Government of Tamilnadu

Department of Employment and Training

Course : TNPSC Group I Mains Material

Subject : General Aptitude & Mental Ability

Topic : **Area and Volume**

Area and Volume

Area of the square

= side × side sq.units

= base x height sq.units

= area of the rhombus

Perimeter of a Square

Perimeter of a square = Total boundary of the square

= side + side + side + side

= (4 × side) units

If the side of a square is 's' units, then Perimeter of the square, P = (4 × s) units = 4s units.

Area of a Rectangle

Area of any rectangle = (length x breadth) square units.

= (l x b) sq. units.

Perimeter of a Rectangle

Perimeter of a rectangle = Total boundary of the rectangle

= length + breadth + length + breadth

= 2 length + 2 breadth

= 2 (length + breadth)

Let us denote the length, breadth and the perimeter of a rectangle as l, b and P respectively. Perimeter of the rectangle, P= 2 x (l + b) units

Perimeter of a Triangle

Perimeter of a triangle = Total boundary of the triangle

= side 1 + side 2 + side 3

If three sides of a triangle are taken as a, b and c, then the Perimeter of the triangle, P = (a + b + c) units.

Area of a Right Angled Triangle

In a right angled triangle one of the sides containing the right angle is treated as its base (b units) and the other side as its height (h units).

When a rectangular sheet is cut along its diagonal, two right angled triangles are obtained.

- Area of two right angled triangles = Area of the rectangle
- $2 \times$ Area of a right angled triangle = $l \times b$
- Area of the right angled triangle = $1/2\ (l \times b)$ sq. units.

The length and breadth of the rectangle are respectively the base (b) and height (h) of the right angled triangle.

Hence, area of the right angled triangle = $1/2\ (b \times h)$ sq. units.

Area of Irregular Shapes

The area of the shapes like triangle, square etc., are found by standard formulae. But we can find the approximate area of shapes like leaves as follows. Place a leaf on a graph sheet and trace its boundary. Now observe the squares of size 1 cm x 1 cm inside of this boundary. We get complete squares (Green), partial but bigger than half squares (Orange) and half squares (Blue). The smaller than half squares which have negligible area are omitted. Now the approximate area of the leaf

= (Number of full squares + Number of more than half squares + $1/2 \times$ Number of half squares) sq. units

= $(14 + 6 + 1/2 \times 2)$ sq. cm = 21 sq. cm

Rhombus

Take four sticks of equal length and four connectors. Connect four sticks to form a square. Then, try to make any two opposite vertices closer. such that opposite sides remain parallel to each other to get a new shape called rhombus.

Hence, we conclude that, in a parallelogram, if all the sides are equal then it is called a rhombus.

In a rhombus, (i) all the sides are equal (ii) opposite sides are parallel (iii) diagonals divide the rhombus into 4 right angled triangles of equal area. (iv) the diagonals bisect each other at right angles.

Draw a perpendicular line from one vertex to the opposite side. Cut the triangle and shift the triangle to the other side of the rhombus. What shape do you see? It is a square. Hence, the area of the rhombus is the same as that of the square.

Area of the rhombus if the diagonals are given

Let us find, area of the rhombus ABCD by splitting it into two triangles. Here AB = BC = CD = DA and diagonals AC (d_1) and BD (d_2) are perpendicular to each other. Area of the rhombus ABCD = Area of triangle ABC + Area of triangle ADC

$= \frac{1}{2} \times AC \times OB + x\ AC \times OD$

$= \frac{1}{2} \times AC\ (OB + OD)$

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times d_1 \times d_2$ sq. units

Therefore, area of the rhombus = ½ (product of diagonals) square units.

Trapezium

A parallelogram with one pair of non-parallel sides is known as a Trapezium. The distance between the parallel sides is the height of the trapezium. Here the sides AD and BC are not parallel, but AB is parallel to DC.

Isosceles Trapezium

If the non - parallel sides of a trapezium are equal (AB = CD) then, it is known as an isosceles trapezium.

Area of the Trapezium

ABCD is a trapezium with parallel sides AB and DC measuring 'a'units and 'b' units respectively. Let the distance between the two parallel sides be 'h' units. The diagonal BD divides the trapezium into two triangles ABD and BCD.

Area of the trapezium = area Δ of ABD + area of Δ BCD

$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$ [since the two triangles ABD and BCD have same heights]

Formula:

1. Area of Triangle = ½ bh Sq. units
2. Quadrilateral Area = ½ × d × (h_1 + h_2) Sq. units
3. Perpendiculars = bh
4. Rhombus Area = ½ × d_1 × d_2 Sq. units
= b × h Sq. units

1. Find area of a square of side 15cm.

Solution:

Area = a^2 = 15 × 15
= 225 cm^2.

2. Find area of a rectangle of length 12 cm and breath 7 cm.

Solution:

Area of rectangle = $l \times b$
= 12 × 7 = 84 cm^2.

3. Find area of right angled triangle whole base is 18 cm and height is 12 cm.

Solution:

Area = ½ bh sq. units
= ½ × 18 × 12
= 108 cm^2.

4. Find area and perimeter of rectangle field of length 15 m and breath 10 m.

Solution:

Area of rectangle = $l \times b$
= 15 × 10
= 150 m2

Perimeter of rectangle = 2($l + b$)
= 2(15 + 10) = 2(25)
= 50 m.

5. **Find the length of the rectangular black board, whole perimeter is 6 m and breath is 1 m.**

Solution:

$$2(l + b) = 6$$
$$2(l + 1) = 6$$
$$2l + 2 = 6$$
$$2l = 4$$
$$l = 2 \text{ m.}$$

6. **In a right triangle ground, the side adjacent to the right angle are 50 m and 80 m. Find the cost of centering the ground at ₹ 5/m.**

Solution:

Area of right angled triangle ground = ½ bh

$= \frac{1}{2} \times 50 \times 80 = 2000 \text{ m}^2.$

Cost of centering = 5 × 2000 = ₹ 10,000.

7. **If the length of a rectangle is 12 cm and breadth is 10 cm, find the perimeter?**

Solution:

Perimeter of rectangle = $2\,(l + b)$

= 2 (12 + 10) = 2(22) = 44 cm.

8. **Find perimeter of triangle whole sides are 4 cm, 3 cm, 5 cm.**

Solution:

Perimeter = (a + b + c) units

= 4 + 3 + 5 = 12 cm.

9. **Find the side of equilateral triangle of perimeter 129 cm.**

Solution:

$$3a = 129$$
$$a = 129/3 \qquad a = 43 \text{ cm.}$$

10. **Find the cost of fencing a square plot of side 12 m at the rate of ₹ 15 per meter?**

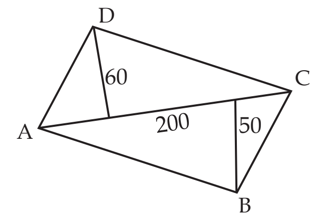
Solution:

Side of a square plot = 12 m; Perimeter of square plot = 4 × 12 = 48 m

Cost of fencing the plot at the rate of ₹ 15/meter = 48 × 15

= ₹ 720.

11. A plot of -land is in the form of a quadrilateral where one of its diagonals is 200 m long. The two on either side of this diagonal are 50 m and 60 m away from this diagonal. What is the area of the plot of land?



Solution:

Area of quadrilateral = $\frac{1}{2} \times d \times (h_1 + h_2)$

$= \frac{1}{2} \times 2000 \times (50 + 60)$

$= \frac{1}{2} \times 2000 \times 110$

$= 11000 \text{ m}^2$.

12. The area of a quadrilateral is 525 sq.m The perpendiculars from two vertices to the diagonal are 15 m and 20m. What length of this diagonal?

Solution:

Area of quadrilateral = 525 m^2

$$\frac{1}{2} \times d \times (h_1 + h_2) = 525$$

$$\frac{1}{2} \times d \times (15 + 20) = 525$$

$$d = \frac{525 \times 2}{35}$$

$$d = 30 \text{ m.}$$

13. Area of a quadrilateral is 54 cm^2. The perpendiculars from two opposite vertices to the diagonal are 4 cm, 5cm. What is the length of this diagonal?

Solution:

$$\text{Area} = \frac{1}{2} \times d \times (h_1 + h_2)$$

$$54 = \frac{1}{2} \times d \times (4 + 5)$$

$$54 = \frac{1}{2} \times d \times 9$$

$$d = \frac{54 \times 2}{9}$$

$$d = 12 \text{ cm.}$$

14. A flower garden is in the shape of a rhombus. The length of its diagonal are 18 m and 25 m. Find the area of the flower garden?

Solution:

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 18 \times 25$

$= 225 \text{ m}^2$.

15. The side of a triangular park are in the ratio 9 : 10 : 11 and its perimeter is 300 m, find the area of triangle park.

Solution:

Perimeter of triangle park = 300 m

$$9\text{ k} + 10\text{k} + 11\text{ k} = 300$$

$$30\text{ k} = 300 \quad \therefore \quad \text{k} = 10\text{ m.}$$

$$s = \frac{90+100+110}{2} = \frac{300}{2} = 150\text{ m.}$$

$$\text{Area triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{150(150-90)(150-100)(150-110)}$$

$$= \sqrt{150\times60\times50\times40}$$

$$= 50 \times 20 \times 3\sqrt{2}$$

$$= 3000 \times 1.414$$

$$= 4244\text{ m}^2.$$

16. The length of sides of a triangle field are 28 m, 15 m and 41 m calculate the area of the filed, find the cost of levelling thefiled at the rate of ₹ 20 per m^2.

Solution:

$$s = \frac{a+b+c}{2}$$

$$= \frac{28+15+41}{2} = \frac{84}{2} = 42\text{ m}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

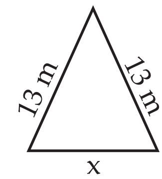
$$= \sqrt{42(42-28)(42-15)(42-41)}$$

$$= \sqrt{42\times14\times27\times1}$$

$$= 126\text{ m}^2.$$

Cost of levelling = 20 × 126 = ₹ 2,520.

17. An advertisement board is in the form of an isosceles triangle with perimeter 36 m and each of the equal sides are 13 m. find the cost of painting it at ₹ 17.50 per square meter.



Solution:

Perimeter $a + b + c = 36$ m $\qquad s = 36/2 = 18$

$13 + 13 + x = 36$

$x = 10$

$$\text{Area} = \sqrt{18(18-13)(18-13)(18-10)}$$

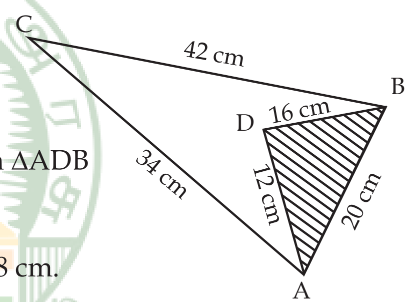
$$= \sqrt{18 \times 5 \times 5 \times 8}$$

$$= 60 \text{ m}^2.$$

Cost of levelling $= 60 \times 17.50$

$= ₹\ 1{,}050.$

18. Find the area of unchanged region.



Solution:

$$AB = \sqrt{12^2 + 16^2} = 20 \text{ cm}$$

Area of unchanged portion = Area of ΔABC – Area ΔADB

i) Area of Δ ABC

$$s = \frac{42+34+20}{2} = \frac{96}{2} = 48 \text{ cm.}$$

$$\text{Area} = \sqrt{48(48-42)(48-34)(48-20)}$$

$$= \sqrt{48 \times 6 \times 14 \times 28} = 336 \text{ cm}^2.$$

ii) Area of Δ ADB

$$s = \frac{12+16+20}{2} = 24 \text{ cm.}$$

$$\text{Area} = \sqrt{24(24-12)(24-16)(24-20)}$$

$$= \sqrt{24 \times 12 \times 8 \times 4} = 96 \text{ cm}^2.$$

Area of un shaded portion $= 336 - 96$

$= 240 \text{ cm}^2.$

19. The side of the triangular ground are 22 m, 120 m and 122 m, find the area and cost of levelling (find area) the ground at the rate of ₹ 20 per m^2.

Solution:

$$s = \frac{22+120+122}{2} = 132 \text{ m.}$$

$$= \sqrt{132(132-22)(132-120)(132-122)}$$

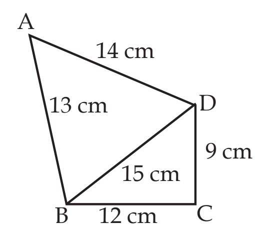
$$= \sqrt{132\times110\times12\times10} = 1320 \text{ m}^2.$$

Cost of levelling = 130 × 20

= ₹ 6,400.

20. Find the area of quadrilateral ABCD where sides are AB = 13 cm, BC = 12 cm, CD = 9 cm, AD = 14 cm, diagonal ∟ D = 15 cm

Solution:



Area of quadrilateral = Area of Δ ABD + Area Δ BCD

i) Area of Δ ABD

$$s = \frac{13+14+15}{2} = 21$$

$$\text{Area} = \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21\times8\times7\times6} = 84 \text{ cm}^2.$$

ii) Area of Δ BCD

$$s = \frac{15+9+12}{2} = 18$$

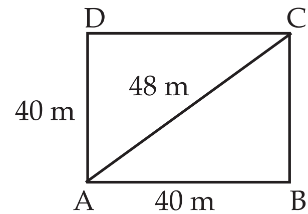
$$\text{Area} = \sqrt{18(18-15)(18-9)(18-12)}$$

$$= \sqrt{18\times3\times9\times6} = 84 \text{ cm}^2.$$

Area of un shaded portion = 84 + 54

= 138 cm^2.

21. A land is in the shape of rhombus. The perimeter of the land is 160 m and one of the of diagonal is 48 m. Find the area of the land?



Solution:

Perimeter of land = 160

$4a = 160$

$a = 40$ m.

Area of Δ ABC

$$s = \frac{40+40+48}{2} = 64 \text{ m}^2.$$

$$\text{Area} = \sqrt{64(64-40)(64-40)(64-48)}$$

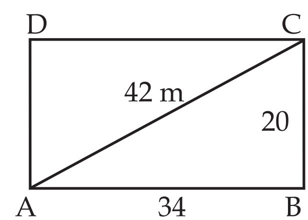
$$= \sqrt{64 \times 24 \times 24 \times 16} = 768 \text{ m}^2.$$

Area of Rhombus = 2 × Area of Δ ABC

= 2 × 768

= 1536 m^2.

22. The adjacent sides of a parallelogram measures 34 m, 20 m and measure of diagonal is 42 m, find the area of parallelogram.



Solution:

Area of parallelogram = Area of Δ ABC + Area of Δ ACD

Area of Δ ABC

$$s = \frac{34+42+20}{2} = 48 \text{ m}^2.$$

$$\text{Area} = \sqrt{48(48-34)(48-42)(48-20)}$$

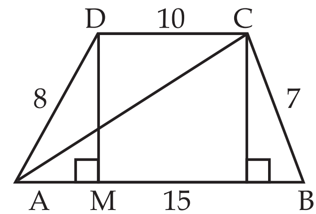
$$= \sqrt{48 \times 14 \times 6 \times 28} = 336 \text{ m}^2.$$

Area of Rhombus = 2 × Area of Δ ABC

= 2 × 336

= 672 m^2.

23. The parallel sides of a trapezium are 15 m and 10 m long and its non parallel sides are 8 and 7 m long. Find area of trapezium.



Solution:

Area of trapezium = ½ h (a + b)

In Δ DMA

$$AD^2 = AM^2 + MD^2$$
$$8^2 = (5 - x)^2 + h^2$$
$$h^2 = 8^2 - (5 - x)^2 \rightarrow (1)$$

In Δ BMC

$$AD^2 = AM^2 + MD^2$$
$$7^2 = h^2 + x^2$$
$$h^2 = 7^2 - x^2 \rightarrow (2)$$

Equaling (1) and (2)

$$8^2 - (5 - x)^2 = 7^2 - x^2$$
$$64 - 49 - (25 - 10x + x^2) = -x^2$$
$$64 - 49 - 25 + 10x - x^2 = -x^2$$
$$10x = 74 - 64$$
$$10x = 10$$
$$x = 1$$

$$h^2 = 7^2 - x^2 = 7^2 - 1$$
$$= 48$$
$$h = 4\sqrt{3}$$

Area of trapezium $= \frac{1}{2} \times 4\sqrt{3}\ (a + b)$

$= \frac{1}{2} \times 4\sqrt{3}\ (15 + 10)$

$= 86.60\ m^2$.

- Formula used to find the are of the circle = πr^2 sq. units.
- The radio of the area of the circle to the area of thr semicircle is 2 : 1

$$\pi r^2 = \frac{\pi r^2}{2} \Rightarrow 2:1$$

- Area of a circle of radius 'n' units is πr^2 sq. units.

1. Find the area of circle of radius 21 cm.

Solution:

Radius, r = 21 cm;

$$\text{Area} = \pi r^2 = \frac{22}{7} \times 21 \times 21 = 184.74 \text{ cm}^2.$$

2. The area of the circular region is 2464 cm^2. Find its radius and daimeter.

Solution:

$$\pi r^2 = 2464$$

$$r^2 = \frac{2464 \times 7}{22} = 12 \times 7 = 784$$

$$r^2 = 784 \Rightarrow r = 28 \text{ cm.}$$

Diameter (d) = 2 × r = 2 × 28 = 56 cm.

3. Find the length of the rope by with a cow must be bothered on circle that it may be able to graze an area of 9856 cm.

Solution:

Area of circle = 9856 $\Rightarrow \pi r^2 = 9856$

$$r^2 = \frac{9856 \times 7}{22} = 3136.$$

$$r = \sqrt{3136} = 56 \text{ cm.}$$

4. A Sprinkle placed at the centre of a flower garden sprays water covering a circular area. If the area watered is 1386 cm^2. Find its radius and diameter.

Solution:

Area to be watered, $1386 = \pi r^2$

$$r^2 = \frac{1386 \times 7}{22} = 441$$

$r^2 = \sqrt{441} = 21$ cm. Diameter = 42 cm.

5. A gardener walks around a circular park of distance 154 m. If he wants to level the park at the rate ₹ 25 per sq. meter. How much amount will be need?

Solution:

Circumference of circle, $2\pi r = 154$

$$r = \frac{154 \times 7}{2 \times 22}$$

$$r = 24.5.$$

Area $= \pi r^2$, $= \frac{22}{7} \times 24.5 \times 24.5 = 18865$ m^2.

Cost of levelling = 1886.5 × 25 = ₹ 47,162.50

6. Thenmozhi wants to level her circular flower garden whore diameter is 49 m at the rate of ₹ 150 per m^2.

Solution:

$r = \frac{49}{2} = 24.5$; Area $= \pi r^2$

$$= \frac{22}{7} \times \frac{49}{2} \times \frac{49}{2} = 1886.5 \text{ m}^2.$$

Cost of levelling = 1886.5 150 = ₹ 2,82,975.

7. The floor of the circular swimming pool whore radius is 7 m has to be covered at the rate of ₹ 18 per m^2. Find total cost of cementing the floor?

Solution:

Area $= \pi r^2 \Rightarrow = \frac{22}{7} \times 7 \times 7 = 154$ m^2.

Cost of cementing = 154 × 18 = ₹ 2,772.

1. Area of semicircle $= \frac{1}{2} \times$ Area of circle

$$= \frac{1}{2} \times \pi r^2$$

$$= \frac{\pi r^2}{2} \text{ sq. unit.}$$

2. Perimeter of semicircle $= \frac{1}{2} \times$ Circumference of circle $+ 2r$

$$= \frac{1}{2} \times 2\pi r + 2r$$

$$= \pi r + 2r = r(\pi + 2) \text{ units}$$

3. Area of quadrant $= \frac{1}{4} \times$ Area of circle

$$= \frac{\pi r^2}{4} \text{ sq. unit}$$

4. Perimeter of quadrant $= \frac{1}{4} \times$ Circumference of circle $+ 2r$

$$= \frac{1}{4} \times 2\pi r + 2r$$

$$= \frac{2\pi r}{4} + 2r = \frac{\pi r}{2} + 2r$$

$$= r\left(\frac{\pi}{2} + 2\right) \text{ units}$$

1. Find the perimeter and area of semicircle where radius is 14 cm?

Solution:

Radius r = 14 cm

$$\text{Area} = \frac{\pi r^2}{2} \Rightarrow = \frac{22}{7} \times \frac{14 \times 14}{2}$$

$$= 308 \text{ cm}^2.$$

$$\text{Perimeter} = r(\pi + 2) \Rightarrow = 14\left(\frac{22}{7} + 2\right)$$

$$= 14\left(\frac{22+14}{7}\right)$$

$$= 2(36) = 72 \text{ cm.}$$

2. The length of a chain used as the boundary of a semicircle park is 36 m, find area of park.

Solution:

Length of the boundary = Perimeter of semicircle

$$36 = r(\pi + 2)$$

$$36 = r\left(\frac{36}{7}\right)$$

$$r = 7 \text{ m.}$$

$$\text{Area of park} = \frac{22}{7} \times \frac{7 \times 7}{2}$$

$$= 77 \text{ m}^2.$$

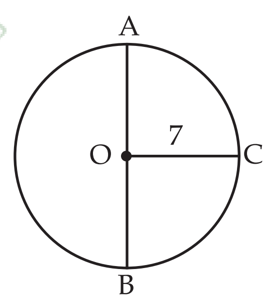
3. Find ares of semicircle AOB and quandrant BOC in the given figure.

Solution:

$$\text{Area of semicircle} = \frac{\pi r^2}{2} \text{ sq. units}$$

$$= \frac{22}{7} \times \frac{7 \times 7}{2}$$

$$= 77 \text{ cm}^2.$$

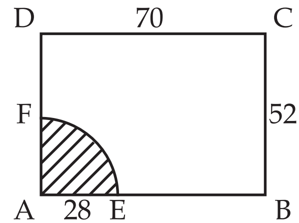


$$\text{Area of quadrant} = \frac{\pi r^2}{4} \text{ sq. units}$$

$$= \frac{22}{7} \times \frac{7 \times 7}{4} = 38.5 \text{ cm}^2.$$

4. A horse is tethered to one corner of a rectangular field of dimension 70 m by 52 m by a rope 28 m long for grazing. How much area can the horse graze in side? HOw much area is left un grazed?

Solution:



$$\text{Area of quadrant} = \frac{\pi r^2}{4} \text{ sq. units}$$

$$= \frac{22}{7} \times \frac{28 \times 28}{4}$$

$$\text{Grazing area} = 616 \text{ m}^2.$$

Area left un grazed = Area of rectangle ABCD − Area of the quadrant AEF

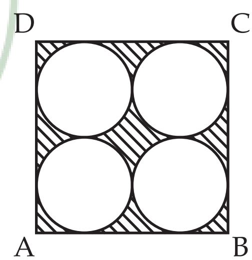
$$\text{Area of rectangle} = l \times b$$

$$= 70 \times 52 = 3640 \text{ m}^2.$$

$$\text{Area left un grazed} = 3640 - 616 = 3024 \text{ m}^2.$$

5. In the given figure, ABCD is a square of side 14 cm, find the area of shaded portion

Solution:



Side of a square = 14 cm

Area of a squre = 14 ×14 = 196 cm^2

$$\text{Radius of circle} = \frac{14}{4} = \frac{7}{2} \text{ cm}$$

$$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$\text{Area of 4 circle} = 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 154 \text{ cm}^2.$$

Area of shaded portion = Area of square − Area of 4 circle

$$= 196 - 154$$

$$= 42 \text{ cm}^2.$$

6. A copper wire is in the form of circle with radius 35 cm. It is bent into a square --------------- the side of the square

Solution:

radius, r = 35

Perimeter of circle = Perimeter of square

Perimeter of circle = $2\pi r$ units

$$= 2 \times \frac{22}{7} \times 35 = 220$$

Perimeter of square $= 4a = 220$

$a = 55$ cm

Side of a square = 55 cm.

7. A 14 cm circle athletic track conceits of two straight sections each 120 m long joined by semicircular ends with inner radius is 35 m. Calculate the area of the track.

Solution:

Radius of inner semicircle = 35

Radius of outer semicircle = 35 + 14 m( ) = 49 m

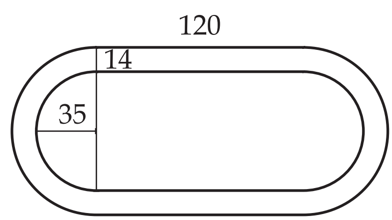
Area of track = Area of 2 semicircular + Area of rectangular track

Area of rectangular track $= 2 \times (l + b)$

$= 2\,(120 + 14) = 3360\ m^2$.

Area of semicircular track = 2 × Area outer semicircle − Area of inner semicircle

$$= 2 \times \left(\frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2\right)$$



$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times (R^2 - r^2)$$

$$= \frac{22}{7} \times (35^2 - 49^2)$$

$$= \frac{22}{7} \times (35+49) - (49 - 35)$$

$= 3696\ m^2$.

Area of track $= 3360 + 3696$

$= 7056\ m^2$.

8. A square park has each side of 100 m. At each corner of the park there is a flower bed in the form of a quadrant of radius 14 m, find the area of the remaining portion of the park.

Solution:

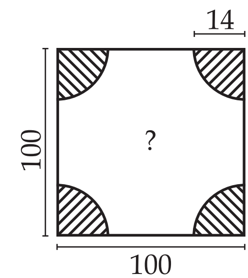
Area of remaining portion = Area of square − $4 \times \frac{\pi r^2}{4}$

$= a^2 - 4 \times \frac{\pi r^2}{4}$

$= (100 \times 100) - \frac{22}{7} \times 14 \times 14$

$= 10000 - 616$

$= 9384\ m^2$.



Surface Area of Cube and Cuboid

1. LSA of Cuboid = $2\,(l + b)\,h$ sq. units.
   (Front and Back; Left and Right)
2. TSA of Cuboid = $2\,(l\,b + bh + lh)$ sq. units.
   (Top and bottom; Left and Right; Front and Back)

1. A cloned wooden box is in the farm of cuboid. Its length, breadth and height are 6 m, 1.5 m and 300 cm. Find TSA and cost of painting its entire outer surface at the rate of ₹ 50 per m^2.

Solution:

$l = 6$ m; $b = 1.5$ m; $h = 3$ m

Painting area of woodenbox = TSA of wood

TSA $= 2\,(l\,b + bh + lh)$

$= 2\,(6 \times 1.5 + (1.5 \times 3) + (3 \times 6)$

$= 2 \times 31.5 = 63\ m^2$.

Cost of painting $= 63 \times 50$

= ₹ 3,150.

2. The length, breadth and height of a hall are 25 m, 15 m, 5 m respectively. Find the cost of renovating its floor and facer walls at the rate of ₹ 80 per m^2.

Solution:

Facer walls = LSA of cuboid = $2(l + b)h$

$= 2(25 + 15)5$

$= 80 \times 5 = 400\ m^2$.

Area of floor = $l \times b = 25 \times 15$

$= 365\ m^2$.

Total renovating area $= 775\ m^2$

Cost of renovating $= 80 \times 775 =$ ₹ 62,000.

Cube and its Surface Area

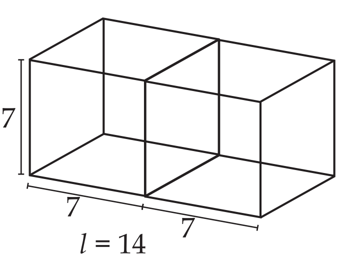
1. TSA of cube = $6a^2$ sq. units
2. LSA of cube = $4a^2$ sq. units

Volume of Cuboid and Cube

1. Volume of Cuboid = $l \times b \times h$ cu. units
2. Volume of Cube = a^3 cu. units

3. Two identical cubes of side 7 cm are forced end to end. Find the total surface area and LSA of new resulting cuboid.

Solution:



Side of a cube = 7 cm

Now resulting cuboid length = 7 + 7 = 14 cm

breadth = 7 cm; height = 7 cm

LSA $= 2(l + b)h$

$= 2(14 + 7) \times 7 = 294\ cm^2$.

TSA $= 2(l b + bh + lh)$

$= 2(14 \times 7) + (7 \times 7) + (14 \times 9)$

$= 2(98 + 49 + 98)$

$= 2 \times 245 = 490\ cm^2$.

4. **Dimensions of hall is 10 m × 9 m × 8 m. Find the cost of white washing the walls and ceiling at the rate of ₹ 8.50 per m².**

Solution:

Area of walls = LSA of hall

$$\text{LSA} = 2(l + b)h$$
$$= 2(19) \times 8 = 304 \text{ m}^2.$$

Area of ceiling $= l \times b$

$$= 10 \times 9 = 90 \text{ m}^2.$$

Total area white washing = 394 m^2

Cost of white washing = 394 × 850

= ₹ 3,349.

1. **The length, breadth and height of cuboid is 120 mm, 10 cm, 8 cm respectively. Find volume of 10 such cuboid**

Solution:

l = 120 mm = 120/10 = 12 cm; b = 10; h = 8

Volume of cuboid = $l \times b \times h$

$$= 12 \times 10 \times 8$$
$$= 960 \text{ cm}^3.$$

Volume of 10 cuboid = 9600 cm^3.

2. **Dimension of a fish tank are 3.8 m × 2.5 m × 1.6 m. How many liters of water it can hold?**

Solution:

Length of fish tank = 3.8 m; breadth = 2.5; h = 1.6 m

Volume of fins tank = 3.8 × 2.5 × 1.6

= 15.2 m^3 ∴ 1 cm^3 = 1 ml

= 15.2 × 1000 liters ∴ 1 m^3 = 1000 liters

= 15200 liters ∴ 1000 cm^3 = 1 liter

3. The dimension of sweet box are 22 cm × 18 cm × 10 cm. How many such boxes can be packed in a carton of dimension 1 m × 88 cm × 63 cm?

Solution:

Volume of sweet box = $l \times b \times h = 22 \times 18 \times 10 \text{ cm}^3$

Volume of carton = $100 \times 88 \times 63 \text{ cm}^3$

$$\text{No. of sweet boxes} = \frac{\text{Volume of carton}}{\text{Volume of sweet box}}$$

$$= \frac{100 \times 88 \times 63}{22 \times 18 \times 10}$$

$$= 140 \text{ boxes}$$

4. A of cube is 864 cm², find the volume

Solution:

TSA = $6a^2 = 864/6$

$a^2 = 144$

$a = 12$

Volume = a^3

$= 12 \times 12 \times 12$

$= 1728 \text{ cm}^3$.

5. A cubical tank can hold 64000 liters of water. Find the length of the side in meter

Solution:

let 'a' be the side of cubical tank

Volume of tank = 64000 liters

$a^3 = 64000$

$= 64000/1000$ $\quad\therefore$ (1000 liters = 1 m^3)

$a^3 = 64 \text{ m}^3$

$a = 4 \text{ m}$

6. The side of a metallic cube is 12 cm. It is melted and formed into cubical where length and breadth are 18 cm and 16 cm respectively. Find height of cuboid.

Solution:

Volume of cuboid ($l \times b \times h$) = Volume of cube(a^3)

$18 \times 16 \times h = 12 \times 12 \times 12$

$$h = \frac{12 \times 12 \times 12}{18 \times 16}$$

$h = 6 \text{ cm}$.

7. The dimensions of a brick are 24 cm × 12 cm × 8 cm. How many such brick will be required to build a wall of 20 m length, 48 cm breadth and 6 m height.

Solution:

Volume of brick = $24 \times 12 \times 8$

Volume of wall = $2000 \times 48 \times 600$

$$\text{No. of bricks} = \frac{\text{Volume of wall}}{\text{Volume of brick}}$$

$$= \frac{2000 \times 48 \times 600}{24 \times 12 \times 8} = 25000 \text{ bricks.}$$

8. A metallic cube with side 15 cm is melted and formed into a cubical. If the length and height of cuboid is 25 cm and 9 cm respectively. Find the breadth of the cuboid.

Solution:

Volume of cube = Volume of cuboid

$$a^3 = l \times b \times h$$

$$15 \times 15 \times 15 = 25 \times 9 \times h$$

$$h = \frac{15 \times 15 \times 15}{25 \times 9}$$

$$h = 15 \text{ cm.}$$

Cylinder

1. CSA = $2\pi rh$ sq units
2. TSA = CSA + (Arfea of top + Area of bottom)

   TSA = $2\pi r(h + r)$ sq units

9. A cylindrical drum has a height of 20 cm and base radius of 14 cm, find the curved surface area and TSA?

Solution:

CSA = $2\pi rh$ sq units

$$= 2 \times \frac{22}{7} \times 14 \times 20$$

$= 1760 \text{ cm}^2$.

TSA = $2\pi r(h + r)$ sq. units

$$= 2 \times \frac{22}{7}(20 \times 14) \times 14$$

$$= 2992 \text{ cm}^3.$$

10. A garden roller where length is 3 m long and where diameter is 2.8 m is rolled to level a garden. How much area it will cover in 8 revolution?

Solution:

Area covered in one revolution = Carved surface area of cylinder

$= 2\pi rh$

Area covered in 1 revolution $= 26.4 \text{ m}^2$.

Area covered in 8 revolution $= 26.4 \times 8$

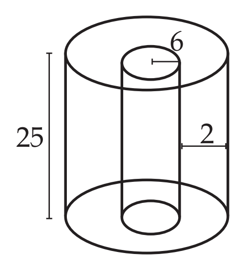
$= 211.2 \text{ m}^2$.

Hollow Cylinder

1. CSA $= 2\pi(R + r)h$ sq units
2. TSA $= 2\pi(R + r)(R - r + h)$

11. If one litre of paint cover 10 m 2, how many liters of paint is required to paint the metal the internal and external surface areas of a cylinder tunnel where thickness is 2 m, internal radius is 6 m and height is 25 m.

Solution:



h = 25 m; thickness = 2 m; thickness = 2m;

Intenal radius, r = 6 m; External radius R = 6 + 2 = 8 m

CSA of cylindrical tunnel = CSA of hollow cylinder

$= 2\pi(R + r)h$

$$= 2 \times \frac{22}{7} \times (8 + 6) \times 25 = 2200 \text{ m}^2.$$

Area covered by one liter of paint $= 10 \text{ m}^2$.

Number of liters required to paint the tunnel $= \frac{2200}{10} = 220$.

Right Circular Cone

1. CSA $= \pi r l$ sq units
2. TSA $= \pi r(l + r)$ sq units

12. The radius of a conical tent is 7 m and heights 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

Solution:

Radius, r = 7m; height, h = 24 m

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{49 + 576}$$
$$= \sqrt{625}$$
$$= 25 \text{ m.}$$

CSA $= \pi r l$ sq. units

$$= \frac{22}{7} \times 7 \times 25$$
$$= 550 \text{ m}^2.$$

$$\text{Now, length of the canvas} = \frac{\text{Area of the canvas}}{\text{Width}}$$
$$= \frac{550}{4}$$
$$= 137.5 \text{ m.}$$

13. From a solid cylinder where height is 2.4 cm and diameter 1.4 cm, a conical ---------- of the same height and base is hollowed out, find the TSA of remaining

Solution:

TSA = CSA of cylinder + CSA of cone + Area of the bottom

$$= 2\pi r h + \pi r l + \pi r^2 \text{ sq. units}$$

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{.049 + 5.76}$$
$$= \sqrt{6.25}$$
$$l = 2.5 \text{ cm}$$

Area of remaining solid = $2\pi h + \pi r l + \pi r^2$ sq. units

$= \pi r(2h + l + r)$

$= \frac{22}{7} \times 0.7\,[(2 \times 0.4) + 2.5 + 0.7]$

$= 17.6\ m^2$.

Sphere

1. Surface area of sphere $4\pi r^2$ sq. units
2. CSA of hemisphere = $2\pi r^2$ sq units
3. TSA of hemisphere = $3\pi r^2$ sq units
4. CSA of hollow hemisphere = $2\pi\,(R^2 + r^2)$ sq units
5. TSA of hollow hemisphere = $\pi\,(3R^2 + r^2)$ sq units

14. Find the diameter of a sphere where surface are 154 m^2.

Solution:

Surface area = $154\ m^2$

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{49}{4}$$

$$r = \frac{7}{2} \qquad \therefore \text{Diameter} = 7\ m.$$

15. The internal and external ratio of a hollow hemisphere shell are 3 m and 5 m respectively, find TSA and CSA of the shell?

Solution:

R = 5 m; r = 3 m

CSA of shell $= 2\pi\,(R^2 + r^2)$ sq. units

$= 2 \times \frac{22}{7} \times (25 + 9) = 213.71\ m^2$.

TSA of shell $= \pi\,(3R^2 + r^2)$ sq. units

$= \frac{22}{7}(75 + 9) = 264\ m^2$.

16. A sphere, a cylinder and cone are of same radius where as cone and cylinder are of same height, find ratio of their curved surface area.

Solution:

Required ratio = CSA of the sphere : CSA of cylinder + CSA of cone

$$[\, l = \sqrt{r^2 + h^2} = \sqrt{2r^2} = \sqrt{2}\, r \,]$$

$$= 4\pi r^2 : 2\pi rh : \pi rl$$

$$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$$

Frustum of Right Circular Cone

1. CSA of frustum = $\pi(R + r)l$ sq units
2. TSA of frustum = $\pi(R + r)l + \pi R^2 + \pi r^3$ sq units

Volume of Frustum

1. Volume of frustum = $\frac{\pi h}{3}(R^2 + Rr + r^2)$ cu. units

17. Slant height of a frustum of a cone is 5 cm and the ratio of its ends are 4 cm and 1 cm, find its curved surface area.

Solution:

CSA of frustum = $\pi(R + r)l$ sq units

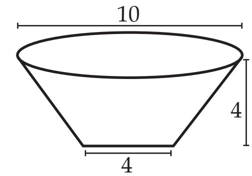
$$= \frac{22}{7}(4 + 1) \times 5$$

$$= \frac{570}{7}$$

CSA $= 73.57\ cm^2$.

18. An industrial metallic bucket is in the shape of the frustum of a right circular cone where top and bottom diameter are 10 m , 4 m and where height 14 cm, find the curved and total surface area of the bucket.

Solution:



$$l = \sqrt{h^2 + (R - r)^2}$$

$$= \sqrt{4^2 + (5 - 2^2)}$$

$$= \sqrt{16 + 9}$$

$$l = 5 \text{ m.}$$

$$\text{CSA} = \pi(R + r)l$$

$$= \frac{22}{7} \times (5 + 2) \times 5$$

$$= 110 \text{ m}^2.$$

$$\text{TSA} = \pi(R + r)l + \pi R^2 + \pi r^3$$

$$= \frac{22}{7}[(5 + 2)5 + 25 + 4]$$

$$= \frac{1408}{7}$$

$$= 201.14 \text{ m}^2.$$

19. A solid iron cylinder has total surface area of 1848 cm^2. Its CSA is 5/6 of TSA. Find radius and height of iron cylinder.

Solution:

TSA of cylinder = $2\pi r\,(h + r)$

CSA of cylinder = $2\pi rh$

CSA = 5/6 of TSA

$$2\pi rh = \frac{5}{6}[2\pi r(h + r)]$$

$$\pi rh = \frac{5}{6}[\pi r(h + r)]$$

$$h = \frac{5}{6}(h + r)$$

$$6h = 5h + 5r$$

$$h = 5r$$

$$\text{TSA} = 2\pi r(h + r) = 1848$$

$$2 \times \frac{22}{7} \times r(5 + r) = 1848$$

$$2 \times \frac{22}{7} \times 6r^2 = 1848$$

$$6r^2 = \frac{1848 \times 7}{44} = \frac{29}{6}$$

$$r^2 = 49$$

Radius = 7m; height = 5r = 35 m.

20. 4 persons line in a conical tent where slant height is 19 cm. If each person equal 22 cm^2 of the floor area then find height of the tent?

Solution:

$l = 19$ cm

$4 \times 22 = 88$ cm^2 (floor area)

$$\pi r^2 = 88$$

$$r^2 = \frac{88 \times 7}{22} \Rightarrow 28$$

$$r = \sqrt{28}$$

$$h^2 = l^2 - r^2$$

$$h^2 = 19^2 - \left(\sqrt{28}\right)^2$$

$$= 361 - 28 \Rightarrow = 333$$

$$h = \sqrt{333}$$

$$h = 18.25 \text{ cm.}$$

21. A girl wishes to prepare birthday caps in the form of right circular cone for her birthday party using sheet of paper whole area is 5720 cm^2, how many caps can be made with radius 5 cm and height 12 cm

Solution:

Sheet of paper, area = 5720 cm^2

$$\text{Number of cape to be made} = \frac{\text{Area of sheet paper}}{\text{CSA of cone}}$$

$$= \frac{5720}{\pi r l} = \frac{5720}{\frac{22}{7} \times 5 \times 13}$$

$$= \frac{5720}{204.2}$$

$$= 28 \text{ caps.}$$

$$= \sqrt{r^2 + h^2}$$

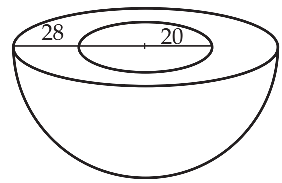
$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

22. The internal and external diameter of a hollow hemisphere vessel are 20 cm and 28 cm respectively, find the cost to paint the vessel all over at ₹ 0.4 per cm^2.

Solution:



$R = \frac{28}{2} = 14; \; r = \frac{20}{2} = 10$

TSA of hollow hemisphere $= \pi(3R^2 - r^2)$

$= \frac{22}{7}[(3 \times 196) + (100)]$

$= \frac{22}{7}(888)$

$= 2162.28 \text{ cm}^2.$

Cost to paint $= 2162.28 \times 0.14$

$=$ ₹ 302.71

23. The ratio of radius of two right circular cones of same height is 1 : 8, find the ratio of their carved surface area when height of each cone is 3 times the radius of smaller cone.

Solution:

$r_1 : r_2 = 1 : 3$

$h_1 = 3r_1$

$\frac{\text{CSA of } 1^{st} \text{ cone}}{\text{CSA of } 2^{nd} \text{ cone}} = \frac{\pi r_1 l_1}{\pi r_2 l_2}$

$= \frac{x \,.\, x\sqrt{10}}{3x \,.\, x\sqrt{18}} \Rightarrow = \frac{\sqrt{10}}{3\sqrt{18}}$

$= \frac{\sqrt{2} \times \sqrt{5}}{3 \times 3 \times \sqrt{2}} \Rightarrow = \frac{\sqrt{5}}{9}$

$= \sqrt{5} : 9$

$l = \sqrt{(3x)^2 + x^2}$

$= \sqrt{10x^2} = \sqrt{(10)}x$

$l = \sqrt{(3x)^2 + (3x)^2}$

$= \sqrt{(18)}x$

Volume of right Cylinder Cone

1. Volume of Cone $= \frac{1}{3}\pi r^2 h$ cu. units

24. Volume of a solid right circular cone is 11088 cm^3. if the height is 24 cm, then find radius of the cone.

Solution:

Volume = 11088 cm^3

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = \frac{11088 \times 3 \times 7}{22 \times 24}$$

$$r^2 = 441$$

$$r = 21 \text{ cm.}$$

Volume of Sphere

1. Volume of sphere = $\frac{4}{3}\pi r^3$ cu. units
2. Volume of hollow sphere = $\frac{4}{3}\pi (R^3 - r^3)$ cu. units
3. Volume of solid hemisphere = $\frac{2}{3}\pi r^3$ cu. units
4. Volume of hollow sphere = $\frac{2}{3}\pi (R^3 - r^3)$ cu. units

25. The Volume of a solid hemisphere is 29106 cm^3. Another hemisphere where volume is 2/3 of the above is carved out. Find the radius of the new hemisphere.

Solution:

Volume of hemisphere = 29106 cm3

Volume of new hemisphere = $\frac{2}{3}$ (Volume of original sphere)

$$= \frac{2}{3} \times 29106 = 19404 \text{ cm}^3.$$

$$\text{Volume } 19404 = \frac{2}{3}\pi r^3$$

$$r^3 = \frac{19404 \times 3 \times 7}{22 \times 2}$$

$$r = 21 \text{ cm..}$$

26. Calculate the weight of a hollow ball plate sphere if the inner radius is 7 cm and thickness is 1 mm and whre density is 17.3 g/cm^3.

Solution:

Inner r = 7 cm; thickness = 1 mm = 1/10 cm

$$\text{Outer r} = 7 + \frac{1}{10} = \frac{71}{10} = 7.1 \text{ cm}$$

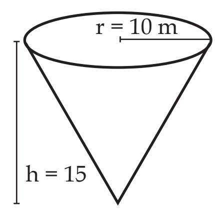
$$\text{Volume of hollow sphere} = \frac{4}{3}\pi\,(R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7}\ (35791 - 343) = 62.\,48 \text{ cm}^3.$$

But, weight of balls in 1 cm^3 = 17.3 gm

Total weight = 17.3 × 62.48 = 1080.90 gm.

27. A conical container is fully filled with petrol the radius is 10 m and height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. m/per minute in how many minutes the container will be emptied (amount round off to be need minutes)



Solution:

$$\text{Volume of container} = \frac{1}{3}\pi\, r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 15$$

$$= 1571.42 \text{ m}^3.$$

Container will be emptied by = Volume of container/Volume of petrol release

$$= \frac{1571.42}{25} = 62.85 = 63 \text{ minutes.}$$

28. A solid sphere and a solid hemisphere have equal TSA. Prove that the ratio their volume is $3\sqrt{3}:4$

Solution:

TSA of sphere = $4\pi r^2$

TSA of hemisphere = $3\pi r^2$

$$4\pi r_1^2 = 3\pi r_1^2$$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{3}{4} \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

Ratio volume of sphere and hemisphere =

$$\frac{4/3\,\pi r_1^3}{2/3\,\pi r_2^3} = \frac{4r_1^3}{2r_2^3}$$

$$\frac{2r_1^3}{r_2^3} = \frac{2\times(\sqrt{3})^3}{(2)^3}$$

$$= \frac{2\times 3\sqrt{3}}{84}$$

$$= 3\sqrt{3}:4$$ Proved.