Government of Tamilnadu

Department of Employment and Training

Course : TNPSC Group I Mains Material

Subject : General Aptitude & Mental Ability

Topic : **HCF and LCM**

Commissioner,
Department of Employment and Training.

HCF and LCM

COMMON FACTORS

Consider the numbers 45 and 60. Use of divisibility tests will also help us to find the factors of 45 and 60. The factors of 45 are 1,3,5,9,15 and 45 and the factors of 60 are 1,2,3,4,5,6,10,12,15,20, 30 and 60. Here, the common factors of 45 and 60 are 1,3,5 and 15.

As factors of a number are finite, we can think of the **Highest Common Factor** of numbers, shortly denoted as **HCF**.

HIGHEST COMMON FACTOR (HCF)

Think about the situation:

Situation: Consider the rods of length 8 feet and 12 feet. We have to cut these rods into pieces of equal length. How many pieces can we get? What will be the length of the longest piece that is common for both the rods? The rod of 8 feet can be divided into small rods of length 1 foot or 2 feet or 4 feet (These are factors of 8). The rod of 12 feet can be divided into small rods of length 1 foot or 2 feet or 3 feet or 4 feet or 6 feet (These are factors of 12).

The length of the pieces that are common to both the rods (as given above) are of length 1 foot, 2 feet and 4 feet (i.e., common factors of 8 and 12). Hence, the HCF of 8 and 12 is the length of the longest rod i.e., **4 feet** that can be cut equally from the rods of length 8 feet and 12 feet. So, the **Highest Common Factor (HCF)** of two numbers is the largest factor that is common to both of them.The Highest Common Factor of the numbers x and y can be written as HCF (x,y).

The Highest Common Factor (HCF) is also called as the Greatest Common Divisor (GCD) or the Greatest Common Factor (GCF).

- HCF $(1, x) = 1$
- HCF $(x, y) = x$, if y is a multiple of x. For example, HCF $(3, 6) = 3$.
- If the HCF of two numbers is 1, then the numbers are said to be **co-primes** or **relatively prime.** Here, the two numbers can both be primes as (5, 7) or both can be composites as (14, 27) or one can be a prime and other a composite as (11, 12).

Common Multiples

- Let us now write the multiples of 5 and 7
- Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70,…

- Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70,…
- Here, the common multiples of 5 and 7 are 35 and 70 and will go on without ending.
- As multiples of a number are infinite, we can think of the **Least Common Multiple** of numbers, shortly denoted as **LCM**.

Least Common Multiple (LCM)

Think about the situation:

Write the multiplication table of 4 and 5 (upto 10).

Observing the multiplication tables, can you find the multiples (product of numbers) that are the same in the 4th table and 5th table?. If yes, what are they? Yes, they are **20, 40,**…etc. From the multiples of 4 and 5, we can easily find that 20 is the least common multiple of 4 and 5.

4th Table	5th Table
1x4 = 4	1x5 = 5
2x4 = 8	2x5 = 10
3x4 = 12	3x5 = 15
4x4 = 16	4x5 = **20**
5x4 = **20**	5x5 = 25
6x4 = 24	6x5 = 30
7x4 = 28	7x5 = 35
8x4 = 32	8x5 = **40**
9x4 = 36	9x5 = 45
10x4 = **40**	10x5 = 50

Situation

Consider the red and the blue coloured floor mats of length 4 units and 5 units as follows.

5 units	4 units

Five red coloured floor mats of 4 units each can be arranged as follows. Its total length is 5 x 4 = 20 units.

4 units	4 units	4 units	4 units	4 units

Four blue coloured floor mats of 5 units each can be arranged as follows. Its total length is also the same 4 x 5 = 20 units.

5 units	5 units	5 units	5 units

Note that the 5 floor mats each of length 4 units are required to equal 4 floor mats each of length 5 units and that is, the length 20 units is the smallest common length that can be matched by both sizes. From the above, it shows that the least common multiple of 4 and 5 is 4x5=20.

The **Least Common Multiple** of any two non-zero whole numbers is the smallest or the lowest common multiple of both the numbers. The Least Common Multiple of the numbers x and y can be written as LCM (x,y). We can find the least common multiple of two or more numbers by the following methods.

1. Division Method
2. Prime Factorisation Method

1. Find HCF of numbers 40 and 56

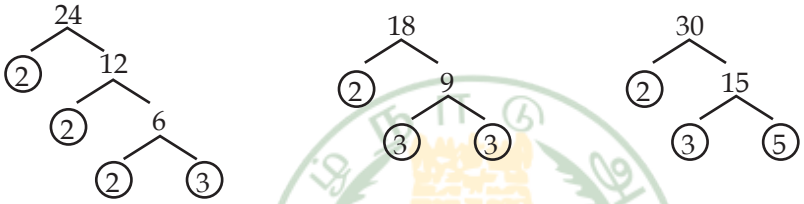
Solution:

2	40,	56
2	20,	28
2	10,	14
	5,	7

HCF = $2 \times 2 \times 2 = \mathbf{8}$.

2. HCF of 18, 24 and 30 by factor tree method

Solution:



HCF of 24, 18 and 30 is $2 \times 3 = \mathbf{6}$.

3. LCM of 156 and 124

Solution:

2	156,	124
2	78,	62
3	39,	31
13	13,	31
31	1,	31
	1,	1

LCM = $2 \times 2 \times 3 \times 13 \times 31 = \mathbf{4836}$.

4. What is the greatest number that will divide 62, 78 and 109 leaving remainder 2, 3 and 4 respectively?

Solution:

(62 − 2), (78 − 3) (109 − 4) = 60, 75, 105

HCF = 15.

15 is the greatest number will divide 62, 78 and 109 leaving remainder 2, 3 and 4 respectively.

5	60,	75,	105
3	12,	15,	21
	4,	5,	7

5. A book seller has 175 English books, 245 Science books and 385 Maths books. He wants to sell the books in a box, subject wise in equal numbers, what will be the greatest number of boxes required? Also find the number of books for each subject in a box?

Solution:

5	175,	245,	385
7	35,	49,	77
	5,	7,	11

HCF = 5×7 = **35**. Greatest number of boxes = 35. The number of books for each subject in a box is 170 English, 238 Science and 374 Maths books.

6. Find the smallest number that can be divided by 254 and 508 which leaves remainder 4

Solution:

2	254,	508
2	127,	254
127	127,	127
	1,	1

Smallest number - **LCM**
Greatest number - **HCF**

Remainder same in each case:
Add remainder to the LCM (Smallest number)

LCM = $4 \times 127 = 508$.

Required number = 508 + 4 = **512**.

7. What is the smallest 5-digit number is exactly divisible by 72 and 108?

Solution:

Let smallest 5 digit number = 10,000

LCM of 72 and 108

2	72,	108
2	36,	54
3	18,	27
3	6,	9
	2,	3

		46
216	10000	
	9936	
	164	

LCM = 216.

10000 ÷ 216 = gives remainder 164, Quotient 46

Next multiple 216×47 = 10,152 is the required number.

The smallest 5 digit number is exactly divisible by 72 & 108 is 10, 152.

8. There are 4 mobile phones in a room. All the 4 mobile phones will ring together thereafter 1st one rings every 15 minutes, the second one rings every 20 minutes, 3rd one ring 25 minutes, 4th one rings every 30 minutes. At what time, will the 4 mobile phones ring together again?

Solution:

5	15,	20,	25,	30
2	3,	4,	5,	6
3	3,	2,	5,	3
	1,	2,	5,	1

LCM = $5 \times 2 \times 3 \times 2 \times 5 = 300$ minutes.

= 5 hours

4 Mobile phones will ring together again at 10.00 am.

9. LCM of two numbers is 432 and their HCF is 36. If one of the numbers is 108, then find other numbers.

Solution:

LCM × HCF = Product of 2 number

$$432 \times 36 = 108 \times x$$

$$x = \frac{432 \times 36}{108}$$

$$x = \mathbf{144.}$$

10. Find the ratio of HCF and LCM of the numbers 18 and 30

Solution:

2	18,	30
3	9,	15
	3,	5

HCF = $2 \times 3 = 66$, LCM = $2 \times 3 \times 3 \times 5 = 90$

HCF : LCM = 6 : 90 = **1 : 15.**

11. HCF of two successive even numbers is 2

Solution:

HCF = 2, 4

2	2,	4
	1,	2

2	4,	6
	2,	3

HCF = **2**.

12. LCM of two co-prime numbers is 5005. If one of the numbers is 65, find the other number

Solution:

LCM × HCF = Product of 2 number

$$5005 \times 1 = 65 \times x$$

$$x = \frac{5005}{65}$$

$$x = 77.$$

13. LCM of 26, 39, 52 is

Solution:

2	26,	39,	52
13	13,	39,	26
	1,	3,	2

LCM = 2 × 13 × 6 = **156**.

14. Least number that should be added to 57, so that the sum is exactly divisible by 2, 3, 4 and 5 is

Solution:

LCM of 2, 3, 4, 5

2	2,	3,	4,	5
	1,	3,	2,	5

LCM = 2 × 3 × 10 = 60

The required number = 60 ⇒ 57 + 3 (3 should be added).

15. Find HCF and LCM of the numbers 154, 198, 286

Solution:

2	154,	198,	286
11	77,	99,	143
	7,	9,	13

LCM = 2 × 11 × 7 × 9 × 13 = **18018.**

HCF = 2 × 11 = **22**.

16. LCM of 3 and 9 is 9, then their HCF is

Solution:

3	3,	9
	1,	3

HCF = 3.

17. What is the greatest possible volume of a vessel that can beused to measure exactly, the volume of milk in cans (in full capacity) of 80 liter, 100 liter and 120 liter?

Solution:

2	80,	100,	120
2	40,	50,	60
5	20,	25,	30
	4,	5,	6

HCF = $2 \times 2 \times 5$ = **20 Liters.**

18. 19. The traffic lights at 3 different road junctions change after every 40 seconds, 60 seconds, 72 seconds respectively. If they changed simultaneously together at 8 a.m. at junction, at what time will they simultaneously change together again?

Solution:

2	40,	60,	72
2	20,	30,	36
2	10,	15,	18
3	5,	15,	9
5	5,	5,	3
	1,	1,	3

LCM = $2 \times 2 \times 2 \times 15 \times 3$ = **360 seconds. (6 minutes)**

Change together again at 8.06 am.

19. LCM of two numbers is 210, HCF is 14. How many such pairs are possible?

Solution:

HCF = 14; LCM = 210

$$14a \times 14b = 14 \times 210$$

$$ab = \frac{210 \times 14}{14 \times 14}$$

$ab = 15$. Products of 15 are (1, 15) and (3, 5)

20. LCM of 2 numbers is 6 times there HCF If the HCF is 12 and one of the numbers is 36, find other number

Solution:

LCM (a, b) = 6 HCF (a, b)

LCM = 6 × 12, LCM (a,b) = 72

$$72 \times 12 = 36 \times x$$

LCM × HCF = Product of 2 number

$$x = \frac{72 \times 12}{36}$$

$x = 24.$

21. (71, 81) is co-prime (or) not?

Solution:

HCF - (71, 81) is 1

If HCF is 1, then 2 number are co-prime.

22. HCF of 2 numbers is 2 and their LCM is 154. If the difference between the numbers is 8, then sum ₹

Solution:

$2a \times 2b = 2 \times 154$

$$ab = \frac{2 \times 154}{2 \times 2}$$

ab = 77

$(4 + b)\, b = 77$

$b^2 + 4b - 77 = 0$

b = 11, −7

$2a - 2b = 8$

$2(a - b) = 8$

$a - b = 4$

a = 4 + b

The number be 2a, 2b, If b = 11; a = 15, If b = 7; a = 11

1. 2a + 2b = 22 + 30 = 52.

2. 2a + 2b = 14 + 22 = 36.

23. Greatest four digit number exactly divisible by 8, 9 , 12 is

Solution:

LCM 8, 9, 12

3	8,	9,	12
2	8,	3,	4
2	4,	3,	2
	2,	3,	1

LCM = $3 \times 3 \times 8 = 72$.

Greatest 4 digit number = $9999 \div 72$

Remainder 63.

Required number is $9999 - 63 = 9936$.

	138
72	9999
	72
	279
	216
	639
	576
	63

24. Find the length of the longest rope that can be used to measure exactly the rope of length of 1 m 20 cm; 3 m 60cm and 4m?

Solution:

($\therefore$ 1 m = 100 cm)

1 m 20 cm = 120 cm

3 m 60 cm = 360 cm

4 m = 400 cm

2	120,	360,	400
2	60,	180,	200
2	30,	90,	100
5	15,	45,	50
	3,	9,	10

LCM = $2 \times 2 \times 2 \times 5 = 40$ cm.