a. 0
b. 1
c. 2
d. 3

Solution: The value of $cos\theta$ ranges from -1 to 1 intermediate values being less than 1. That is $-1 \leq \cos\theta \leq 1$. This gives, $0 \leq \cos^2\theta \leq 1$.

Thus assuming $cos^2\theta = \frac{1}{x}$, we get $x \geq 1$, that is, a positive real number with minimum value 1. Substituting in the given expression, we get,

$E = \frac{1}{x} + x = \frac{x^2+1}{x}$, where $x \geq 1$.

As $x \geq 1$ and numerator grows much faster than the denominator with increasing value of x, the expression will have minimum value when x has minimum value of 1. So minimum value of expression will be 2.

Answer: Option c : 2.

Q.21 . If $cos\theta + sec\theta = 2$ $(0^0 \leq \theta \leq 90^0)$ then the value of $cos10\theta + sec11\theta$ is,

a. 0
b. 1
c. 2
d. -1

Solution: After initial examination we find the target expression a little hard to break. Recalling that simpler initial expression is to be used for evaluating more complex expression, we concentrate on the first expression.

$cos\theta + sec\theta = 2$

Or, $cos^2\theta - 2cos\theta + 1 = 0$

Or, $cos\theta = 1$, giving $\theta = 0^0$. So given expression = 2.

Answer: Option c: 2.

Q.22 If $tan\theta = \frac{3}{4}$ and θ is acute then, $cosec\theta$ is equal to,

a. $\frac{5}{3}$
b. $\frac{5}{4}$
c. $\frac{4}{3}$
d. $\frac{4}{5}$

Solution: $cot\theta = \frac{4}{3}$, or, $cot^2\theta = cosec^2\theta - 1 = \frac{16}{9}$, or, $cosec^2\theta = \frac{25}{9}$, or, $cosec\theta = \frac{5}{3}$.

Answer: Option a: $\frac{5}{3}$.

Q.23 If $\dfrac{sin\theta + cos\theta}{sin\theta - cos\theta} = 3$ then the numerical value of $sin^4\theta - cos^4\theta$ is,

a. $\frac{1}{2}$
b. $\frac{2}{5}$
c. $\frac{3}{5}$
d. $\frac{4}{5}$

Solution: Adding 1 to both sides, subtracting 1 from both sides and taking ratio of the two we get, $tan\theta = 2$.

Target expression,

$$\begin{aligned} E &= (sin^2\theta + cos^2\theta)(sin^2\theta - cos^2\theta) \\ &= (sin^2\theta - cos^2\theta) \\ &= 1 - 2cos^2\theta \end{aligned}$$

Again $tan\theta = 2$, or, $tan^2\theta = 4 = sec^2\theta - 1$, or, $sec^2\theta = 5$, or, $cos^2\theta = \frac{1}{5}$

Thus $E = 1 - \frac{2}{5} = \frac{3}{5}$

Answer: Option c: $\frac{3}{5}$.

Q.24). The minimum value of $2sin^2\theta + 3cos^2\theta$ is,

a. 0
b. 3
c. 2
d. 1

Solution: Target expression,

$E = 2 + cos^2\theta$

Minimum value of $cos^2\theta$ being 0, minimum value of target expression is 2.

Answer: Option c: 2.

Q.25 The simplified value of $(sec\theta - cos\theta)^2 + (cosec\theta - sin\theta)^2 - (cot\theta - tan\theta)^2$ is,

a. $\frac{1}{2}$
b. 0
c. 2
d. 1

$$(sec\theta - cos\theta) = sec\theta(1 - cos^2\theta)$$

$= sec\theta sin^2\theta = sin\theta tan\theta$, and similarly,

$$(cosec\theta - sin\theta) = cosec\theta(1 - sin^2\theta)$$

$$= cosec\theta cos^2\theta = cos\theta cot\theta.$$

Let us focus on the first two terms then and apply the rich concept and technique of factoring out the inverse,

$(sec\theta - cos\theta)^2 = sec^2\theta(1 - cos^2\theta)^2 = sin^2\theta tan^2\theta$.

Similarly,

$(cosec\theta - sin\theta)^2 = cos^2\theta cot^2\theta$.

Now we sum up these terms with the expanded third term and apply **principle of collection of friendly terms.** The pair of terms involving $tan^2\theta$ and the second pair involving $cot^2\theta$ are combined together.

$$E = 2 - tan^2\theta(1 - sin^2\theta) - cot^2\theta(1 - cos^2\theta)$$

$$= 2 - (sin^2\theta + cos^2\theta)$$

$$= 1.$$

Answer: Option d: 1.

Q.26

If $\frac{sin\theta + cos\theta}{sin\theta - cos\theta} = \frac{5}{4}$, then the value of $\frac{tan^2\theta + 1}{tan^2\theta - 1}$ will be,

a. $\frac{41}{40}$
b. $\frac{40}{41}$
c. $\frac{25}{16}$
d. $\frac{41}{9}$

Applying the powerful **componendo dividendo technique** on the input expression we get directly,

$$\frac{sin\theta}{cos\theta} = \frac{5+4}{5-4} = 9,$$

Or, $tan\theta = 9$

Substituting in the target expression,

$$E = \frac{9^2+1}{9^2-1} = \frac{82}{80} = \frac{41}{40}$$

Answer: Option a : $\frac{41}{40}$.

Q.27

If $sin\theta + cosec\theta = 2$, then the value of $sin^{100}\theta + cosec^{100}\theta$ is,

a. 100
b. 3
c. 2
d. 1

$sin\theta + cosec\theta = 2$,

Or, $sin^2\theta - 2sin\theta + 1 = (sin\theta - 1)^2 = 0$

Or, $sin\theta = 1$, and so, $cosec\theta = 1$.

Putting these convenient values in the target expression we get,

$E = 1 + 1 = 2$

Answer: Option c: 2.

Q.28

The greatest value of $sin^4\theta + cos^4\theta$ is,

a. 1
b. $\frac{1}{2}$
c. 3
d. 2

Using algebraic maxima finding technique we transform part of the given expression to a square of sums.

$sin^4\theta + cos^4\theta$

$= (sin^2\theta + cos^2\theta)^2 - 2sin^2\theta cos^2\theta$,

$= 1 - 2sin^2\theta cos^2\theta$.

The maximum of this expression can only be 1 when, the second term is zero, or when either $sin\theta$ or $cos\theta$ is 0.

This is a quick method as it has used the trigonometric relations along with the algebraic maxima technique elegantly.

Answer: Option a: 1.

Q.29 If $\frac{sin\theta}{x} = \frac{cos\theta}{y}$, then $sin\theta - cos\theta$ is,

a. $x - y$
b. $\frac{x-y}{\sqrt{x^2+y^2}}$
c. $\frac{y-x}{\sqrt{x^2+y^2}}$
d. $x + y$

$\frac{sin\theta}{x} = \frac{cos\theta}{y}$,

Or, $cot\theta = \frac{y}{x}$,

Or, $cot^2\theta + 1 = cosec^2\theta = \frac{x^2+y^2}{x^2}$,

Or, $sin^2\theta = \frac{x^2}{x^2+y^2}$,

Or, $sin\theta = \frac{x}{\sqrt{x^2+y^2}}$.

Similarly,

$cos\theta = \frac{y}{\sqrt{x^2+y^2}}$.

Thus target expression,

$sin\theta - cos\theta = \frac{x-y}{\sqrt{x^2+y^2}}$.

Answer: Option b: $\frac{x-y}{\sqrt{x^2+y^2}}$.

Q.30 If $tan\theta - cot\theta = 0$ find the value of $sin\theta + cos\theta$.

a. $\sqrt{2}$
b. 0
c. 1
d. 2

With the form of the given expression we will go straight for equating the tan and cot with the intention of getting an equation in sin and cos.

So,

$tan\theta = cot\theta$,

Or, $sin^2\theta = cos^2\theta$.

We know this equality occurs only when $sin\theta = cos\theta = \frac{1}{\sqrt{2}}$ at $\theta = 45^0$.

So, $sin\theta + cos\theta = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

Answer: Option a : $\sqrt{2}$.

Q.31 If $sin21^0 = \frac{x}{y}$ then $sec21^0 - sin69^0$ is,

a. $\frac{y^2}{x\sqrt{y^2 - x^2}}$
b. $\frac{x^2}{y\sqrt{y^2 - x^2}}$
c. $\frac{x^2}{y\sqrt{x^2 - y^2}}$
d. $\frac{y^2}{x\sqrt{x^2 - y^2}}$

In our problem situation applying this concept we have, $sin21^0 = cos69^0$,

So, $sin21^0 = cos69^0 = \frac{x}{y}$,

Or, $1 - cos^2 69^0 = sin^2 69^0 = \frac{y^2 - x^2}{y^2}$,

Or, $sin69^0 = \frac{\sqrt{y^2 - x^2}}{y}$.

The target expression,

$sec21^0 - sin69^0 = cosec69^0 - sin69^0$

$= \frac{1 - sin^2 69^0}{sin69^0}$

$= \frac{cos^2 69^0}{sin69^0}$,

$= \frac{x^2}{y^2} \times \frac{y}{\sqrt{y^2 - x^2}}$

$= \frac{x^2}{y\sqrt{y^2 - x^2}}$.

Answer: Option b: $\frac{x^2}{y\sqrt{y^2 - x^2}}$.

Q.32

If $\frac{sec\theta + tan\theta}{sec\theta - tan\theta} = \frac{5}{3}$ then $sin\theta$ is,

a. $\frac{3}{4}$
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. $\frac{1}{4}$

As usual detecting the possibility of taking out $sec\theta$ as a factor from both numerator and denominator we take up the transformation,

$$\frac{sec\theta + tan\theta}{sec\theta - tan\theta} = \frac{5}{3},$$

Or, $\dfrac{sec\theta(1 + sin\theta)}{sec\theta(1 - sin\theta)} = \dfrac{5}{3},$

Or, $\dfrac{1 + sin\theta}{1 - sin\theta} = \dfrac{5}{3}.$

This is ripe form for applying the Componendo dividendo technique. We thus get,

$$sin\theta = \frac{5-3}{5+3} = \frac{1}{4}$$

Answer: Option d: $\dfrac{1}{4}$.

Q.33 If $(1 + sinA)(1 + sinB)(1 + sinC) = (1 - sinA)(1 - sinB)(1 - sinC)$, then the expression on each side of the equation equals,

a. 1
b. $tanA.tanB.tanC$
c. $cosA.cosB.cosC$
d. $sinA.sinB.sinC$

Observing the possibility of getting $1 - cos^2\theta$ for each of the three expressions on one side if we multiply them together we first equate the expressions to a dummy variable p,

$(1 + sinA)(1 + sinB)(1 + sinC)$

$= (1 - sinA)(1 - sinB)(1 - sinC) = p$ which results in two equations,

$(1 + sinA)(1 + sinB)(1 + sinC) = p$ and,

$(1 - sinA)(1 - sinB)(1 - sinC) = p$.

Multiplying the two together,

$p^2 = (1 - sin^2A)(1 - sin^2B)(1 - sin^2C)$

$= cos^2A.cos^2B.cos^2C$.

Or, each target expression $p = cosA.cosB.cosC$.

Answer: Option c: $cosA.cosB.cosC$.

Q.34 If $\theta = 60^0$, then, $\frac{1}{2}\sqrt{1 + sin\theta} + \frac{1}{2}\sqrt{1 - sin\theta}$ is,

a. $cos\frac{\theta}{2}$

b. $cot\frac{\theta}{2}$

c. $sec\frac{\theta}{2}$

d. $sin\frac{\theta}{2}$

Substituting value of θ,

$\sqrt{1+sin\theta}$

$=\sqrt{1+\frac{\sqrt{3}}{2}}$

$=\sqrt{\frac{2+\sqrt{3}}{2}}$

The problem expression is thus $\sqrt{1+sin\theta}$ in which we need to eliminate the square root applying the powerful surd techniques of transforming a two term surd expression to a squ sum of a two term surd expression.

As the surd $\sqrt{3}$ doesn't have 2 as a coefficient, we multiply and diivide by 2, getting,

$\sqrt{1+sin\theta}$

$=\sqrt{\frac{(\sqrt{3}+1)^2}{4}}$

$=\frac{\sqrt{3}+1}{2}$,

Similarly,

$\sqrt{1-sin\theta}=\frac{\sqrt{3}-1}{2}$, and the target expression,

$E=\frac{\sqrt{3}}{2}=cos30^0=cos\frac{\theta}{2}$

Answer: Option a: $cos\frac{\theta}{2}$.

Q.35 The value of $tan1^0 tan2^0 tan3^0 \ldots\ldots tan89^0$ is,

a. $\sqrt{3}$
b. 0
c. 1
d. $\frac{1}{\sqrt{3}}$

Solution - Problem analysis

From ***Complementary Trigonometric functions concepts*** we know,

$sin\left(\frac{\pi}{2}-\theta\right)=cos\theta$

$cos\left(\frac{\pi}{2}-\theta\right)=sin\theta$, and so,

$tan\left(\frac{\pi}{2}-\theta\right)=cot\theta$, where θ is acute.

With this knowledge, we find in the product a matching pair of $tan89^0$ for $tan1^0$. As $tan89^0=cot1^0$, the product of the pair results into a 1.

Likewise, we discover the pairs, $tand2^0$ and $tan88^0$, $tan3^0$ and $tan87^0$, continuing up to the pairing of $tan44^0$ and $tan46^0$. Each of these pairs contributes 1 to the result evaluation.

Finally, only $tan45^0=1$ is left alone.

Answer: c: 1.

Q.36 The value of $cot18^0\left(cot72^0cos^222^0+\frac{1}{tan72^0sec^268^0}\right)$ is,

a. $\frac{1}{\sqrt{3}}$
b. 3
c. 1
d. $\sqrt{2}$

Solution - Problem analysis

In this second problem also using the ***Complementary Trigonometric functions concepts,*** we observe presence of two distinct values of angles as, $cot72^0=tan18^0$, $tan72^0=cot18^0$ and $sec68^0=cosec22^0$. Thus only two distinct angle values of 18^0 and 22^0 are present. So we decide to transform all functions to these two values.

Solution - Simplifying actions

$$E=cot18^0\left(tan18^0cos^222^0+\frac{1}{cot18^0cosec^222^0}\right)$$

$=cos^222^0+sin^222^0=1$, taking the $cot18^0$ factor inside the brackets.

Answer: c: 1

Q.37 If $asin\theta+bcos\theta=c$, then the value of $acos\theta-bsin\theta$ is,

a. $\pm\sqrt{-a^2+b^2+c^2}$
b. $\pm\sqrt{a^2-b^2+c^2}$
c. $\pm\sqrt{a^2-b^2-c^2}$
d. $\pm\sqrt{a^2+b^2-c^2}$

This urges us to start the solution process by squaring the given expression,

$asin\theta + bcos\theta = c$

Or, $a^2 sin^2\theta + 2absin\theta cos\theta + b^2 cos^2\theta = c^2$

Or, $a^2(1 - cos^2\theta) + 2absin\theta cos\theta + b^2(1 - sin^2\theta) = c^2$

Or, $a^2 + b^2 - c^2 = a^2 cos^2\theta - 2absin\theta cos\theta + b^2 sin^2\theta$

Or, $(acos\theta - bsin\theta)^2 = a^2 + b^2 - c^2$

Or, $acos\theta - bsin\theta = \pm\sqrt{a^2 + b^2 - c^2}$.

Answer: d: $\pm\sqrt{a^2 + b^2 - c^2}$.

Q.38 The value of $\left(\frac{cos^2\theta(sin\theta + cos\theta)}{cosec^2\theta(sin\theta - cos\theta)} + \frac{sin^2\theta(sin\theta - cos\theta)}{sec^2\theta(sin\theta + cos\theta)}\right)(sec^2\theta - cosec^2\theta)$ is,

a. 1
b. 2
c. 3
d. 4

Transforming the smaller expression we have,

$$(sec^2\theta - cosec^2\theta) = \frac{(sin^2\theta - cos^2\theta)}{sin^2\theta cos^2\theta}$$

When we take this factor inside the brackets multiplying with the two complex terms, significant simplification is achieved immediately,

$$\left(\frac{cos^2\theta(sin\theta + cos\theta)}{cosec^2\theta(sin\theta - cos\theta)} + \frac{sin^2\theta(sin\theta - cos\theta)}{sec^2\theta(sin\theta + cos\theta)}\right)\frac{(sin^2\theta - cos^2\theta)}{sin^2\theta cos^2\theta}$$

$= (sin\theta + cos\theta)^2 + (sin\theta - cos\theta)^2$, all other factors having been canceled out

$= 2(sin^2\theta + cos^2\theta)$, the middle terms having been canceled out

$= 2$

Answer: b: 2.

Q.39

$$\frac{tan\theta}{1-cot\theta}+\frac{cot\theta}{1-tan\theta}$$ is equal to,

a. $1-tan\theta-cot\theta$
b. $1+tan\theta+cot\theta$
c. $1-tan\theta+cot\theta$
d. $1+tan\theta-cot\theta$

Thus the target expression,

$$E=\frac{x}{1-\frac{1}{x}}+\frac{\frac{1}{x}}{1-x}$$

$$=\frac{x^2}{x-1}-\frac{1}{x(x-1)}$$

$$=\frac{x^3-1}{x(x-1)}$$

Now we will use the algebraic expression, $x^3-1=(x-1)(x^2+x+1)$. Thus,

$$E=\frac{x^2+x+1}{x}$$

$$=x+1+\frac{1}{x}$$

$$=1+tan\theta+cot\theta$$

Answer: b: $1+tan\theta+cot\theta$.

Q.40

If $tan\theta=\frac{sin\alpha-cos\alpha}{sin\alpha+cos\alpha}$ then $sin\alpha+cos\alpha$ is,

a. $\pm\sqrt{2}sin\theta$
b. $\pm\sqrt{2}cos\theta$
c. $\pm\frac{1}{\sqrt{2}}cos\theta$
d. $\pm\frac{1}{\sqrt{2}}sin\theta$

Squaring both sides of the equation and adding 1,

$$tan^2\theta + 1 = \frac{(sin\alpha - cos\alpha)^2}{(sin\alpha + cos\alpha)^2} + 1$$

$$sec^2\theta = \frac{(sin\alpha - cos\alpha)^2 + (sin\alpha + cos\alpha)^2}{(sin\alpha + cos\alpha)^2}.$$

The numerator in the RHS is in the form of a standard rich concept which we name as ***Middle term cancellation concept*** as detailed below.

When squares of sum and subtraction expressions in two variables are added, the middle terms are canceled out leaving twice the sum of squares of the two variables.

$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$.

When applied to trigonometry we have,

$(sin\theta + cos\theta)^2 + (sin\theta - cos\theta)^2$,

$= 2(sin^2\theta + cos^2\theta)$, the middle terms $2sin\theta cos\theta$ cancel out

$= 2$, a much more simplified result.

Thus from the last result we have,

$$sec^2\theta = \frac{2}{(sin\alpha + cos\alpha)^2},$$

Or, $(sin\alpha + cos\alpha)^2 = 2cos^2\theta$,

Or, $sin\alpha + cos\alpha = \pm\sqrt{2}cos\theta$.

Answer: b: $\pm\sqrt{2}cos\theta$.

Q.41 If $cos^2\alpha - sin^2\alpha = tan^2\beta$, then $cos^2\beta - sin^2\beta =$

a. $tan^2\alpha$
b. $cot^2\alpha$
c. $cot^2\beta$
d. $tan^2\beta$

$cos^2\alpha - sin^2\alpha = tan^2\beta$,

Or, $sec^2\beta = 1 + cos^2\alpha - sin^2\alpha = 2cos^2\alpha$,

Or, $cos^2\beta = \frac{1}{2cos^2\alpha}$, and

$$sin^2\beta = 1 - cos^2\beta = 1 - \frac{1}{2cos^2\alpha} = \frac{2cos^2\alpha - 1}{2cos^2\alpha}.$$

So the target expression,

$$cos^2\beta - sin^2\beta = \frac{1}{2cos^2\alpha} - \frac{2cos^2\alpha - 1}{2cos^2\alpha}$$

$$= \frac{2(1 - cos^2\alpha)}{2cos^2\alpha}$$

$$= tan^2\alpha$$

Answer: a: $tan^2\alpha$.

Q.42

If $tan\alpha = ntan\beta$, and $sin\alpha = msin\beta$ then $cos^2\alpha$ is,

a. $\frac{m^2 - 1}{n^2 - 1}$
b. $\frac{m^2 + 1}{n^2 + 1}$
c. $\frac{m^2}{n^2 + 1}$
d. $\frac{m^2}{n^2}$

As we have decided $cot\beta$ to be the target function we directly take up the first expression to get $cot^2\beta$ in terms of purely α functions.

$tan\alpha = ntan\beta$,

Or, $cot\beta = \dfrac{n}{tan\alpha}$

Or, $cot^2\beta = \dfrac{n^2}{sec^2\alpha - 1} = \dfrac{n^2cos^2\alpha}{1 - cos^2\alpha}$.

We proceed to convert the $sec^2\alpha$ to $cos^2\alpha$ with an eye fixed on the target expression where we have only $cos^2\alpha$.

Taking similar actions on the second expression we have,

$sin\alpha = msin\beta$,

Or, $cosec^2\beta = \dfrac{m^2}{sin^2\alpha} = \dfrac{m^2}{1 - cos^2\alpha}$,

In the same vein we convert the $sin^2\alpha$ to $cos^2\alpha$ as that is in the target expression.

Again converting $cosec^2\beta$ to $cot^2\beta$,

$cot^2\beta + 1 = \dfrac{m^2}{1 - cos^2\alpha}$,

Or, $cot^2\beta = \dfrac{m^2}{1 - cos^2\alpha} - 1 = \dfrac{m^2 - 1 + cos^2\alpha}{1 - cos^2\alpha}$

Equating the two as planned, we get,

$\dfrac{n^2cos^2\alpha}{1 - cos^2\alpha} = \dfrac{m^2 - 1 + cos^2\alpha}{1 - cos^2\alpha}$,

Or, $n^2cos^2\alpha = m^2 - 1 + cos^2\alpha$

Or, $cos^2\alpha = \dfrac{m^2 - 1}{n^2 - 1}$,

Answer: a: $\dfrac{m^2 - 1}{n^2 - 1}$.

Q.43 If A, B and C are the three angles of a triangle, then the incorrect relation among the following is,

a. $cos\frac{A+B}{2}=sin\frac{C}{2}$
b. $sin\frac{A+B}{2}=cos\frac{C}{2}$
c. $cot\frac{A+B}{2}=tan\frac{C}{2}$
d. $tan\frac{A+B}{2}=sec\frac{C}{2}$

In a triangle, the sum of three angles is 180^0, that is,

$A+B+C=180^0$,

Or, $\frac{A+B}{2}=\left(\frac{\pi}{2}-\frac{C}{2}\right)$.

The same will be the relations for the other pairs of angles $(B+C)$ and $(C+A)$.

With these relations, applying the ***complementary trigonometric functions concept*** of

$sin\left(\frac{\pi}{2}-\theta\right)=cos\theta$

$cos\left(\frac{\pi}{2}-\theta\right)=sin\theta$

$tan\left(\frac{\pi}{2}-\theta\right)=cot\theta$, and

$cot\left(\frac{\pi}{2}-\theta\right)=tan\theta$,

we would easily be able to identify the odd wrong relation out of the four.

Just observing the choices we find that the first three relations follow the ***complementary trigonometric functions concept*** perfectly. But in the fourth relation, sec is not the complementary function of tan, rather cot is. So this is the wrong one.

Answer: d: $tan\frac{A+B}{2}=sec\frac{C}{2}$.

Q.44 If θ is a positive acute angle and $tan2\theta tan3\theta = 1$ then the value of $\left(2cos^2\frac{5\theta}{2} - 1\right)$ is,

a. 0
b. 1
c. $-\frac{1}{2}$
d. $\frac{1}{2}$

$tan2\theta tan3\theta = 1$,

Or, $tan2\theta = cot3\theta = tan(90^0 - 3\theta)$

Or, $2\theta = 90^0 - 3\theta$,

Or, $5\theta = 90^0$,

Or, $\theta = 18^0$, as expected.

Substituting this value in target expression we get,

$\left(2cos^2\frac{5\theta}{2} - 1\right) = 2cos^2 45^0 - 1 = 0$.

Answer: a: 0.

Q.45 The $cosec39^0 = p$, the value of, $\frac{1}{cosec^2 51^0} + sin^2 39^0 + tan^2 51^0 - \frac{1}{sin^2 51^0 sec^2 39^0}$ is,

a. $p^2 - 1$
b. $\sqrt{p^2 - 1}$
c. $1 - p^2$
d. $\sqrt{1 - p^2}$

Solution - Problem analysis and execution

From ***Complementary Trigonometric functions concepts*** we know,

$$sin\left(\frac{\pi}{2}-\theta\right)=cos\theta$$

$$cos\left(\frac{\pi}{2}-\theta\right)=sin\theta$$, and so,

$$tan\left(\frac{\pi}{2}-\theta\right)=cot\theta$$, where θ is acute.

With this knowledge and analyzing the terms of the target expression we decide to transform the requisite functions giving the target expression,

$$E=sin^2 51^0+sin^2 39^0+tan^2 51^0-\frac{cos^2 39^0}{sin^2 51^0}$$

$$=cos^2 39^0+sin^2 39^0+cot^2 39^0-\frac{cos^2 39^0}{cos^2 39^0}$$

$$=1+(cosec^2 39^0-1)-1$$

$$=p^2-1$$

Answer: a: p^2-1.

Q.46 If $sec\theta=x+\frac{1}{4x}$, where $(0^0<\theta<90^0)$, then $sec\theta+tan\theta$ is,

a. $\frac{x}{2}$
b. $2x$
c. $\frac{2}{x}$
d. x

$sec\theta = x + \frac{1}{4x}$,

Or, $sec\theta = \frac{1}{2}\left(2x + \frac{1}{2x}\right)$.

As we know, $sec^2\theta - 1 = tan^2\theta$, squaring and subtracting 1 from both sides of the above equation we get,

$$sec^2\theta - 1 = \frac{1}{4}\left(4x^2 + 2 + \frac{1}{4x^2}\right) - 1,$$

Or, $tan^2\theta = \frac{1}{4}\left(4x^2 - 2 + \frac{1}{4x^2}\right)$

$$= \frac{1}{4}\left(2x - \frac{1}{2x}\right)^2,$$

Or, $tan\theta = \frac{1}{2}\left(2x - \frac{1}{2x}\right)$.

Adding $sec\theta$ and $tan\theta$ expressions finally we get,

$sec\theta + tan\theta = 2x$

Answer: b: $2x$.

Q.47 If $tan\theta = 1$, then the value of $\frac{8sin\theta + 5cos\theta}{sin^3\theta - 2cos^3\theta + 7cos\theta}$ is,

a. $2\frac{1}{2}$
b. 2
c. 3
d. $\frac{4}{5}$

$$E = \frac{8sin\theta + 5sin\theta}{sin^3\theta - 2sin^3\theta + 7sin\theta}$$

$$= \frac{13}{-sin^2\theta + 7}.$$

At this point we realize that we also have to evaluate $sin^2\theta$ from the input value $tan\theta = 1$. This is the ***second way to use the same input expression.*** Thus we have,

$tan\theta = 1$,

Or, $cot^2\theta = 1$,

Or, $cosec^2\theta - 1 = 1$,

Or, $sin^2\theta = \frac{1}{2}$.

Substituting,

$$E = \frac{13}{-\frac{1}{2} + 7}$$

$$= \frac{13}{\frac{13}{2}}$$

$$= 2$$

Answer: b: 2.

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Q.48 If $7sin\theta = 24cos\theta$, where $0 < \theta < \frac{\pi}{2}$, then the value of $14tan\theta - 75cos\theta - 7sec\theta$ is,

a. 1
b. 3
c. 2
d. 4

.

$7sin\theta = 24cos\theta$,

Or, $tan\theta = \frac{24}{7}$.

This leaves basically $cos\theta$ in the target expression. So we need a second input tranformation applying ***Multiple input use technique,***

$7sin\theta = 24cos\theta$,

Or, $tan\theta = \frac{24}{7}$,

Or, $tan^2\theta + 1 = \frac{24^2}{7^2} + 1$,

Or, $sec^2\theta = \frac{576}{49} + 1 = \frac{625}{49}$,

Or, $sec\theta = \frac{25}{7}$, as $0 < \theta < \frac{\pi}{2}$, $sec\theta$ is positive.

So,

$cos\theta = \frac{7}{25}$.

Using these values in the target expression we have,

$$E = 14tan\theta - 75cos\theta - 7sec\theta,$$

$$= 14 \times \frac{24}{7} - 75 \times \frac{7}{25} - 7 \times \frac{25}{7}$$

$$= 48 - 21 - 25$$

$$= 2.$$

Answer: c: 2.

Q.49

The minimum value of $sin^2\theta + cos^2\theta + sec^2\theta + cosec^2\theta + tan^2\theta + cot^2\theta$ is equal to,

a. 1
b. 7
c. 3
d. 5

$$E = 3 + 2(tan^2\theta - 2 + cot^2\theta) + 4$$

$$= 7 + (tan\theta - cot\theta)^2.$$

The square term of this expression will always be positive except when $tan\theta = cot\theta$.

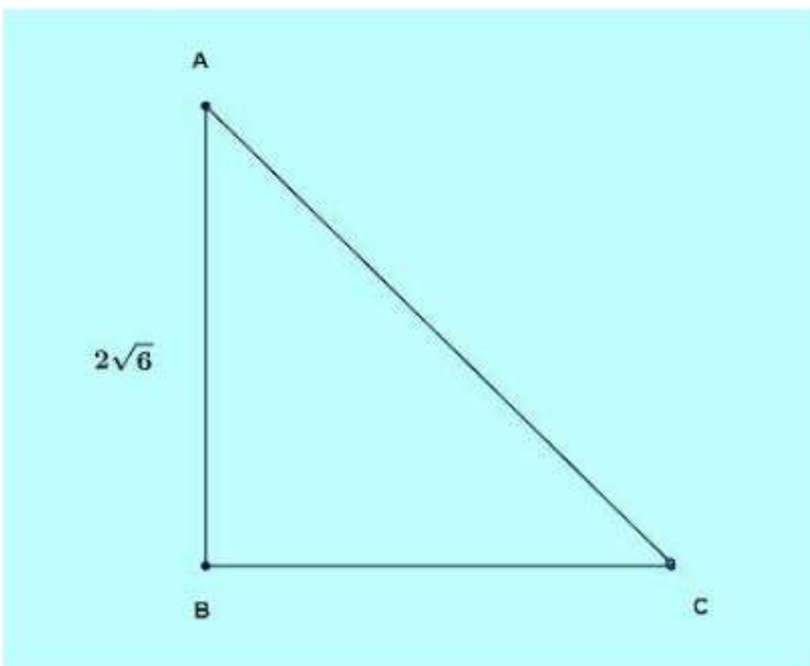
Thus the minimum value of the given expression is 7.

Answer: b: 7.

Q.50 In a right $\triangle ABC$ with right angle at $\angle ABC$, if $AB = 2\sqrt{6}$ and $AC - BC = 2$ then, $secA + tanA$ is,

a. $\frac{\sqrt{6}}{2}$
b. $2\sqrt{6}$
c. $\sqrt{6}$
d. $\frac{1}{\sqrt{6}}$

The following figure describes the problem,



As the length of a side is given and the target expression is in usual form of sum of basic trigonometric functions that primarily are ratios of side lengths of a right triangle, we decide to go for finding the length of sides using Pythagoras theorem rather than trigonometric simplification of target expression.

In $\triangle ABC$,

$AB = 2\sqrt{6}$ and,

$AC - BC = 2$.

Multiplying both sides by $(AC + BC)$ we get,

$AC^2 - BC^2 = 2(AC + BC)$,

Or, $AB^2 = 2(AC + BC)$,

Or, $AC + BC = \frac{1}{2}(2\sqrt{6})^2 = 12$.

Adding this with the equation of $AC - BC = 2$ we get,

$2AC = 14$,

Or, $AC = 7$.

And subracting,

$2BC = 10$,

Or, $BC = 5$.

So,

$secA = \dfrac{AC}{AB} = \dfrac{7}{2\sqrt{6}}$, and

$tanA = \dfrac{BC}{AB} = \dfrac{5}{2\sqrt{6}}$, giving,

$secA + tanA = \dfrac{12}{2\sqrt{6}} = \sqrt{6}$.

Answer: c: $\sqrt{6}$.

Q.51

If $tan2\theta . tan4\theta = 1$, then the value of $tan3\theta$ is,

a. $\sqrt{3}$
b. 0
c. $\dfrac{1}{\sqrt{3}}$
d. 1

$tan2\theta.tan4\theta = 1$,

Or, $tan2\theta = \frac{1}{tan4\theta} = cot4\theta$.

From ***Complementary Trigonometric functions concepts*** we know,

$sin\left(\frac{\pi}{2} - \theta\right) = cos\theta$

$cos\left(\frac{\pi}{2} - \theta\right) = sin\theta$, and so,

$tan\left(\frac{\pi}{2} - \theta\right) = cot\theta$, where θ is acute.

So,

$tan2\theta = cot4\theta = tan\left(\frac{\pi}{2} - 4\theta\right)$,

Or, $2\theta = \left(\frac{\pi}{2} - 4\theta\right)$,

Or, $6\theta = \frac{\pi}{2}$,

Or, $3\theta = \frac{\pi}{4} = 45^0$, and

$tan3\theta = tan45^0 = 1$.

Answer: d: 1.

Q.52

If $sin\frac{\pi x}{2} = x^2 - 2x + 2$, then the value of x is,

a. 0
b. -1
c. 1
d. none of these

$sin\frac{\pi x}{2} = x^2 - 2x + 2 = (x-1)^2 + 1$.

As for any real value of x the square of sum term will be positive it will make the value of the RHS larger than 1 which is not possible for any value of a sin function. So the square term must be 0 and so, $x = 1$.

Answer: c: 1.

Q.53

If $2sin\theta + cos\theta = \frac{7}{3}$, then the value of $(tan^2\theta - sec^2\theta)$ is,

a. 0
b. $\frac{7}{3}$
c. $\frac{3}{7}$
d. -1

As a rule we examine the target expression and this time it handed us the solutuon in one step without going into any complexity of using the awkward input expression,

$(tan^2\theta - sec^2\theta) = tan^2\theta - (1 + tan^2\theta) = -1$

Answer: d: -1.

Q.54

If $(rcos\theta - \sqrt{3})^2 + (rsin\theta - 1)^2 = 0$, then the value of $\frac{rtan\theta + sec\theta}{rsec\theta + tan\theta}$ is,

a. $\frac{4}{5}$
b. $\frac{\sqrt{3}}{4}$
c. $\frac{\sqrt{5}}{4}$
d. $\frac{5}{4}$

$r^2(sin^2\theta + cos^2\theta) = r^2 = 4,$

Or, $r = 2$.

Substituting this value of r in any of the two equations we get the value of θ as 30^0.

Using these values of $r = 2$ and $\theta = 30^0$ in the target expression we have the simplified target expression as,

$$E = \frac{2tan30^0 + sec30^0}{2sec30^0 + tan30^0}$$

$$= \frac{\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{\frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{4}{\sqrt{3}}}{\frac{5}{\sqrt{3}}}$$

$$= \frac{4}{5}.$$

Answer: a: $\frac{4}{5}$.

Q55. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 30^o and 45^o respectively. If the lighthouse is 100 m high, the distance between the two ships is:

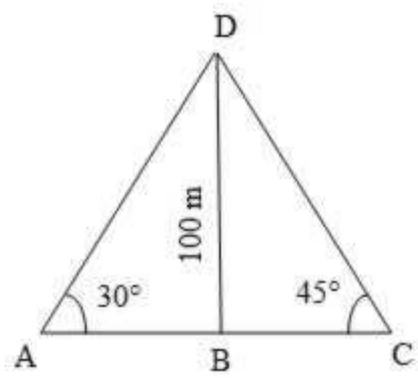
A. 300 m

B. 173 m

C. 273 m

D. 200 m

Answer: **Option** C



Let BD be the lighthouse and A and C be the positions of the ships.
Then, BD = 100 m, $\angle BAD = 30°$, $\angle BCD = 45°$

$$\tan 30^\circ = \frac{BD}{BA}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BA}$$
$$\Rightarrow BA = 100\sqrt{3}$$

$$\tan 45^\circ = \frac{BD}{BC}$$
$$\Rightarrow 1 = \frac{100}{BC}$$
$$\Rightarrow BC = 100$$

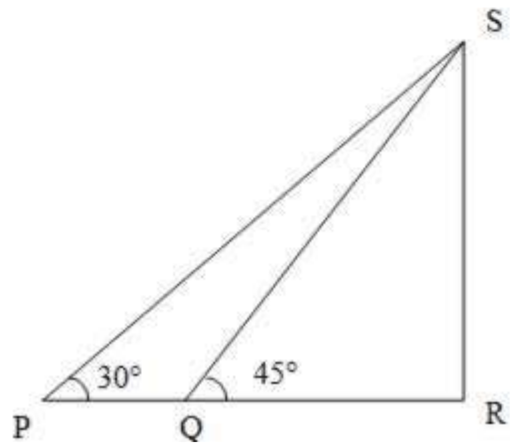
Distance between the two ships
= AC = BA + BC
$= 100\sqrt{3} + 100$
$= 100(\sqrt{3} + 1)$
= 100(1.73+1) = 100 × 2.73 = 273 m

Q56. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of the elevation becomes 45°. What is the distance between the base of the tower and the point P?

A. 9 units
B. $3\sqrt{3}$ units
C. Data inadequate
D. 12 units

Answer: Option **C**

Explanation:



$$\tan 45^\circ = \frac{SR}{QR}$$

$$\tan 30^\circ = \frac{SR}{PR} = \frac{SR}{(PQ + QR)}$$

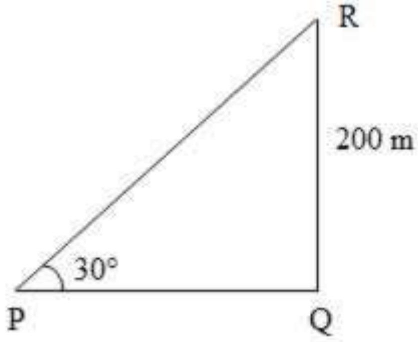
Two equations and 3 variables. Hence we can not find the required value with the given data.

(Note that if one of SR, PQ, QR is known, this becomes two equations and two variables and if that was the case, we could have found out the required value.)

Q57. From a point P on a level ground, the angle of elevation of the top tower is 30°. If the tower is 200 m high, the distance of point P from the foot of the tower is:

A. 346 m
B. 400 m
C. 312 m
D. 298 m

Answer: Option **A**

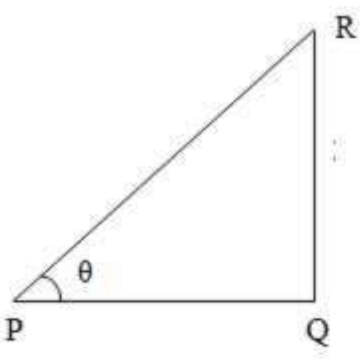
Explanation: 

$$\tan 30^\circ = \frac{RQ}{PQ}$$
$$\frac{1}{\sqrt{3}} = \frac{200}{PQ}$$
$$PQ = 200\sqrt{3} = 200 \times 1.73 = 346 \text{ m}$$

Q58. The angle of elevation of the sun, when the length of the shadow of a tree is equal to the height of the tree, is:

A. None of these
B. 60°
C. 45°
D. 30°

Answer: Option **C**

Explanation: 

Consider the diagram shown above where QR represents the tree and PQ represents its shadow

We have, QR = PQ
Let ∠QPR = θ

$$\tan\theta = \frac{QR}{PQ} = 1 \quad (since\ QR = PQ)$$
=> θ = 45°

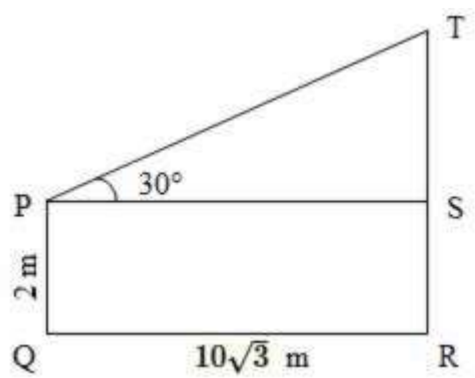
i.e., required angle of elevation = 45°

Q59. An observer 2 m tall is 10√$\boxed{3}$103 m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is:

A. None of these
B. 12 m
C. 14 m
D. 10 m

Answer: Option **B**

Explanation:



SR = PQ = 2 m
PS = QR = $10\sqrt{3}$ m

$$\tan 30^\circ = \frac{TS}{PS}$$
$$\frac{1}{\sqrt{3}} = \frac{TS}{10\sqrt{3}}$$
$$TS = \frac{10\sqrt{3}}{\sqrt{3}} = 10\text{ m}$$

TR = TS + SR = 10 + 2 = 12 m

Q60.

26. A flagstaff is placed on top of a building. The flagstaff and building subtend equal angles at a point on level ground which is 200 m away from the foot of the building. If the height of the flagstaff is 50 m and the height of the building is h, which of the following is true?

A. $h^3 - 50h^2 + (200)^2h + (200)^2 50 = 0$ B. None of these

C. $h^3 + 50h^2 + (200)^2h - (200)^2 50 = 0$ D. $h^3 - 50h^2 - (200)^2h + (200)^2 50 = 0$

Answer: Option **C**

Explanation:

Let AD be the flagstaff and CD be the building.

Assume that the flagstaff and building subtend equal angles at point B.

Given that AD = 50 m, CD = h and BC = 200 m

Let $\angle ABD = \theta$, $\angle DBC = \theta$ ($\because$ flagstaff and building subtend equal angles at a point on level ground).

Then, $\angle ABC = 2\theta$

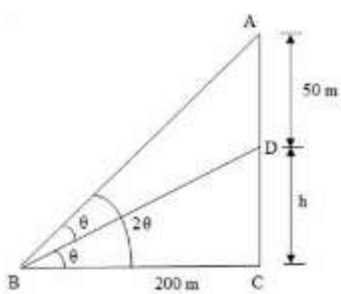
From the right $\triangle$ BCD,

$$\tan\theta = \frac{DC}{BC} = \frac{h}{200} \quad \cdots (eq:1)$$

From the right $\triangle$ BCA,

$$\tan 2\theta = \frac{AC}{BC} = \frac{AD + DC}{200} = \frac{50 + h}{200}$$

$$\Rightarrow \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{50 + h}{200} \qquad \left(\because \tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}\right)$$



$$\Rightarrow \frac{2\left(\frac{h}{200}\right)}{1-\frac{h^2}{200^2}} = \frac{50+h}{200} \quad (\because \text{substituted value of } \tan\theta \text{ from eq:1})$$

$$\Rightarrow 2h = \left(1-\frac{h^2}{200^2}\right)(50+h)$$

$$\Rightarrow 2h = 50 + h - \frac{50h^2}{200^2} - \frac{h^3}{200^2}$$

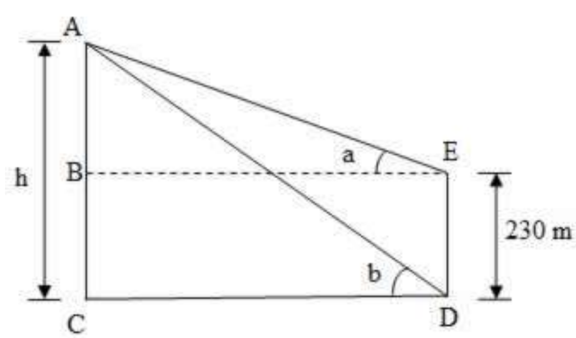
$$\Rightarrow 2\left(200^2\right)h = 50(200)^2 + h(200)^2 - 50h^2 - h^3 \quad (\because \text{multiplied LHS and RHS by } 200^2)$$

$$\Rightarrow h^3 + 50h^2 + (200)^2h - 50(200)^2 = 0$$

Q61. From the foot and the top of a building of height 230 m, a person observes the top of a tower with angles of elevation of b and a respectively. What is the distance between the top of these buildings if tan a = 5/12 and tan b = 4/5

A. 400 m
B. 250 m
C. 600 m
D. 650 m

Answer: Option **D**



Explanation:

Let ED be the building and AC be the tower.

Given that ED = 230 m, ∠ADC = b, ∠AEB = a
Also given that tan a = 5/12 and tan b = 4/5

Let AC = h

Required Distance = Distance between the top of these buildings = AE

From the right △ ABE,

$$\tan(a) = \frac{\text{AB}}{\text{BE}}$$

$$=> \frac{5}{12} = \frac{\text{(h-230)}}{\text{BE}}$$ [∵ tan(a)=5/12(given), AB = (AC-BC) = (AC-ED) = (h-230)]

$$=> \text{BE} = \frac{12\text{(h-230)}}{5} \quad \cdots (eq:1)$$

$$\tan(b) = \frac{AC}{CD}$$

$$=> \frac{4}{5} = \frac{h}{CD} \quad [\because \tan(b) = 4/5(\text{given}), AC=h]$$

$$=> CD = \frac{5h}{4} \quad \cdots (eq:2)$$

From the diagram, BE = CD

$$\Rightarrow \frac{12(h\text{-}230)}{5} = \frac{5h}{4} \text{ (from eq:1 \& eq:2)}$$
$$\Rightarrow 48h - (4 \times 12 \times 230) = 25h$$
$$\Rightarrow 23h = (4 \times 12 \times 230)$$
$$\Rightarrow h = \frac{(4 \times 12 \times 230)}{23} = 480 \text{ m} \quad \cdots (eq:3)$$

AB = (AC - BC)
= (480 - 230) [∵ Since AC=h=480(from eq:3) and BC=ED=230 m(given)]
= 250 m

In the triangle ABE, tan(a) = 5/12. Let's figure out the value of sin(a) now.

Consider a triangle with opposite side = 5 and adjacent side = 12 such that tan(a) = 5/12

$$\text{hypotenuse} = \sqrt{5^2 + 12^2} = 13$$
$$\text{i.e., } \sin(a) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5}{13}$$

We have seen that sin(a) = $\frac{5}{13}$

$$\Rightarrow \frac{AB}{AE} = \frac{5}{13}$$
$$\Rightarrow AE = AB \times \frac{13}{5}$$
$$= 250 \times \frac{13}{5} = 650 \text{ m}$$

i.e., Distance between the top of the buildings = 650 m

Q62.

28. A vertical pole fixed to the ground is divided in the ratio 1:9 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a place on the ground, 15 m away from the base of the pole, what is the height of the pole?

A. $60\sqrt{5}$ m

B. $15\sqrt{5}$ m

C. $15\sqrt{3}$ m

D. $60\sqrt{3}$ m

Answer: Option **A**

Explanation:

Let CB be the pole and point D divides it such that BD : DC = 1 : 9

Given that AB = 15 m

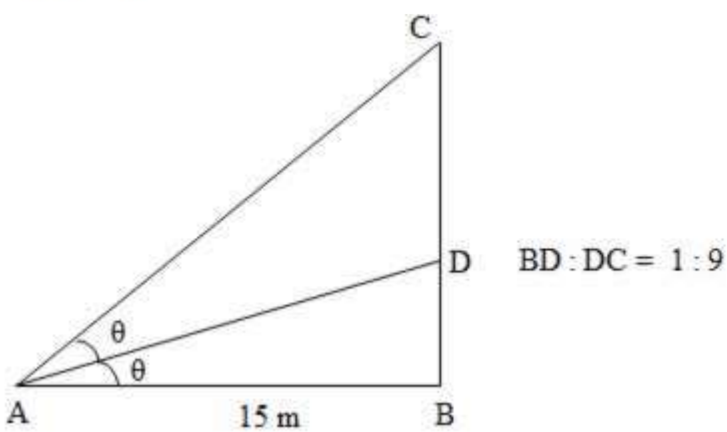
Let the the two parts subtend equal angles at point A such that
$\angle CAD = \angle BAD = \theta$

From "Angle Bisector Theorem", we have
$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{9} = \frac{15}{AC} \quad [\because BD : DC = 1 : 9 \text{ and } AB = 15(\text{given})]$$

=> AC = 15×9 m ...(eq: 1)

From the right $\triangle$ ABC,
$$CB = \sqrt{AC^2 - AB^2} \quad (\because \text{Pythagorean theorem})$$
$$= \sqrt{(15 \times 9)^2 - 15^2} \quad (\because AC=15\times9(\text{eq:1}) \text{ and } AB=15 \text{ m(given)})$$
$$= \sqrt{15^2 \times 9^2 - 15^2}$$
$$= \sqrt{15^2\left(9^2 - 1\right)} = \sqrt{15^2 \times 80}$$
$$= \sqrt{15^2 \times 16 \times 5} = 15 \times 4 \times \sqrt{5}$$
$$= 60\sqrt{5} \text{ m}$$

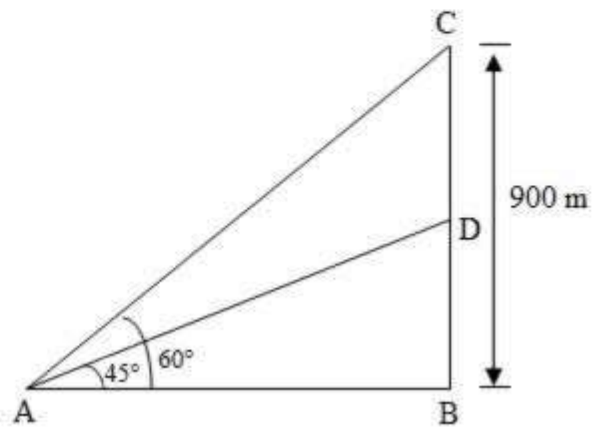


Q63. An aeroplane when 900 m high passes vertically above another aeroplane at an instant when their angles of elevation at same observing point are 60° and 45° respectively. Approximately, how many meters higher is the one than the other?

A. 381 m B. 169 m
C. 254 m D. 211 m

Answer: Option **A**

Explanation:



Let C and D be the position of the aeroplanes.

Given that CB = 900 m, ∠CAB = 60°, ∠DAB = 45°

From the right Δ ABC,

$\tan 60^\circ = \frac{CB}{AB}$

$\sqrt{3} = \frac{900}{AB}$

$AB = \frac{900}{\sqrt{3}}$

$= \frac{900 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{900\sqrt{3}}{3} = 300\sqrt{3}$

From the right Δ ABD,

$\tan 45^\circ = \frac{DB}{AB}$

$1 = \frac{DB}{AB}$

$DB = AB = 300\sqrt{3}$

Required height

= CD = (CB-DB)

$= \left(900 - 300\sqrt{3}\right) = (900 - 300 \times 1.73)$

$= (900 - 519) = 381$ m

Q64. When the sun's altitude changes from 30° to 60°, the length of the shadow of a tower decreases by 70m. What is the height of the tower?

A. 35 m
B. 140 m
C. 60.6 m
D. 20.2 m

Answer: Option **C**

Explanation:

Let AD be the tower, BD be the initial shadow and CD be the final shadow.

Given that BC = 70 m, ∠ABD = 30°, ∠ACD = 60°,

Let CD = x, AD = h

From the right △ CDA,

$$\tan 60^\circ = \frac{AD}{CD}$$

$$\sqrt{3} = \frac{h}{x} \quad \cdots (eq:1)$$

From the right △ BDA,

$$\tan 30^\circ = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{70+x} \quad \cdots (eq:2)$$

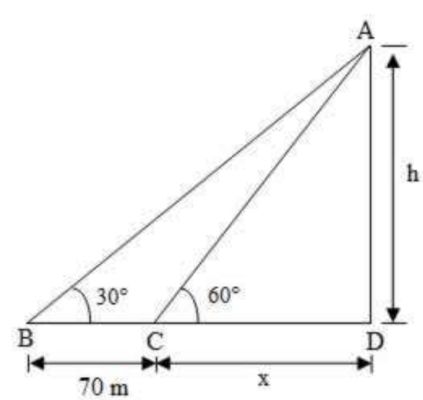
$$\frac{eq:1}{eq:2} \Rightarrow \frac{\sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{\left(\frac{h}{x}\right)}{\left(\frac{h}{70+x}\right)}$$

$$\Rightarrow 3 = \frac{70+x}{x}$$
$$\Rightarrow 2x = 70$$
$$\Rightarrow x = 35$$

Substituting this value of x in eq:1, we have

$$\sqrt{3} = \frac{h}{35}$$

$$\Rightarrow h = 35\sqrt{3} = 35 \times 1.73$$
$$= 60.55 \approx 60.6$$



Data Interpretation

Data Interpretation is one of the easy sections of one day competitive Examinations. It is an extension of Mathematical skill and accuracy. Data interpretation is nothing but drawing conclusions and inferences from a comprehensive data presented numerically in tabular form by means of an illustration, viz. Graphs, Pie Chart etc. Thus the act of organising and interpreting data to get meaningful information is Data Interpretation.

A good grasp of basic geometric as well as arithmetic formulae is must to score high in this section. Familiarity with graphical representation of data like Venn diagrams, graphs, pie charts, histogram, polygon etc. should be thought. Once the data are grasped well, questions based on tables and graphs take little time.

In some competitive examinations data are presented in more than one table or graphs.

The aim is to test not only quantitative skill but also relative, comparative and analytical ability. The crux of the matter is to find a relationship between the two tables or graphs before attempting the questions.

Some Useful Hacks:

1 . Data Interpretation questions are based on information given in tables and graphs. These questions test your ability to interpret the information presented and to select the appropriate data for answering a question.

2 . Get a general picture of the information before reading the question. Read the

given titles carefully and try to understand its nature.

3 . Avoid lengthy calculations generally, data interpretation questions do not require to do extensive calculations and computations.

Most questions simply require reading the data correctly and carefully and putting them to use directly with common sense.

4 . Breakdown lengthy questions into smaller parts and eliminate impossible choices.

5 . Use only the information given and your knowledge of everyday facts, such as the number of hours in a day, to answer the questions based on tables and graphs.

6 . Answer the questions asked and not what you think the questions should be.

7 . Be careful while dealing with units.

8 . To make reading easier and to avoid errors observe graphs keeping them straight.

9 . Be prepared to apply basic mathematical rules, principles and formulae.

10. Since one of the major benefits of graphs and tables is that they present data in a form that enables you to readily make comparisons, use this visual attribute of graphs and tables to help you answer the questions. Where possible, use your eyes instead of your computational skills.

Tables

Tables are often used in reports, magazines and newspaper to present a set of numerical facts. They enable the reader to make comparisons and to draw quick conclusions. It is one of the easiest and most accurate ways of presenting data. They require much closer reading than graphs of charts and hence are difficult and time consuming to interpret.

One of the main purposes of tables is to make complicated information easier to understand. The advantage of presenting data in a table is that one can see the information at a glance.

While answering questions based on tables, carefully read the table title and the column headings. The title of the table gives you a general idea of the type and often the purpose of the information presented.

The column headings tell you the specific kind of information given in that column. Both the table title and the column headings are usually very straight forward.

Graphs

There may be four types of graphs.

1) **Circle Graphs**: Circle graphs are used to show how various sectors are in the whole. Circle graphs are sometimes called Pie Charts. Circle graphs usually give the percent that each sector receives In such representation the total quantity in question is distributed over a total angle of 360°.

While using circle graphs to find ratios of various sectors, don't find the amounts each sector received and then the ratio of the amounts. Find the ratio of the percent’s, which is much quicker.

2) **Line Graphs**: Line graphs are used to show how a quantity changes continuously. If the line goes up, the quantity is increasing; if the line goes down, the quantity is decreasing; if the line is horizontal, the quantity is not changing.

3) **Bar Graphs**: Given quantities can be compared by the height or length of a bar graph. A bar graph can have either vertical or horizontal bars. You can compare different quantities or the same quantity at different times. In bar graph the data is discrete. Presentation of data in this form makes evaluation of parameters comparatively very easy.

4) **Cumulative Graphs**: You can compare several categories by a graph of the cumulative type. These are usually bar or line graphs where the height of the bar or line is divided up proportionally among different quantities.

Data Interpretation :-

As all of you know Data can be written or represented in 4 Forms

1. **Numrical** :- Data in numerical form
2. **Table Form** :- Data in Tabular form
3. **Mixed form** :- Data in Mix Form
4. **Graphical** form Like Line, Bar graph etc.

Must Have Hacks for DI and DA:-

1. Always keep in your Mind that in the circular Graph or Charts 100% =360 Degree .

2. Formulas and Full Concept of Percentage , Average, Approx. Value and Ratio.

3. While solving keeps in mind that which Type of Graph is using in the question this will help you to get the correct answer.

4. First Read the Question and try to interrupt in the question as the Topics says DATA INTERPRETATION. Because every bank wants the PO who can handle and Play with numbers of Figures just in few seconds and not just calculate it with closed eyes.

5. The whole of the DA concept is based on Calculation so if you want to be perfect in DA and DI don't even use calculator at home while practicing. One who can calculate fast and accurate even Large numbers without calculator is 50% ready for the Any Aptitude test.

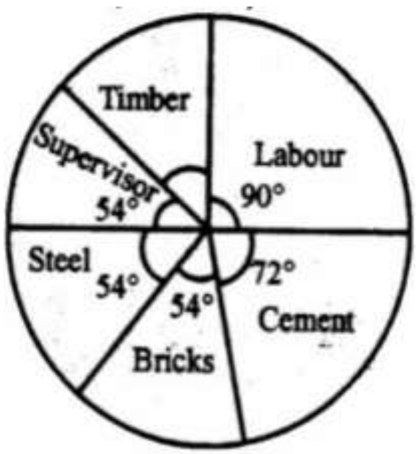
6. Try to use your mind while solving DA questions. Try to write only important Lines on the paper and Others just write and calculate in your Mind. This can be easily done by practicing More and More at Home.

7. Don't feel confusing and no need to Try and Solve all the questions only Focus on questions with you are more familiar and you find yourself in such a way that you can solve the question accurately.

8. Try to solve all types of Questions while practicing this will not confuse you in the exam.

Data Interpretation Set-1: Pie Chart

The following pie chart cost of constructing one house. The total cost was Rs. 6 lakhs.



1. The amount spent on cement is

   A. Rs.2,00,000
   B. Rs.1,60,000
   C. Rs.1,20,000
   D. Rs.1,00,000

2. The amount spent on labour exceeds the amount spent on steel by

   A. 5% of the total cost
   B. 10% of the total cost
   C. 12% of the total cost
   D. 15% of the total cost

3. The amount spent on cement, steel and supervision is what percent of the total cost of construction?

   A. 40%
   B. 45%
   C. 50%
   D. 55%

4. The amount spent on labour exceeds the amount spent on supervision by

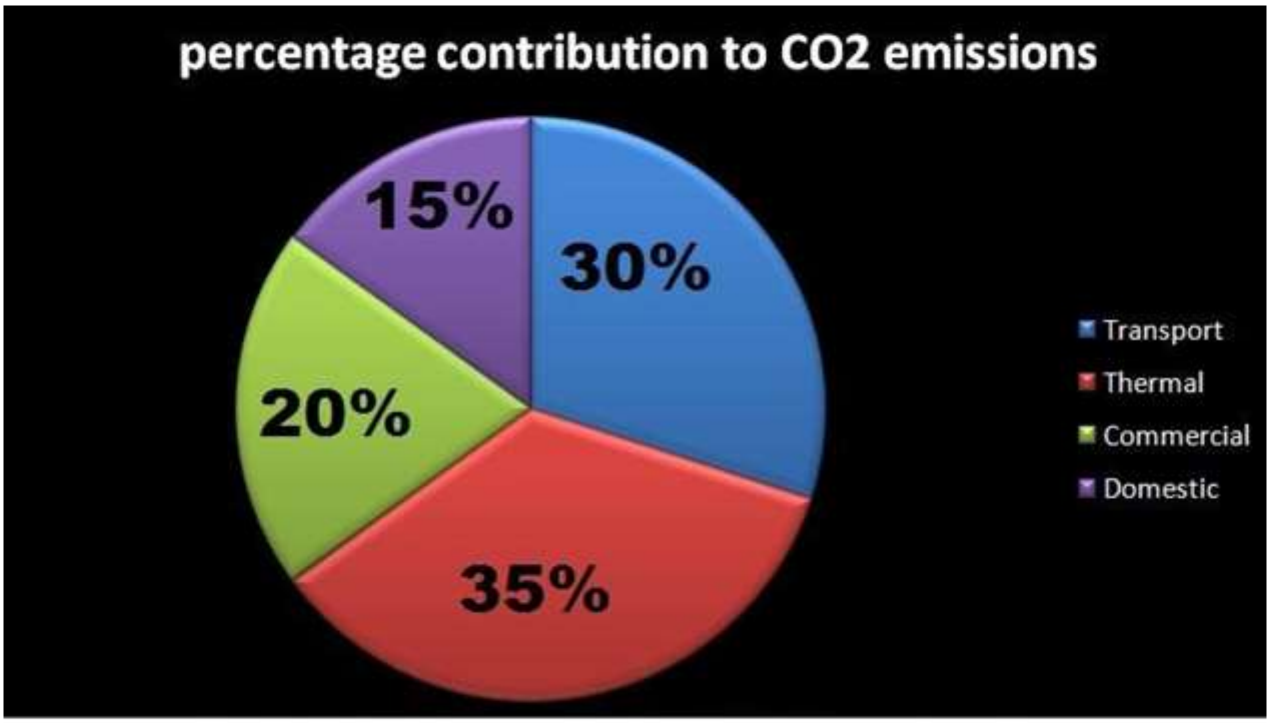
   A. Rs.2,00,000
   B. Rs.16,000
   C. Rs.1,20,000 D. Rs.60,000

5. The amount spent on Timber is

   A. Rs.60,000
   B. Rs.90,000
   C. Rs.1,20,000
   D. Rs.36,000

DI Set-2: Pie-Chart CO2 Emission

The total annual CO_2 emissions from various sectors are 5 mmt. In the Pie Chart given below, the percentage contribution to CO_2 emissions from various sectors is indicated.



1. Which of the following sectors together emit 2.5 mmt of CO_2 every year? a. Thermal and Transport b. Domestic and Commercial c. Transport and Commericial d. Commercial and Thermal	2. Which of the following sectors have emission difference of 1 mmt between them. a. Domestic and Commercial b. Transport and Commercial c. Thermal and Domestic d. Thermal and Transport.

3. Emission of Domestic sector is how much % of Transport and Commercial sector combined?

a. 20%
b. 15%
c. 30%
d. 35%

4. In the next year, if emission from Commercial sector decreases by 20% of its current emission, while other sectors continue to emit same amount of CO_2 as earlier then What will be the new % contribution of Thermal power sector (approximately)?

a. 31.9%
b. 39.8%
c. 42.7%
d. 36.5%

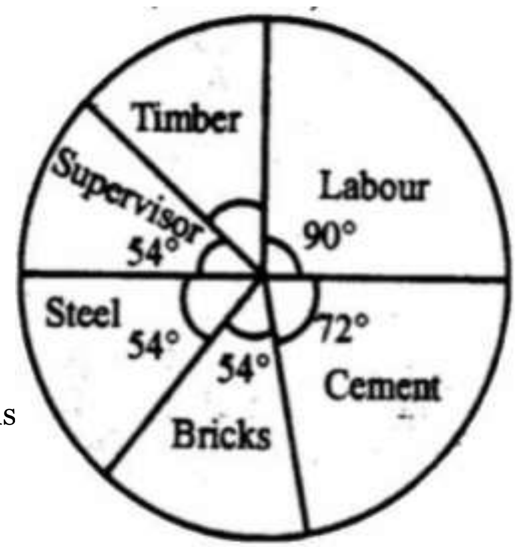
5. Which of the following is incorrectly matched?

a) Domestic=54 degrees.
b) Thermal=125 degrees.
c) commercial =72 degrees.
d) transport=108 degrees.

Solutions for the DI questions

Solution DI Set#1: Pie Chart with Explainations/Shortcuts

You might be tempted to transform degrees into percentages and find absolute Rupee value of each item (timber, labour etc). But in the exam, **<u>don't waste time</u>** by calculating everything. Only calculate the stuff that is asked.



1. The amount spent on cement is

   a. Rs.2,00,000
   b. Rs.1,60,000
   c. Rs.1,20,000
   d. Rs.1,00,000

Total is Rs. 6,00,000 (=360 degrees of the circle) Cement is 72 Degrees.

Apply the ratio principle

Degrees	Value
72 (Cement)	M?
360(total)	6 lakhs

Divide left column on one side and right column on the other side. 72/360= M/6 lakhs

M=(72*6)/360=1.2 lakhs.

2. The amount spent on labor exceeds the amount spent on steel by
   A. 5% of the total cost
   B. 10% of the total cost
   C. 12% of the total cost
   D. 15% of the total cost

There is no need to find absolute values. Just observe the degrees.

Item	Degrees
Labour	90
Steel (Base %)	54
Difference	36

So the difference between Labour and Steel is 36 degrees. And total cost is (6lakh=) 360 degrees.

So the percentage= 36/360 x 100 =10% of the total cost. Answer (B)

3. The amount spent on cement, steel and supervision is what percent of the total cost of construction?

A. 40%

B. 45%

C. 50%

D. 55%

Again no need to find absolute values. Just observe the degrees.

	degrees
Cement	72
Steel	54
Supervision	54
Total	180

He is asking, "what percent of the total cost of construction?" So total cost (360 degrees) is the "base"

%=(180/360) x 100

=50%

4. The amount spent on labour exceeds the amount spent on supervision by

A. Rs.2,00,000 B. Rs.16,000 C. Rs.1,20,000 D. Rs.60,000

Again no need to find absolute values for both items. Just observe the degrees.

Difference between labour and supervision = 90-54 =36 degrees

Shortcut: 36 degrees =10% of a circle. (because total is360 degrees)

So the amount spend = 10% of total (6,00,000)= one decimal leftwards=60,000.

If shortcut is not clicking your mind, no problem. Go by ratio principle

Degrees	Value
36 (difference between labor and supervision)	M?
360 (total)	6 lakhs

Divide left column on one side and right column on the other side.
36/360= M/6 lakhs

M=(36*600000)/360 M=60,000

Answer is (D)

5. The amount spent on Timber is

A. Rs.60,000 B. Rs.90,000 C. Rs.1,20,000 D. Rs.36,000

Observe that degree/percentage of Timber is not given the chart. So, first task is to find degrees of timber.

In a circle Total=360 degrees. So Timber's degree

=360 MINUS the degrees of remaining items

=360-(54+54+54+72+90)

=36 degrees.

Now use the concept given in previous question, 36 degrees= 10% of total cost=60,000 rupees. Answer (A)

Solution: Set #2: CO2 Emission Pie Chart

1. Which of the following sectors together emit 2.5 mmt of CO2 every year?
    a. Thermal and Transport
    b. Domestic and Commercial
    c. Transport and Commercial
    d. Commercial and Thermal

Total emission is 5. And 2.5mmt is 50% (half) of 5 (total emission)

From the given circle, transport + commercial, represent half circle (50%) Similarly Thermal+Domestic is also 50%. But it is not given in options. So answer is (C): Transport + Commercial.

2. Which of the following sectors have emission difference of 1 mmt between them.

    a. Domestic and Commercial
    b. Transport and Commercial
    c. Thermal and Domestic
    d. Thermal and Transport.

- ☐ Total is 5mmt (100%) So 1mmt=1 x 100/5=20%.
- Find out which two sectors have % difference of 20%?
- Thermal minus Domestic=35-15=20%. So (C) is the answer.

3. Emission of Domestic sector is how much % of Transport and Commercial sector combined?

    a. 20%
    b. 15%
    c. 30%
    d. 35%

Observe the chart Required % =(15/50) x 100 =30%

Answer is (**C**)

4. In the next year, if emission from Commercial sector decreases by 20%

while other sectors continue to emit same amount of CO_2 as earlier then What will be the new % contribution of Thermal power sector?

Right now Commercial sector emits 20% of 5mmt=1/5 x 5=1mm

Next year 20% decrease=20% of 1mm=0.2mm will be less emitted. But other Sectors remains the same.

So total emission next year = 5 minus

0.2=4.8mm. And Share of thermal power =

(35% of 5)/4.8

=0.35*5/4.8

=36.5%

5. Which of the following is incorrectly matched?

a) Domestic = 54 degrees.
b) Thermal = 125 degrees.
c) Commercial =72 degrees.
d) Transport = 108 degrees.

100%=360 degrees So 1%=3.6 degree.

Multiply every sector's % with 3.6 and you'll get corresponding degree.

Ans (B) because correct degree of Thermal should be 35 x 3.6=126 degrees.

Let's practice more

1. Introduction: Tabular presentation

A table is a set of data arranged in rows and columns and is one of the most common way of putting information across to people. A table consists of several boxes with information inside. The first row and the first column are generally used to denote the titles. While any type of data can be presented in table form, that too in a very accurate manner, interpreting the data in table form is more difficult and time consuming than the other modes, all of which are basically pictorial or graphical in presentation.

2. Hacks on Solving Table Chart Problems:

A: Read the data very carefully, as the smallest detail may change the meaning of the question completely. Similarly, the instructions have to be understood carefully to prevent wasting time in calculating data that is not required, and also to find out exactly what is the answer that is sought.

B: Try to understand the data provided carefully, before jumping to answer the questions. The questions are designed to be deceptive, and proper understanding of the requirements is a must. If the Data provided is of the combined variety or if there are more than one data table/charts/graphs, try to understand the relation between the given tables.

For Example, one table may talk about absolute sales figures, while the other table may talk of sales as a percentage of production. Hence, any question on excess production or Goods in stock, will require data from both tables.

C: Be very careful of the units used in the tables, and the units in which the answers (options) are provided. A mistake in the units may yield an entirely different answer. Also be careful of whether the answer is required in decimal or percentage. Such errors are common and easily avoidable.

Here is an example consisting tabular data:

Example 1:

Category of Assistance	Average number receiving per month		Total cost per help year (in crores of Rs.)		Cost paid by Centre for the year (in crores of Rs.)	
	1995	**1996**	**1995**	**1996**	**1995**	**1996**
A	36097	38263	38.4	34.8	18.4	17.4
B	6632	5972	5.0	3.2	2.6	1.6
C	32545	31804	76.4	59.4	13.0	10.0
D	13992	11782	26.4	42.6	6.6	10.6
E	21275	228795	216.6	242.8	55.0	62.6

Example 1.1:

The category receiving the least percentage help from the centre (in the entire data) is:

(A) Category B in 1995 **(B)** Category C in 1996 **(C)** Category B in 1996 **(D)** Category D in 1995

Solution:

In this type of question, it is better to examine the alternatives given rather than trying to find the least percentage from the table.

Let us now calculate the required percentage of the given alternatives:

(A) Category B in I995 =(2.65.0)×100=52% (Even without calculation, you can eliminate this choice.)

(B) Category C in 1996 was =(10.059.4)×100=16.8%

(C) Category B in 1996 was =(1.63.2)×100=50% (Even without calculation, you can eliminate this choice.)

(D) Category D in 1995 =(55.0216.6)×100=25.4%

From this we arrive at the answer **(B)** since this is the least percentage.

Example 1.2:

The difference between the average costs paid by the Centre during 1995 and 1996 is

(A) Rs. 66 lakh **(B)** Rs. 13.2 crore **(C)** Rs. 132 lalth **(D)** Rs. 13.2 lakh

Solution:

Adding all the cost figures in the 1995 column, i.e. 18.4+2.6+13.0+6.6+55.0, you get 95.6.

The average in 1995: =95.6+ Number of categories =95.6+5 = Rs. 19.12 Crore

Similarly, the average in 1996: =(17.4+1.6+10.0+10.6+62.6)5 = Rs. 20.44 Crore

The difference = Rs. (20.44–19.12) Cr = Rs. 1.32 Cr = Rs. 132 **lakh**

The correct answer is **(C)**.

(Note how the answer needed conversion from crores to lakhs).

Example 1.3:

Monthly cost to the city receiving E category assistance in 1996 is most nearly:

(A) Rs. 1.8 crore less than that in 1995 **(B)** Rs. 2.1 crore more than that in 1995 **(C)** Rs. 2.1 cnore less than that in 1995 **(D)** Rs. 1.8 crore more than that in 1995

Solution:

Here, straight calculation is only needed. We need to look at the total assistance figures.

In 1995: 216.612=18.05

In 1996: 242.812=20.23

Difference = 2.183 crore ≈ **Rs. 2.1 crore**

The correct answer is **(B).**

Example 1.4:

Assuming that 50% of the persons receiving category B help in 1995 were adults caring for minor children, but the city's contribution towards maintaining these adults was 40% of the total contribution to B program in 1995, average amount paid by the city for each adult per year in 1995 is most nearly:

(A) Rs. 5900 **(B)** Rs. 6000 **(C)** Rs. 7500 **(D)** Rs. 3000.

Solution:

50% of persons receiving B category help during 1995 =3316 City's contribution to maintenance: =5.0×0.4

= Rs. 2 crore =2,00,00,0003316

= Rs. 6031.36

= **Rs. 6000** nearly

The correct choice is **(B)**.

Example 1.5:

Monthly costs to the city of category D during 1995 and 1996 bear a ratio (most nearly)

(A) 2 : 3 **(B)** 5 : 3 **(C)** 3 : 2 **(D)** 3 : 5

Solution:

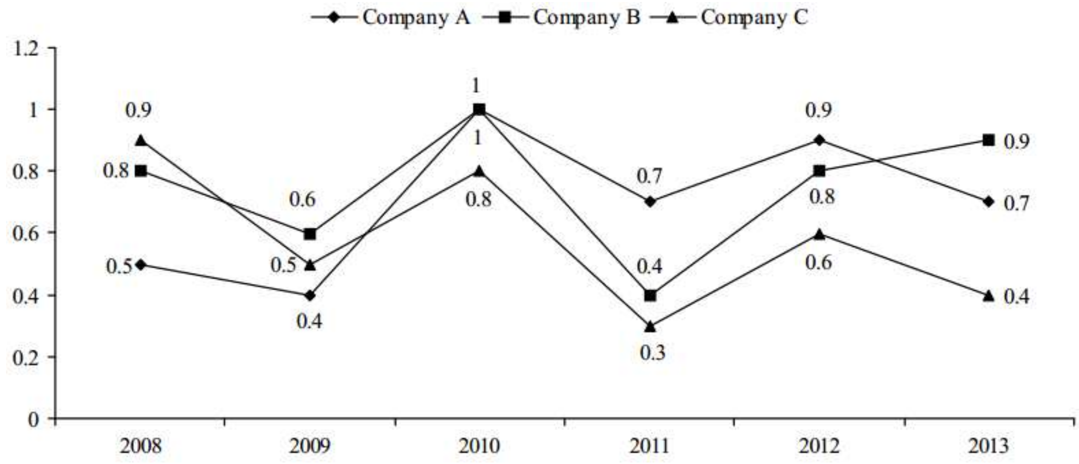
Again, we can straightaway determine the answer through simple calculation.

Since a ratio is required to be calculated, we can avoid the division by 12. Directly from the table we have, total assistance in 1995 and 1996 for Category D as 26.4 and 42.6.

Hence the ratio is 26.4:42.6=3:5 nearly.

Example 1

Following line graph shows the ratio of expenditure to income of three companies A, B and C during the period 2008–2013.



As mentioned above – Reading the headings are important otherwise you will not be able to understand what these lines are all about.

Please observe that – Along Y-Axis are the ratios; Along X-Axis are the years; In between are the lines.

Following Line Graph shows the ratio of expenditure to income of three companies A, B and C.

Learn a few things from the heading:

(1) For Company A in 2008, if Expenditure is Rs 0.9, then Income will be Rs 1, and so on.

(2) It's **Expenditure to Income** Ratio expressed as **E:I** and not Income to Expenditure.

(3) To have Profit, Expenditure is to be less than Income. Reverse is for Loss.

(4) Profit and Loss percentages are calculated using the formulas for the same.

- **Profit** = Income – Expenditure
- **Profit Percentage** = [Profit/Expenditure]*100
- **Loss** = Expenditure – Income
- **Loss Percentage** = [Loss/Expenditure]*100

(5) The lower is the E:I ratio, higher is the profit.

The questions of Expenditure and Income seem difficult to solve. But, let's apply the above mentioned points to solve the questions in no time!

<u>Steps to Solve</u>

Question 1: In which of the following years is the percentage loss/profit of Company C the maximum?

[1] 2008

[2] 2009

[3] 2010

[4] 2011

[5] 2012

Hint: From point no. 5, we conclude that **profit is maximum when E:I is minimum** which is 0.3 in 2011.

Hence **answer is [4].**

Question 2: If the expenditure of Company A in 2008 and 2009 together is Rs 60 lakhs, then what is its income in 2008 and 2009 together?

[1] Rs 120 lakhs

[2] Rs 150 lakhs

[3] Rs 66.66 lakhs

[4] Data inadequate

[5] None of these

Hint: E:I for Company A in 2008 and 2009 is 0.5 and 0.4. This means for Rs 0.5 Expenditure in 2008, Income is Rs 1 in 2008 and for Rs 0.4 Expenditure in 2009, Income is Rs 1 in 2009. But combined Expenditure of 60 lakhs is given. **So, ratios being different, it's not possible to calculate the Income from the combined expenditure. Answer is [4].**

Question 3: If the expenditure of Company B in 2008 and 2012 together is Rs 60 lakhs then what is its income in 2008 and 2012 together?

[1] Rs 66.66 lakhs

[2] Rs 75 lakhs

[3] Rs 48 lakhs

[4] 96 Rs lakhs

[5] Data inadequate

Hint: E:I for 2008 and 2012 is 0.8 and 0.8.

Ratios being same, combined Income from the combined Expenditure can be calculated. Income = E/0.8 = 60/0.8 = 75 lakhs.

Answer is [2].

Question 4: In which of the years does Company C gain 100% profit?

[1] 2008

[2] 2009

[3] 2010

[4] 2011

[5] None of these

Hint: For 100% profit, E:I ratio must be 0.5 so that I = E/0.5 = 2E. It's in 2009.

Answer is [2]

Question 5: What is the percentage decrease in the percentage profit of Company C from 2009 to 2010?

[1] 75%

[2] 300%

[3] 62.5%

[4] 160%

[5] None of these

Hint: E:I of Company C in 2009 = 0.5:1

Profit = 1-0.5 = 0.5

Percentage profit of profit of Company C in 2009 =[0.5/0.5]*100 = 100%

E:I of Company C in 2010 = 0.8:1

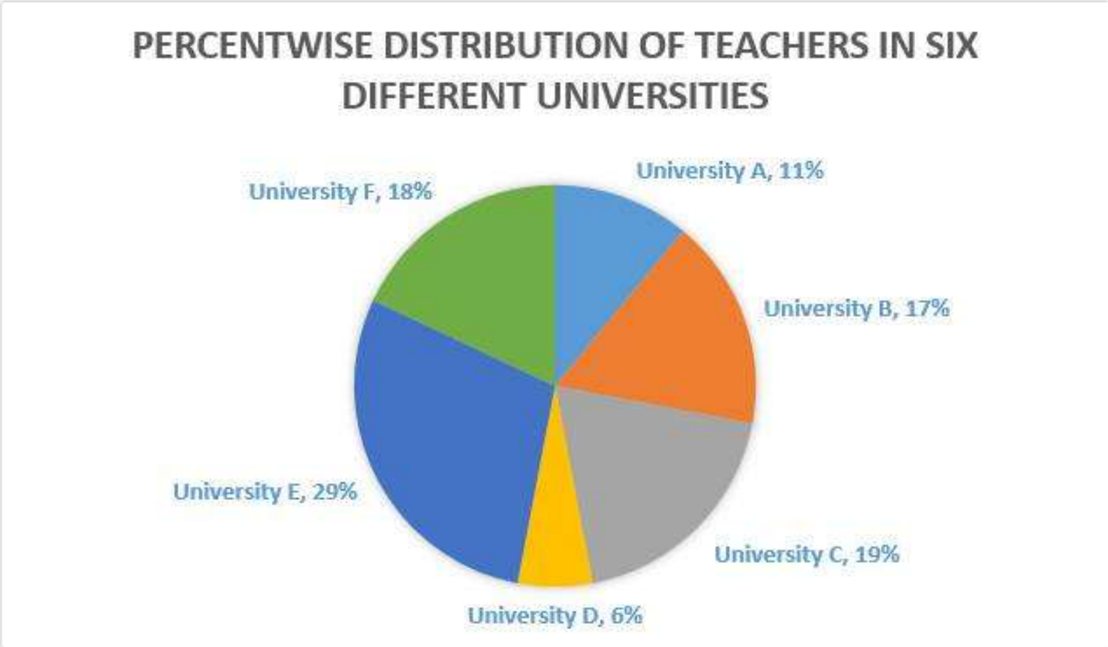
Profit = 1-0.8 = 0.2

Percentage profit of profit of Company C in 2009 =[0.2/0.8]*100 = 25%

Percentage decrease = 75%.

Answer is [1].

Directions: *Study the following pie chart and answer the questions that follows:*



Total Number of Teachers = 6400

(Uttar Bihar Gramin Bank 2012)

Question 1: If one-thirty sixth of the number of teachers from university F is professors and the salary of each professor is Rs 96000, what will be the total salary of all the professors together from university F?

[1] Rs 307.2 lakh

[2] 32.64 lakh

[3] Rs 3.072 lakh

[4] 3.264 lakh

[5] None of these

Solution:-

Number of teachers from university F = 18% of 6400 = 1152

1/36 of 1152 = 32

Total salary = 32*96000 = 3072000 = 30.72 lakh. Answer [5] is correct. *(Note the tricky options [1] and [2])*

Short Tricks:-

Total salary = (6400*18*1*96000)/100*36 = 30.72 Lakhs

<u>Question 2:</u> Difference between the total number of teachers in university A, B and C together and the total number of teachers in university D, E and F together is exactly equal to the number of teachers in which university?

[1] A

[2] B

[3] C

[4] D

[5] F

(You don't even have to calculate the number of teachers. Just presence of mind is needed.)

Solution:-

Number of teachers in university A, B and C = 11+17+19 = 47%

Number of teachers in university D, E and F = 6+29+18 = 53%

Difference = 6% = University D. Answer [4] is correct.

<u>Question 3:</u> What is the average of teachers in university A, C, D and F together?

[1] 854

[2] 3546

[3] 3456

[4] 874

[5] None of these

Solution:-

Again, solving it quickly.

11+19+6+18 = 54%. Average = 54/4 % = [54/400]*6400 = 54*16 = 864. Answer [5] is correct.

<u>Question 4:</u> If twenty five percent of the number of teachers in university C is female, what is the number of male teachers in university C?

[1] 922

[2] 911

[3] 924

[4] 912

[5] None of these

Solution:-

Number of teachers in university C = 19% of 6400 = 19*64 = 1216

25% of this is female. Hence remaining 75% is male.

Number of male teachers = 75% of 1216 = [3/4]*1216 = 912. Option [4] is correct.

Short Tricks:-

Number of male teachers = (6400*19*75)/100*100 = 912

<u>Question 5:</u> Number of teachers in university B is approximately what percent of the total number of teachers in university D and E together?

[1] 55%

[2] 59%

[3] 49%

[4] 45%

[5] 65%

Solution:-

Short Tricks:-

Just solve the percentages.

University B = 17%. University D+E = 6+29 = 35%

Required percentage = [17/35]*100 = approx. 49%. Answer is [3]