Government of Tamilnadu

Department of Employment and Training

Course : TNPSC Group I Mains Material

Subject : General Aptitude & Mental Ability

Topic : **Probability**

Probability

Introduction

To understand the notion of probability, we look into some real life situations that involve some traits of uncertainty.

A life-saving drug is administered to a patient admitted in a hospital. The patient's relatives may like to know the probability with which the drug will work; they will be happy if the doctor tells that out of 100 patients treated with the drug, it worked well with more than 80 patients. This percentage of success is illustrative of the concept of probability; it is based on the frequency of occurrence. It helps one to arrive at a conclusion under uncertain conditions. Probability is thus a way of quantifying or measuring uncertainty.

You should be familiar with the usual complete pack of 52 playing cards. It has 4 suits(Hearts ♥, Clubs ♣, Diamonds ♦, Spades♠), each with 13 cards. Choose one of the suits or cards, say spades. Keep these 13 cards facing downwards on the table. Shuffle them well and pick up any one card. What is the chance that it will be a King? Will the chances vary if you do not want a King but an Ace? You will be quick to see that in either case, the chances are 1 in 13 (Why?). It will be the same whatever single card you choose to pick up. The word 'Probability' means precisely the same thing as 'chances' and has the same value, but instead of saying 1 in 13 we write it as a fraction 1/13 . (It would be easy to manipulate with fractions when we combine probabilities). It is 'the ratio of the favourable cases to the total number of possible cases'.

- Have you seen a 'dice' ? (Some people use the word 'die' for a single 'dice'; we use 'dice' here, both for the singular and plural cases). A standard dice is a cube, with each side having a different number of spots on it, ranging from one to six, rolled and used in gambling and other games involving chance.
- If you throw a dice, what is the probability of getting a five? a two? a seven?
- In all the answers you got for the questions raised above, did you notice anything special about the concept of probability? Could there be a maximum value for probability? or the least value? If you are sure of a certain occurrence what could be its probability? For a better clarity, we will try to formalize the notions in the following paragraphs.

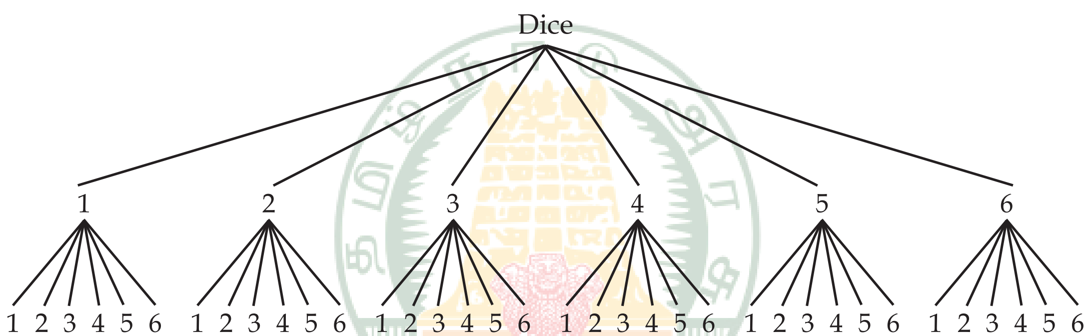
Classical Approach

An urn contains 4 Red balls and 6 Blue balls. You choose a ball at random from the urn. What is the probability of choosing a Red ball?

- The phrase 'at random' assures you that each one of the 10 balls has the same chance (that is, probability) of getting chosen. You may be blindfolded and the balls may be mixed up for a "fair" experiment. This makes the outcomes "equally likely".
- The probability that the Red Ball is chosen is 4/10 (You may also give it as 2/5 or 0.4)
- What would be the probability for choosing a Blue ball? It is 6/10 (or 3/5 or 0.6)

1. Express the sample space for rolling two dice using free diagram.

Solution:



Sample space, = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

2. Two dice rolled, find probability that sum of outcomes is i) equal to 4, ii) greater than 10, iii) less than 13

Solution:

Sample space, S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
.... (6, 6)}

n(S) = **36**.

i) equal to 4

A be event of getting sun equal to 4,

$A = \{(1, 3), (2, 2), (3, 1)\}$; $n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}.$$

ii) greater than 10

B be event of getting greater than 10

$B = \{(5, 6), (6, 5), (6, 6)\}$; $n(B) = 3$

$$P(A) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}.$$

iii) less than 13

all outcomes are less than 13; $n(C) = 36$

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1.$$

3. Two coin turned together . What is the probability of getting different faces on the coins.

Solution:

Sample space, S = { HH, HT, TH, TT }

$n(S) = 4$

A be getting different faces on coins = { HT, TH }

$n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

4. From a well shuffled pack of 52 cards, on card is drawn ab random, find the probability of getting

Solution:

i) getting red card

A be event of getting red card = 26

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}.$$

ii) getting heart card

B be event of getting heart card = 13

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}.$$

iii) getting red king

C be event of getting red king = 2

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}.$$

iv) face card

d be event of getting face card = $4 \times 3 = 12$ (J, K, Q)

$$P(d) = \frac{n(d)}{n(S)} = \frac{12}{52} = \frac{3}{13}.$$

v) number card

E be event of getting number card = $4 \times 9 = 36$ (2, 3, 4, 5, 6, 7, 8, 9, 10)

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}.$$

5. **What is the probability that a leap year selected at random will contains 53 Saturdays.**

Solution:

A leap year has 366 days. It has 52 full weeks and 2 days.

52 Saturday must be in 52 full weeks.

Remaining two days sample space = { (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun) }

n(S) = 7

A be getting 53 Sundays = { (Sat, Sun), (Sun, Mon) } = 2

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

6. **A die is rolled and win is tossed simultaneously. Find probability that a die show an hold number and the coin shows a head.**

Solution:

S = { 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T }

n(S) = 12.

A be event of getting odd number and head = { 1H, 3H, 5H }

n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}.$$

7. A bag contains 6 green balls, some black and red balls, probability of getting a green ball throw the probability of getting a red ball. Find i) number of black balls, ii) total number of balls

Solution:

Number of green balls = 6

Number of red balls = x

Number of black balls = $2x$

Total no. of balls, $\Rightarrow$ $6 + x + 2x = 6 + 3x$

$$P(G) = 3 \times P(R)$$

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

$$6 = 3x$$

$$\boldsymbol{x = 2.}$$

i) No. of black balls = $2 \times 2 = 4$

ii) total no. of balls = $6 + (3 \times 2) = 12.$

8. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the number 1, 2, 3, ... 12. What is the probability that it will point to i) 7, ii) a prime number, iii) a composite number

Solution:

S = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 }

n(S) = 12

i) getting 7

A be event of resting in 7 = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

ii) a prime number

B be event of prime number = { 2, 3, 5, 7, 11 }

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

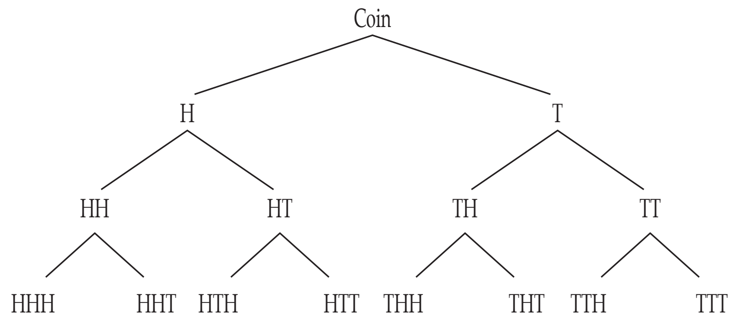
iii) a composite number

B be event of composite number = { 4, 6, 8, 10, 12 } n(C) = 6

$$P(B) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

9. Write the sample space for tossing 3 coins using tree diagram.

Solution:



$$\text{Sample space, S} = \left\{ \begin{array}{llll} \text{HHH,} & \text{HHT,} & \text{HTH,} & \text{HTT} \\ \text{THH,} & \text{THT,} & \text{TTH,} & \text{TTT} \end{array} \right\}$$

10. If A is an event of a random experiment such that P(A) : P($\bar{A}$) = 17 : 15 and n(S) = 640, then find i) P($\bar{A}$), ii) n(A)

Solution:

$$\frac{P(A)}{P(\bar{A})} = \frac{17}{15}; \quad n(S) = 60; \; P(A) = 172; \; P(\bar{A}) = 15x$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) + P(\bar{A}) = 1 \Rightarrow \quad 17x + 15x = 1$$

$$32x = 1, \; x = \frac{1}{32}$$

i) P($\bar{A}$),

$$P(\bar{A}) = 15x = \frac{15}{32}$$

ii) n(A),

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{17}{32} = \frac{n(A)}{640}$$

$$n(A) = \frac{17}{32} \times 640 = 340.$$

11. A coin is toured take, what is the probability of getting two consecutive tools

Solution:

$$\text{Sample space, S} = \left\{ \begin{array}{cccc} \text{HHH,} & \text{HHT,} & \text{HTH,} & \text{HTT} \\ \text{THH,} & \text{THT,} & \text{TTH,} & \text{TTT} \end{array} \right\}$$

$n(S) = 8$

A be event of two consecutive tools = HTT, TTH, TTT

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

12. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wine a prize what is the probability that

Solution:

$n(S) = 100$

i) First player wins the prize

Let A be event of for player wins the prize = { 529, 576, 625, 676, 729, 784, 841, 900, 961 }

$n(A) = 9$

$$P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$$

ii) Second player wins a prize, if the first has won?

Solution:

Let B be second player wins a prize, if first won the prize = 8

here; $n(S) = 999$ ($\therefore$ 1^{st} won the prize)

$n(B) = 8$

$$P(B) = \frac{8}{999}.$$

13. A bag contain 12 blue balls and x red balls. If are ball is drown at random what is the probability that it will be a red ball. If 8 more red balls are pub in the bag and if the probability of drawing a red balls will be twice that of probability in (I) than find *x*

Solution:

n(S) = 12 balls + x red balls = 12 + x balls.

i) Let A be the event of getting a red ball, n(A) = x

$$P(A) = \frac{x}{12+x} \qquad \rightarrow (1)$$

$$P(A) = \frac{1}{4}$$

ii) Now, n(S) = 12 balls + x red balls + 8 red balls

n(S) = 20 + x balls

Let B be event of getting red ball into = x + 8

$$P(B) = \frac{x+8}{20+x}$$

P(B) = 2(P(A))

$$\frac{x+8}{20+x} = \frac{2(x)}{12+x}$$

$$(x + 8)(12 + x) = 40x + 2x^2$$

$$12x + x^2 + 96 + 8x = 40x + 2x^2$$

$$x^2 + 96 + 20x = 40x + 2x^2$$

$$2x^2 - x^2 + 20x - 96 = 0$$

$$x^2 + 20x - 96 = 0$$

$x = 4$; $x = -24$;

$\therefore$ Value of **$x = 4$** $\rightarrow (2)$

Sub (2) in (1) $\Rightarrow$ $P(A) = \frac{4}{16} = \frac{1}{4}$

14. Two unbiased dice are rolled once find probability of getting i) a doubled, ii) product as prime number, iii) sum as prime number, iv) sum as 1

Solution:

n(S) = 36

i) A doubled

let A be event getting a doublet = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

ii) Product as prime number

let B be event getting product as prime number = { (1, 2), (2, 1), (3, 1), (1, 3), (1, 5), (5, 1) }

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

iii) Sum as a prime number

let C be event getting Sum as a prime number = { (1, 1), (1, 2), (2, 1), (5, 2), (2, 5), (1, 6), (6, 1), (3, 4), (4, 3), (6, 5), (5, 6), (1, 4), (4, 1), (2, 3), (3, 2) }

$n(C) = 15$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$$

iv) Sum as 1

let D be event getting Sum as 1 = 0

$P(D) = 0.$

15. 3 fare coin are tossed together. Find probability of getting i) all heads, ii) at most one tail, iii) at most one head, iv) at most two toils

Solution:

n(s) = 8 { (HHH, HHT, HTH, HTT, THH, THT, TTH, TT }

i) All heads

A be event of getting all heads = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) At least one tail

B be event of getting at least one tail = 7

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

iii) At least one head

C be event of getting at least one head = 4 { HTT, THT, TTH, TTT }

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{2}$$

iv) At least two tails

D be event of getting at least two tails = 7

$$P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

at least = less than or equal to

(at most 2 - 0002)

16. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 respectively. They are rolled and sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Solution:

Sample space, =	{(1, 1),	(1, 1),	(1, 2),	(1, 2),	(1, 3),	(1, 3)
	(2, 1),	(2, 1),	(2, 2),	(2, 2),	(2, 3),	(2, 3)
	(3, 1),	(3, 1),	(3, 2),	(3, 2),	(3, 3),	(3, 3)
	(4, 1),	(4, 1),	(4, 2),	(4, 2),	(4, 3),	(4, 3)
	(5, 1),	(5, 1),	(5, 2),	(5, 2),	(5, 3),	(5, 3)
	(6, 1),	(6, 1),	(6, 2),	(6, 2),	(6, 3),	(6, 3)}

i) A be probability of getting sum 2

P(A) = 2 { (1, 1), (1, 1) }

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{36} = \frac{1}{18}.$$

ii) B be event of getting sum 3

= { (1, 2), (1, 2), (2, 1), (2, 1) }

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}.$$

iii) C be event of getting sum 4

= { (1, 3), (1, 3), (3, 1), (3, 1), (3, 2), (3, 2) }

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

iv) D be event of getting sum 5

= { (2, 3), (2, 3), (3, 2), (3, 2), (4, 1), (4, 1) }

$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

v) E be event of getting sum 6

$= \{ (3, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1) \}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

vi) F be event of getting sum 7

$= \{ (4, 3), (4, 3), (5, 2), (5, 2), (6, 1), (6, 1) \}$

$$P(F) = \frac{n(F)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

vii) G be event of getting sum 8

$= \{ (5, 3), (5, 3), (6, 2), (6, 2) \}$

$$P(G) = \frac{n(G)}{n(S)} = \frac{4}{36} = \frac{1}{9}.$$

viii) H be event of getting sum 9

$= \{ (5, 3), (5, 3), (6, 2), (6, 2) \}$

$$P(H) = \frac{n(H)}{n(S)} = \frac{2}{36} = \frac{1}{18}.$$

17. A bag contain 5 red balls, 6 white balls, 7 green balls, 8 black balls. One balls is drawn at random rfom the bag. Find the probability that the ball drawn is i) white, ii) black or red, iii) not white, iv) neither white non black

Solution:

$n(S) = 5+6+7+8 = 26$

i) white

A be event of getting white ball = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}.$$

ii) black or red

B be event of getting black or red = 8 + 5 = 13

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{26} = \frac{3}{13}.$$

iii) not white

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{13}$$
$$= \frac{10}{13}$$

iv) neither white or black

C be event of getting white or black = 6 + 8 = 14

$$P(C) = \frac{14}{26} = \frac{7}{13}.$$

$$P(\bar{C}) = 1 - P(C) = 1 - \frac{7}{13}.$$

$$= \frac{6}{13}.$$

18. The king and queen of diamond, queen and jack of hearts, jack and king of spade are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from remaining cards. Determine the probability that the card is i) a clavor, ii) a queen of red card, iii) a king of black card

Solution:

Queen and jack of hearts = 2

Jack and king of spade = 2

King and queen of diamond = 2

n(S) = 52 − 6 = 46.

i) A clavor

Let A be event of getting a clavor = 13

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}.$$

ii) A queen of red card

Let A be event of getting a queen of red card = 0

iii) A king of black card

Let A be event of getting a king of black card = 1

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}.$$

19. Two customer Priya and Amutha are visiting a particular shop in the same week (Monday to Saturday). Each is equally to visit the shop on any one day as on another day. What is the probability that both will visit the shop on, i) same day, ii) different day, iii) consecutive days

Solution:

Sample space, S = { (Mon, Mon) (Mon, Tue) (Mon, Wed) ....

....(Sat, Sat) }

N(S) = 36.

i) A be event of both will visit shop on same day = 6

{ (Mon, Mon) (Tue, Tue) (Wed, Wed) (Thu, Thu) (Fri, Fri) (Sat, Sat)

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

ii) B be event that both will visit on different days = Total days − Total same day

= 36 − 6 = 30

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{36} = \frac{5}{6}.$$

iii) C be event of both will visit the shop

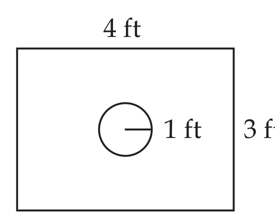
on consecutive days = { (Mon, Tue) (Tue, Wed)

Wed, Thr) (Thr, Fri) (Fri, Sat) }

$$P(C) = \frac{n(C)}{n(S)} = \frac{5}{36}.$$

20. Some boys are playing a game in which the stone throw by than landing in a circular region (given figure) is considered as win and landing other than circular region is considered as loss. What is the probability to win the game?

Solution:



Area of rectangle = $l \times w$

$= 3 \times 4 = 12\ f^2$

Area of circle = лr^2 = л

$$\text{Probability of winning the game} = \frac{\text{Area of Circle}}{\text{Area of Rectangle}}$$

$$= \frac{\pi}{12}$$

21. In a box there are 20 non defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $3/8$ then, find the number of defective bulbs.

Solution:

n(S) = 20 + x

probability of defective bulbs, P(D) = $\frac{3}{8} = \frac{n(D)}{n(S)}$

$$\frac{3}{8} = \frac{n(D)}{20+x} \qquad (\therefore n(D) = x)$$

$$60 + 3x = 8\,n(D)$$

$$60 + 3x = 8x$$

$$60 + 5x = 60$$

x = 12 There are 12 non defective bulbs.

22. In a game, the entry fee is ₹ 150. The game contest of tossing a coin 3 time. Dhana bought a ticket for entry. If one or two heads show she gets her entry fee back. If she throws 3 heads, she receives double the entry fees otherwise she will lone. Find the probability that she i) gets double entry fee, ii) just gets her entry fee, iii) loses the entry fee

Solution:

n(S) = 8

i) A be event of getting double entry fee to get double entry fee, she has to show

S heads = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}.$$

ii) just get free entry to get free entry, she has to show one or two heads

B be event of getting her entry fee

B = { HHT, HTH, HTT, THH, THT, TTH }

n(B) = 6

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}.$$

iii) C be event of loses the entry fee

C = { TTT }

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}.$$