COMPLETE MATHS

ब्रह्मास्त्र

FORMULA BOOK

• CONCEPTS • FORMULAE • SHORT TRICKS

BY - ADITYA RANJAN
EXCISE INSPECTOR

IMPORTANT FOR

SSC, BANK, RAILWAY, DEFENCE, CSAT, OTHER STATE & GOVT. EXAMS

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Maths by Aditya Ranjan

Selected हैं Selection दिलायेंगे

Aditya Ranjan

Maths Expert

Excise Inspector

Motivator & Mentor

Cracked Various exams with Top Rank.


Dedicated to my dear
"ASPIRANTS" & team
"RANKERS' GURUKUL"
who have inspired me
to compile this book.

About Aditya Ranjan

Aditya Ranjan Sir is a renowned Maths Faculty , who has taught lakhs of students through rankers gurukul you tube channel and its offline centre.

His students are selected in various departments in India.

He is the fastest emerging educator in online education.

His free maths & guidance videos have millions of views on you tube.

He has a unique approach of solving new TCS questions . He always focuses on relevant and updated content.

His classes are interactive and he patiently clears the doubts of all his students.

He has got selected at his very early age and his words "**SELECTED है SELECTION दिलायेंगे**" are very popular among students.

His new innovative idea of completing entire maths syllabus for any govt. exam on you tube through "**60 Days 60 Marathon**" is a landmark in online education.

EXAMS QUALIFIED

CGL , CHSL, CPO , CDS (3 TIMES)

**** HE SCORED ****

- **MATHS – 50/50**
- **ENGLISH – 50/50**
- **REASONING – 50/50**

and got selected in **SSC CHSL 2019** with **AIR - 114.**

- He scored the same in CGL 2019 PRE and **227/200 in MAINS (MATHS)** and got selected in SSC CGL 2019 as an **EXCISE INSPECTOR.**

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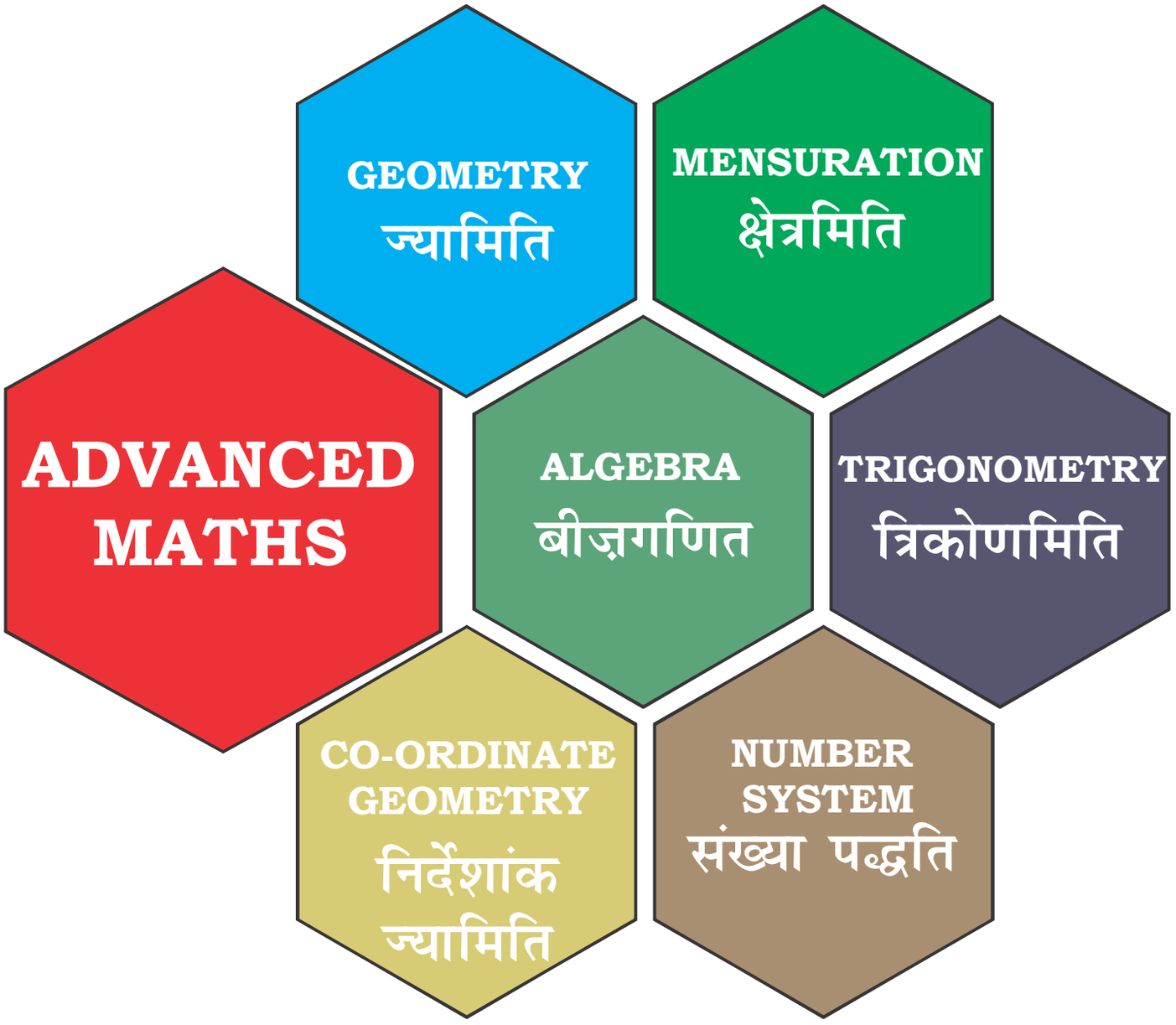
ACKNOWLEDGMENT

मैं उन सभी लोगों का आभारी हूँ जिन्होंने इस पुस्तक को तैयार करने में मेरी मदद की। मैं विशेषकर विक्रमजीत सर का ऋणी हूँ, जिन्होंने हमेशा बड़े भाई की तरह मेरा मार्गदर्शन किया और शायद जिनके बिना इस मुकाम पर पहुँचना संभव नहीं था। साथ में सबसे ज्यादा आभारी Rudra Sir का हूँ, जिन्होंने इस Formula Book को तैयार करने में अपनी जान लगा दी, दिन रात मेहनत करके उन्होंने बच्चों को एक मास्टरपीस दिया है जिससे उनको काफी मदद मिलेगी और जिस तरह से महाभारत के रचयिता वेदव्यास जी है लेकिन इसे लिखा गणेश जी ने है उसी तरह से इस किताब को लिखने वाले हमारे टाइपिस्ट अमित सिंह, अजय कुमार, वीर सिंह और अखिलेश हैं। मैं अपने छात्रों को भी धन्यवाद देना चाहता हूँ जिन्होंने वास्तव में मुझे इस Formula Book को बनाने के लिए प्रेरित किया।

- आदित्य रंजन

INDEX


ADVANCED MATHS
GEOMETRY
ज्यामिति
MENSURATION
क्षेत्रमिति
ALGEBRA
बीज़गणित
TRIGONOMETRY
त्रिकोणमिति
CO-ORDINATE GEOMETRY
निर्देशांक ज्यामिति
NUMBER SYSTEM
संख्या पद्धति

01 GEOMETRY ज्यामिति

Lines & Angles रेखाएं एवं कोण

We have divided the geometry into five sub-topic, which are as follows:

है ज्यामिति को पाँच उप-विषयों में बांटा गया है जो निम्न प्रकार हैं-

- Point, Lines and Angle/ बिंदु, रेखाएं एवं कोण
- Triangle/ त्रिभुज
- Quadrilateral/ चतुर्भुज
- Polygon/ बहुभुज
- Circle/ वृत्त

Points, Line & Angle/ बिंदु, रेखाएं एवं कोण

1. Point : A point is a circle of zero radius

बिंदु शून्य त्रिज्या का एक वृत्त होता है।

Types of Point/ बिंदु के प्रकार

(a) **Collinear Points:** If three or more points situated on a straight line, these points are called collinear points.

यदि तीन या अधिक बिंदु एक सरल रेखा पर स्थित हों, तो वे बिंदु संरेख बिंदु कहलाते हैं।

Example : Points *A*, *B*, and *C* are collinear.

उदाहरण: बिंदु *A*, *B*, एवं *C* संरेख हैं।

(b) **Non-collinear Points:** If three or more points are not situated on a straight line, these points are called non-collinear points.

यदि तीन या अधिक बिंदु एक सरल रेखा पर स्थित नहीं हों, तो वे बिंदु असंरेख बिंदु कहलाते हैं।

2. Lines: A line is made up of a infinite number of points and it has only length i.e., it does not has any thickness (or width). A line is endless so, it can be extended in both directions.

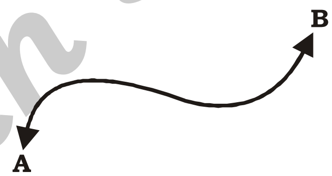
रेखा अनंत बिंदुओं से मिलकर बनी होती है और इसकी केवल लंबाई होती है अर्थात् इसकी कोई मोटाई (या चौड़ाई) नहीं होती। रेखा का कोई अंत नहीं होता इसलिए इसे दोनों दिशाओं में बढ़ाया जा सकता है।

(a) **Straight line:** A line which does not change its direction at any point is called a straight line.

वह रेखा जो किसी भी बिंदु पर अपनी दिशा नहीं बदलती, सरल रेखा कहलाती है।

(b) **Curved line:** A line which changes its direction is called a curved line.

वह रेखा जो अपनी दिशा बदलती है, वक्र रेखा कहलाती है।



(c) **Line Segment:** A line segments has two end points, but generally speaking line segment is called a line.

रेखाखंड के दो अंत बिंदु होते हैं, परंतु सामान्यतः रेखाखंड को रेखा कहा जाता है।

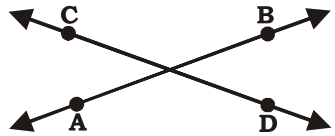
★ **Types of Lines/ रेखाओं के प्रकार**

The lines are basically of five types as discussed below:

रेखाएं मूलतः पाँच प्रकार की होती हैं जो निम्न हैं:

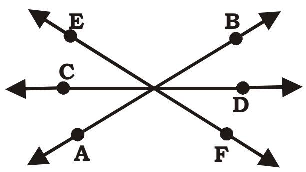
(i) **Intersecting Lines:** If two or more lines intersect each other, then they are called intersecting lines. In the figure *AB* and *CD* are intersecting lines.

यदि दो या अधिक रेखाएं एक-दूसरे को काटती हैं, तो वे प्रतिच्छेदी रेखाएं कहलाती हैं। चित्र में *AB* एवं *CD* प्रतिच्छेदी रेखाएं हैं।



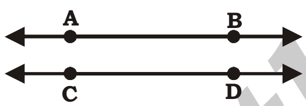
(ii) **Concurrent Lines :** If three or more lines pass through a point, then they are called concurrent lines and the point through which these all lines pass is called point of concurrent.

तc ́ rhu ;k rhu ls vf/kd js[kk,¡ ,d gh fcUnq ls gksdj tkrh gS] rks mUgsa laxkeh js[kk,¡ dgrs gSa rFkk og fcUnq ftlls gksdj lHkh js[kk,¡ xqtjrh gSa] laxkeh fcUnq dgykrk gSA



(iii) **Parallel Lines :** Two straight lines are parallel if they lie in the same plane and do not intersect even if they produced infinitely. Perpendicular distances between two parallel lines are always same at all places.

nks ljy js[kk,¡ lekarj gksrh gSa ;fn os ,d gh lery esa fLFkr gksa vkSj vuar rd c<+kus ij Hkh ,d&nwljs dks izfrPNsn u djsaA nks lekarj js[kkvksa ds chp yacor nwjh lHkh LFkkuksa ij lnSo leku gksrh gSA



In the figure AB and CD are parallel lines.

fn, x, fp= esa AB ,oa CD lekarj js[kk,¡ gSaA

Symbol for parallel lines is ||.

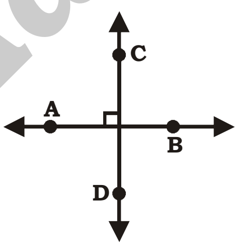
Hence, AB || CD.

lekarj js[kkvksa ds fy, ladsr || gSA

vr% AB || CD.

(iv) **Perpendicular Lines :** If two lines intersect at right angles, then two lines are called perpendicular lines. In the following figure AB and CD are perpendicular lines.

;fn nks js[kk,¡ ,d&nwljs dks ledks.k ij izfrPNsn djrh gSa] rks bu nks js[kkvksa dks yacor js[kk,¡ dgrs gSaA fuEufyf[kr fp= esa AB ,oa CD yacor js[kk,¡ gSaA

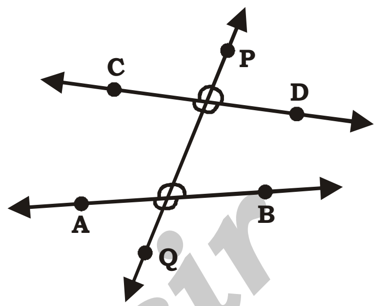


Symbolically it is represented as AB ⊥ CD or we can also say that CD ⊥ AB.

izrhdkRed :i ls bls AB ⊥ CD }kjk iznf'kZr djrs gSa ;k ge ;g Hkh dg ldrs gSa fd CD ⊥ AB gSA

(v) **Transversal Lines :** A line which intersects two or more given lines at distinct points is called a transversal of the given lines.

,d js[kk tks nks ;k nks ls vf/kd js[kkvksa dks fHkUu&fHkUu fcUnqvksa ij izfrPNsn djrh gS] mls nh xbZ js[kkvksa dh fr;Zd js[kk dgrs gSaA



In figure straight lines AB and CD are intersected by a transversal PQ.

fn, x, fp= esa ljy js[kk,¡ AB ,oa CD fr;Zd js[kk PQ }kjk izfrPNsfnr gSaA

IMPORTANT POINTS – LINES

- There is one and only one line passing through two distinct points.

  nks fHkUu fcUnqvksa ls gksdj dsoy ,d vkSj dsoy ,d js[kk gksdj xqtjrh gSA

- Two or more lines are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.

  nks ;k nks ls vf/kd js[kk,¡ leryh; dgykrh gSa ;fn os ,d gh lery esa fLFkr gksa] vU;Fkk mUgsa vleryh; dgk tkrk gSA

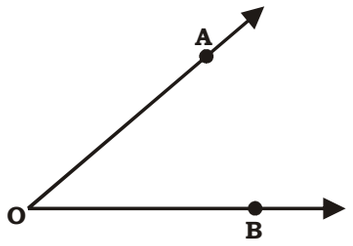
- The intersection of two planes is a line.

  nks lryksa dk izfrPNsn ,d js[kk gksrk gSA

3. **Angles :** An angle is the union of two non-collinear rays with a common initial point. The two rays forming an angle are called arms of the angle and the common initial point is called the vertex of the angle.

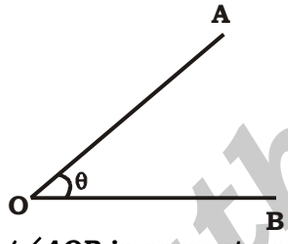
  ,d dks.k ,d gh izkjafHkd fcUnq okyh nks vlajs[k fdj.kksa dk la?k gksrk gSA dks.k cukus okyh nks fdj.kksa dks dks.k dh Hkqtk,¡ dgrs gSa rFkk mHk;fu"B izkjafHkd fcUnq dks.k dk 'kh"kZ dgykrk gSA

The angle *AOB* denoted by $\angle AOB$, is formed by rays *OA* and *OB* and point *O* is the "vertex" of the angle.



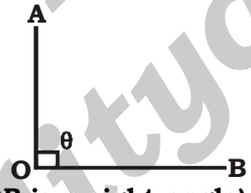
★ Types of Angles/

(i) **Acute Angle:** If the measure of an angle is less than 90°, it is an acute angle.



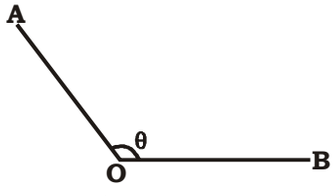
$0° < \theta < 90°$ ($\angle AOB$ is an acute angle)

(ii) **Right Angle :** If measure of an angle is equal to 90°, then it is a right angle.



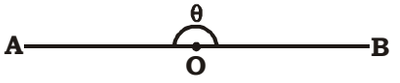
$\theta = 90°$ ($\angle AOB$ is a right angle)

(iii) **Obtuse Angle:** If measure of an angle is more than 90° but less than 180°, then it is a obtuse angle.



$90° < \theta < 180°$ ($\angle AOB$ is an obtuse angle)

(iv) **Straight Angle:** If measure of an angle is equal to 180°, then it is a straight angle.



$\theta = 180°$ ("*AOB* is a straight angle)

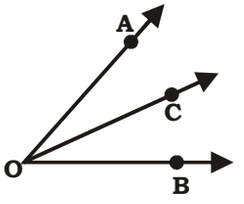
(v) **Reflex Angle:** If measure of an angle is more than 180° but less than 360°, then it is a reflex angle.

$180° < \theta < 360°$ ($\angle AOB$ is a reflex angle)

★ Pairs of Angles/

(i) **Adjacent Angles:** Two angles are called adjacent angles if:

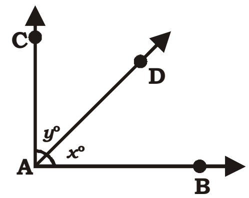
- They have the same vertex,
- They have a common arm,
- Uncommon arms are on either side of the common arm.



- In the figure, $\angle AOC$ and $\angle BOC$ have a common vertex *O*. Also, they have a common arm *OC* and the distinct arms *OA* and *OB*, lie on the opposite side of the line *OC*.

(ii) **Complementary Angles :** If sum of two angles is equal to 90°, then the two angles are called complementary angles.

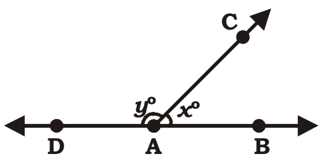
फ् ल` ब्रेर्ल्त्यं ख्लड्डे ला` ब्रे बा` ल्ये व्ह्ल90° हे बनो त्नयलड्डे दिं बा –` ब्रेल्ह्ठ फ् ल` ब्रेल- हा` गव्हम्ह्य



$\angle BAD$ and $\angle DAC$ are complementary angles, if $x° + y° = 90°$

(iii) **Supplementary Angles:** If sum of two angles is equal to 180°, then the two angles are called supplementary angle.

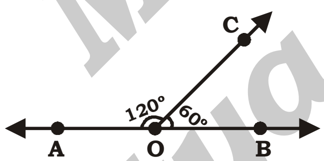
ज्ठ फ् ल` ब्रेर्ल्त्यं ख्लड्डे ला` ब्रे बा` ल्ये व्ह्ल180° हे बनो त्नयद खे दिं बा` ब्रेल फ् ल` ब्रेल- हा` गव्हम्ह्य



$\angle BAC$ and $\angle DAC$ is supplementary angles, if $x° + y° = 180°$

(iv) **Linear Pair of Angles:** Two adjacent angles are said to form a linear pair of angles, if their noncommon arms are two opposite rays.

–` ब्रे बा` लक्न्ड्ड्- ल्यैह्व र्ल लखे लाए ज्खन` ब्रेबन` ब्रे बा` कंद्ध- ल्यैह्व लाश ल्श्र् गव्हम्ह्यक्ष श्ल ज- प्रलखे लाए जै` ल्तिये ह्ये न्लाड्डे द ´न क्ष्याल- क्येल्हे ह्य



- In figure, OA and OB are two opposite rays $\angle AOC$ & $\angle BOC$ are the adjacent angles. Therefore, $\angle AOC$ and $\angle BOC$ form a linear pair.

  ´ख्यल)यलंट्ु ल्डहOA ए क्नOB ´न क्ष्याल- क्येल्हह्ल्∠AOC ए क ∠BOC लाए ज्खन` ब्रेलह्ली ज्। न्ल∠AOC ए क ∠BOC लक्न्ड्ड्- ल्यैह्व ल्श्र् गव्हम्ह्य

- If a ray stand on a line, the sum of the adjacent angles so formed is 180°.

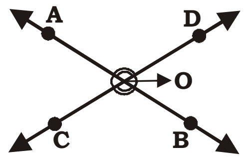
  यं ख्ल` ह्रिग्न- क्येल्लं- ज्प्रलाड्डे ल क्न श्व्प्र्हे लो ली ज्ल इ` क्न्श दि ए ज्खन` ब्रे बा` ल्ये व्ह्ल180° हे ये ल्हऽ

Note:- Conversely, if the sum of two adjacent angles is 180°, then their non-common arms are two opposite rays.

ीज- ल्ञान क्ष्यालयं ख्लड्डे लाए ज्र्ञल` ब्रे बा` ल्ये व्ह्ल180°ल्हव गे ला ि– प्रल)क्लतेयं ि्हल्ये ह्ये न्लाड्डे ल्ञान क्ष्याल- क्येल्हह्य

(v) **Vertically opposite Angles :** If arm of two angles form two pairs of opposite rays, then the two angles are called as vertically opposite angles.

वे प्र्ह्रे ´भ्तै प्ह्रुल-` ब्रेर्ल्त्यं ख्लड्डे ला-` ब्रे बा- प्लतेह्ये लं न क्ष्या ´– क्ये बा- लाड्डे लायैह्व लश्र् ि गप्र्लहे बनो लाड्डे दि बा-` ब्रेलवे प्र्ह्रे ´भ्तै प्ह्रु – हा` गव्हम्ह्य



- In other words, when two lines intersect, two pairs of vertically opposite angles are formed. Each pair of vertically opposite angles are equal.

  ख्ह्न्क्लावे ऋह्रे बा बनक्ष श्लखे लाव्ड्डे न्ला इ़ड ख्न- क्ष्याप्र्लहब्लाग्रे द वे प्र्ह्रे ´भ्तै प्ह्रुल` ब्रे बा लाड्डे लायैह्व ल श्र् ि गव्हम्ह्य वे प्र्ह्रे ´भ्तै पव्ह्रुल` ब्रे ब – प्रल इ्यव ल्यैह्व लश्र् केश्र क्नहे बलहम्ह्य

- In the figure, two lines AB and CD intersect at O. We find that $\angle AOC$ and $\angle BOD$ are vertically opposite angles

  ´ट्ु लब्लाड्डे लाव्ड्डे न्लAB ए कCD ´श्ख्प्O क्ल इ़ड ख्न क्ष्याव्हम्ह्य

  So, $\angle AOC = \angle BOD$

  Similarly, $\angle BOC$ and $\angle AOD$ are vertically opposite angles.

  ीज्प्रल इ़े क्ष्याल∠BOC ए क∠AOD वे प्र्ह्रे ´भ्तै प्ह्रुल` ब्रेल्हम्ह्य

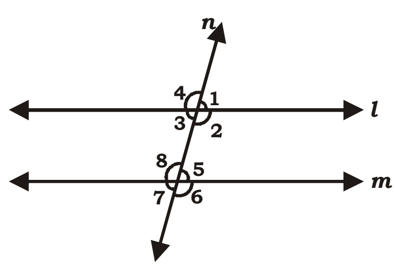
  So, $\angle BOC = \angle AOD$

(vi) **Corresponding Angles :** When two parallel lines are intersected by a transversal. They form pairs of corresponding angles.

ज्प्रेगल` ब्रेर्ल्त्लक्ष श्लखे लाड्डे` मक्लव्ड्डे न्ला – लंगयभ लाव्ड्डे ल्खे के इ़ड ´व्खाल- प्र्लखे गप्र्लहलो लालाद्मेगल` ब्रे बा- लायैह्व लश्र् ि गप्र्लहम्ह्य

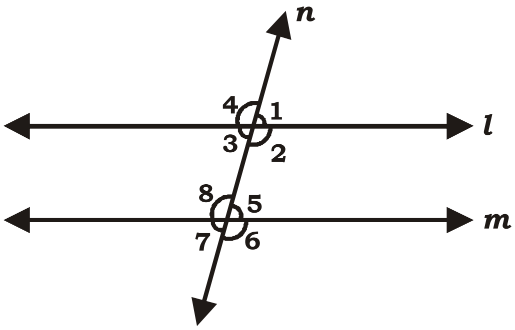
Lines l and m are intersected by the transversal n. Then $\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 3 = \angle 7$ and $\angle 2 = \angle 6$.

क्लव्ड्डे न्लl ए कm ´गयभ लाव्ड्डे लn चे केल इ़ड ´व्खाल- प्र्ली ह्लहबनो द $\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 3 = \angle 7$ ए क $\angle 2 = \angle 6$.



(vii) Alternate Angles : When two parallel lines are intersected by a transversal, they form pairs of Alternate Angles. Each pair of alternate angles are equal.

=–` साकला` ब्दोर्लल्क्श्लोलै` मकलब्दे =ल– लंगयभ लब्दे लके झाड़ ंवक्ागल प्रक्ले गप्रलहलो लालल–` मकल` ब्दो ला लयौबु लर्ािगप्रलह्न =–` मकल` ब्दो ला ला झ्यव लयौबु लरकेश्कलहे मलहह्न



In the given figure alternate interior angles $\angle 3 = \angle 5$ and $\angle 2 = \angle 8$ and alternate exterior angles $\angle 1 = \angle 7$ and $\angle 4 = \angle 6$.

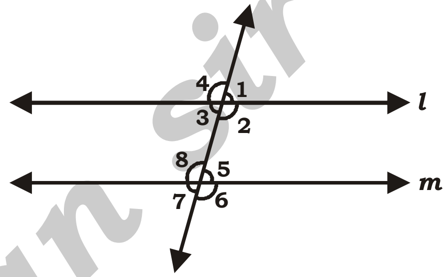
`ख्यला)यलांत्ु` लै ला–` मकलए्मं–` ब्दोल $\angle 3 = \angle 5$ गू` $\angle 2 = \angle 8$ ए` क्ल–` साकलो न्नल` ब्दो $\angle 1 = \angle 7$ गू` $\angle 4 = \angle 6$

Note: Conversely, if a transversal intersects two lines in such a way that a pair of alternate interior angles is equal, then the two lines are parallel.

ोज्–लंन क्ालयं ख्ल` दिलंगयभ लब्दे लो लब्दे ए` ला` ली ज् इ` कल झाड़ ब्लल कप्रलहलं– ल–` मकलए्मं–` ब्दो ला` ल– यौबु लरकेश्कलहे लो लो दि ालब्दे =लै` मकलहे मप्रलहह्न

(viii) Consecutive Interior Angles : When two parallel lines are intersected by a transversal, they form two pairs of interior angles. The pairs of interior angles so formed are supplementary

म` )गलए` मंक ल` ब्दोर्लल्क्श्लोलै` मकलब्दे =ला– `गयभ लब्दे लके कल झाड़ ंवक्ागलहे मप्रलहलो लए` मंक ल` ब्दो ला द खेलयौबु लर्ािगलहलीज्लइ` कलर्ाए` मंक ल` ब्दोलफ लहे मलहह्न

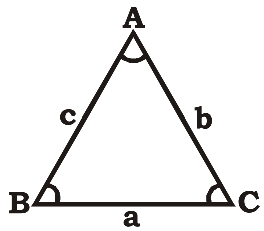


Pair of interior angles are:
$\angle 2 + \angle 5 = 180°$ & $\angle 3 + \angle 8 = 180°$

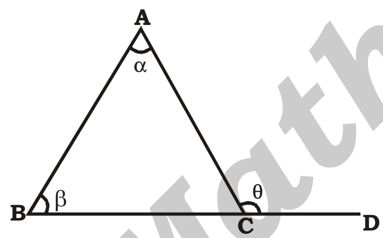
TRIANGLE/f=kHkqt

Properties of triangle/f=kHkqt ds xq.k

1. **Sum of all three angles of a triangle is always 180°, *i.e.* ∠A + ∠B + ∠C = 180°.**
   fdlh Hkh f=kHkqt ds rhuksa dks.kksa dk ;ksx lnSo 180° gksrk gS
   vFkkZr~ ∠A + ∠B + ∠C = 180°

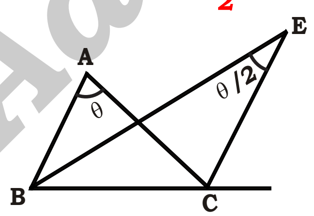


2. **If a side of triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.**
   ;fn f=kHkqt dh ,d Hkqtk dks c<+k;k tk, rks bl izdkj cuus okyk cká dks.k nks vUr% foijhr dks.kksa ds ;ksx ds cjkcj gksrk gSA
   ∠ACD = ∠CAB + ∠ABC



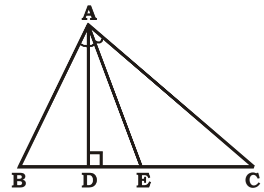
3. **In a triangle the angle between internal bisector of a base angle and external bisector of the other base angle is half of the remaining vertex angle.**

   According to this property $\angle BEC = \frac{\angle A}{2}$.
   fdlh f=kHkqt esa vk/kkj ds ,d dks.k ds vkarfjd lef}Hkktd vkSj vk/kkj ds nwljs dks.k ds cká lef}Hkktd ds chp dk dks.k 'ks"k 'kh"kZ dks.k dk vk/kk gksrk gSA
   bl xq.k ds vuqlkj $\angle BEC = \frac{\angle A}{2}$



4. **In ΔABC, the side BC produced to D and angle bisector of ∠A meets BC at E then**
   ∠ABC + ∠ACD = 2 ∠AEC.
   ΔABC esa Hkqtk BC dks D rd c<+k;k x;k gS vkSj ∠A dk dks.k lef}Hkktd BC dks E ij feyrk gS rks
   ∠ABC + ∠ACD = 2 ∠AEC

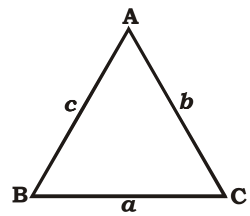
5. **The angle between perpendicular drawn by a vertex to opposite side and angle bisector of the vertex angle is half of difference between other two remaining vertex angles.**
   'kh"kZ ls foijhr Hkqtk ij [khaps x, yac vkSj 'kh"kZ dks.k ds dks.k lef}Hkktd ds chp dk dks.k vU; nks 'ks"k 'kh"kZ dks.kksa ds varj dk vk/kk gksrk gSA



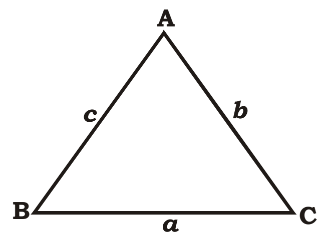
AD ⊥ BC and AE is angle bisector of ∠A

Then $\angle DAE = \left|\frac{\angle B - \angle C}{2}\right|$

6. (i) **Sum of any two sides of a triangle is always greater than the third side, *i.e.***
   fdlh f=kHkqt dh fdUgha nks Hkqtkvksa dk ;ksx rhljh Hkqtk ls lnSo vf/kd gksrk gS vFkkZr~
   $a + b > c$
   $b + c > a$
   $c + a > b$

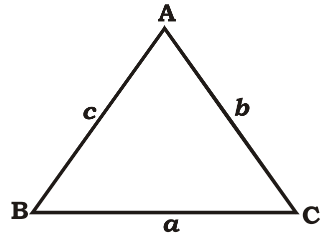


   (ii) **Difference of any two sides of a triangle is always smaller than the third side.**
   fdlh f=kHkqt dh fdUgha nks Hkqtkvksa dk varj rhljh Hkqtk ls lnSo de gksrk gS vFkkZr~
   $|a - b| < c$
   $|b - c| < a$
   $|c - a| < b$



7. (i) **Angles opposite to the equal sides of a triangle are equal.**
   fdlh f=kHkqt dh cjkcj Hkqtkvksa ds lEeq[k dks.k cjkcj gksrs gSaA
   (ii) **The angle opposite to the greater side is always greater than the angle opposite to the smaller side.**
   cM+h Hkqtk ds lkeus dk dks.k NksVh Hkqtk ds lkeus okys dks.k ls lnSo cM+k gksrk gSA

8.



(a) **Sine Rule : Ratio of side and sine of opposite angle of a triangle is equal to double of circum radius.**

ा ये ल-` लं ि यै र्ल लं- ज्प्रलं्ु ते ह्यु लै ब्ला ो हे ल ए क्का ज- त्तं न क्का -` ब्येल- प्र्ला ये ल-` लए ि पे गलं्ु ते ह्यु ल- प्रश्े न्र्लं्ु ा ये ल- द खे व्रे िल- त्र्र के श क्ल हे ये ल्ह३

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

(b) **Cosine Rule : If two sides and angle between sides are given, then we can find the opposite side by Cosine Rule.**

-` ला ये ल-` लं ि यै र्ल लयं ख्लक्षे लते हे लए क्लते हे ए ला- द श्प्र ल-` ल-` ब्येल ख्ये ल्हे क्ला ो ला-` ला ये ल- तं ि यै ल्ज़ल्है ं न क्काल्ते हे ल्के गल- क्क्ष- गल्हु

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

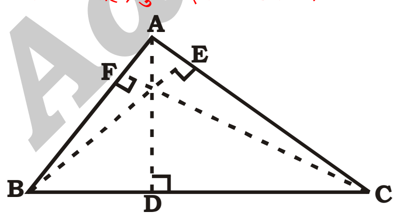
9. **Area of Triangle**

(a) **Area of triangle ABC = $\frac{1}{2}$ × base × height**

ु ते ह्यु लABC -` ल्ह्युद्ु ा = $\frac{1}{2}$ × ए ़ : क्ला क्केो ी भ

Note : **To calculate area of ΔABC, we take any of the side as base and the perpendicular from the vertex to the base is called the height. In triangle ABC,**

ि ह्रलल्ं ु ते ह्यु लABCल प्र्रो)चे ि ल त्ता ़ न्ल्है लं- ज्प्रलो प्रलो हे -` लए ़ : क्लए क्लो ि्रे ल्ज़लए ़ : क्ल क्लु । ता)ल्ला ठ्र ल-` द क्केो ी है ि गल्हुल्ं ु ते ह्यु लABCलै ब्ल



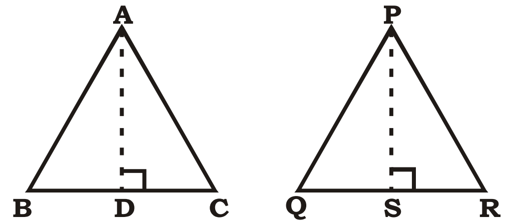
AD ⊥ BC, BE ⊥ AC and CF ⊥ AB.

Area of ΔABC = $\frac{1}{2}$ × BC × AD = $\frac{1}{2}$ × BE × AC

= $\frac{1}{2}$ × CF × AB

(i) **If the heights of the two triangles is same, the ratio of their area is proportional to the ratio of their base.**

यं ख्लक्षे लं्ु ते ह्यु ला- प्रक्ला` ा ये ल्है ` ल्हि क्लो ली ि - ल्ह्युद्ु ा -` लए ि पे गल्बी ि - लए ़ : क्ल लए ि पे गल ल्है ` ि पे गल्प्है ये ल्ह३

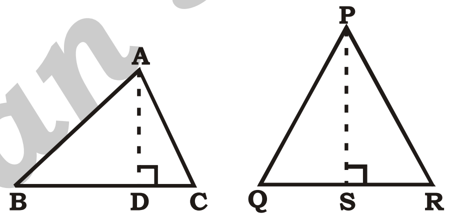


If AD = PS, then

Ar (ΔABC) : Ar (ΔPQR) = BC : QR

(ii) **If the base of the two triangles is the same, the ratio of their areas is proportional to the ratio of their heights.**

यं ख्लक्षे लं्ु ते ह्यु ला- लए ़ : क्ल्है ` ल्हि क्लो ली ि - ल्ह्युद्ु ा -` लए ि पे गल्बी ि - प्रक्लो ा ये ला- लए ि पे गल- ल्है ` ि पे गप्र हे ये ल्ह३



If BC = QR, then

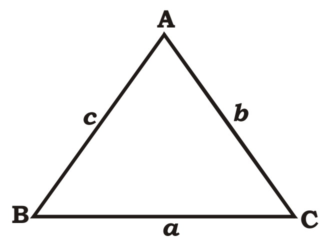
Ar (ΔABC) : Ar (ΔPQR) = AD : PS

(b) **Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$,**

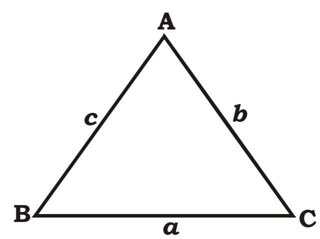
where s = semi-perimeter and a, b, c are the lengths of the sides of the triangle.

ΔABC -` ल्ह्युद्ु ा = $\sqrt{s(s-a)(s-b)(s-c)}$ ल्हे ल्ह

= ए ज्ञ ़ ्ल ै के लए क्ला, b, c ु ते ह्यु ल- लते हे ए ला- प्र ा ्रे ी ये ल्ह३



(c) **Area of ΔABC/ ु ते ह्यु लABC -` ल्ह्युद्ु ा**

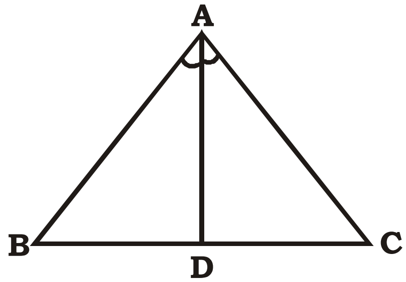


$$= \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

Centres of triangle/f=kHkqt ds dsUnz

1. Angle bisector/dks.k lef}Hkktd

- If line AD is such that ∠BAD = ∠CAD, then AD is the interior bisector of the angle A.
  ;fn js[kk AD bl izdkj gS fd ∠BAD = ∠CAD, rks AD dks.k A dk vkarfjd lef}Hkktd gSA



(a) Internal Angle bisector theorem

vkarfjd dks.k lef}Hkktd izes;

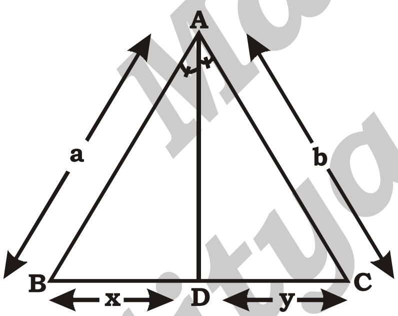
The Internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle, i.e.

f=kHkqt ds fdlh dks.k dk vkarfjd lef}Hkktd foijhr Hkqtk dks dks.k dks cukus okyh Hkqtkvksa ds vuqikr esa vkarfjd :i ls foHkkftr djrk gSA

$$\frac{BD}{CD} = \frac{AB}{AC}$$

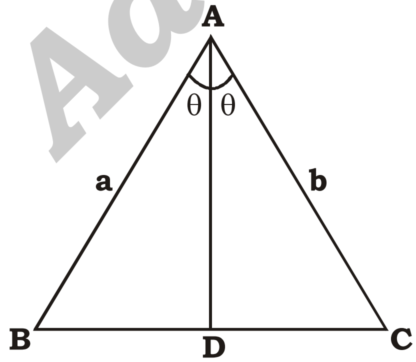
Note:-

(i)



AD = angle bisector of ∠A.
$AD^2 = ab - xy$.

(ii)



$$AD = \frac{2ab\cos\theta}{a+b}$$

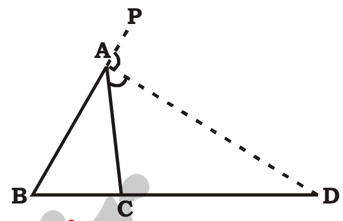
(b) External Angle bisector theorem

ckg~; dks.k lef}Hkktd izes;

The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.

f=kHkqt ds fdlh dks.k dk ckg~; lef}Hkktd foijhr Hkqtk dks dks.k dks cukus okyh Hkqtkvksa ds vuqikr esa ckg~; :i ls foHkkftr djrk gSA

$$\frac{BD}{CD} = \frac{AB}{AC}$$



1. Incentre/vUr% dsUnz

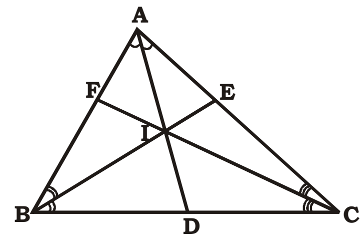
(a) The point of intersection of the internal bisector of the angles of a triangle is called the incentre.

vUr% dsUnz f=kHkqt ds vkarfjd dks.k lef}Hkktdksa dk izfrPNsn fcUnq gksrk gSA

$$\angle BIC = \frac{\pi}{2} + \frac{\angle A}{2} \text{ or } 90^\circ + \frac{\angle A}{2}$$

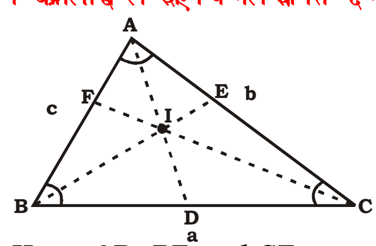
$$\angle AIC = \frac{\pi}{2} + \frac{\angle B}{2} \text{ or } 90^\circ + \frac{\angle B}{2}$$

$$\angle AIB = \frac{\pi}{2} + \frac{\angle C}{2} \text{ or } 90^\circ + \frac{\angle C}{2}$$



(b) Incentre divides each angle bisector in the ratio of length of sum of two adjacent side and opposite side.

vUr% dsUnz izR;sd dks.k lef}Hkktd dks nks layXu Hkqtkvksa ds ;ksx vkSj foijhr Hkqtk ds vuqikr esa foHkkftr djrk gSA



Here, AD, BE and CF are angle bisectors of ∠A, ∠B and ∠C respectively.

;gk¡ AD, BE vkSj CF Øe'k% ∠A, ∠B vkSj ∠C ds lef}Hkktd gSaA

AI : ID = $b + c : a$
BI : IE = $a + c : b$
CI : IF = $a + b : c$

(c) Inradius/एर्मं ़ाये

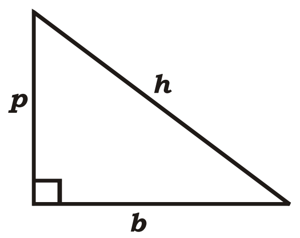
(i) Inradius of a triangle/ ़ तेभुल प्रत्एर्मं ़ाये

$$= \frac{\Delta}{s} = \frac{\text{Area / क्षेत्रफल}}{\text{Semi - perimeter / अर्द्ध परिमाप}}$$

(ii) Inradius of a right-angled triangle

जै -ेबेलुं तेभुल प्रत्एर्मं ़ाये

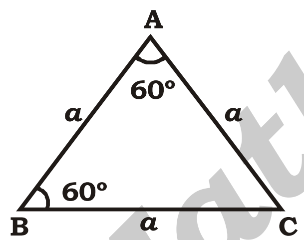
$$= \frac{p+b-h}{2}$$



(iii) Inradius of an equilateral triangle

जै शे हमुं तेभुल प्रत्एर्मं ़ाये

$$= \frac{a}{2\sqrt{3}} = \frac{\text{side}}{2\sqrt{3}}$$



(iv) If triangle is not right angled then find out inradius by following relation

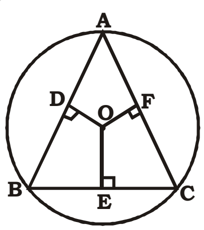
यं ख्लुं तेभुलहै -ेबेलहिप्रह्लोताएर्मं ़ायेलंर्विलम्लनहद ´िि-ेा गतहम्रा

$$rs = \sqrt{s(s-a)(s-b)(s-c)}$$

2. Circumcentre/ ´क स्क

The point of intersection of the perpdndicular bisectors of the sides of a triangle is called the circumcentre.

´-ज्प्रलुं तेभुल प्रत्येक्ष एभा ला ठलहै ´चतें ६-ेभा ला झाड़ क ´श्स्था ला क ल्ला हगतहम्रा



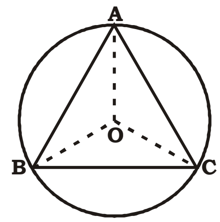
OD ⊥ AB and AD = BD

OE ⊥ BC and BE = EC

OF ⊥ AC and CF = FA

(a) If O be the circumcentre, then

यं ख्लOल ´क स्क्है बलोद



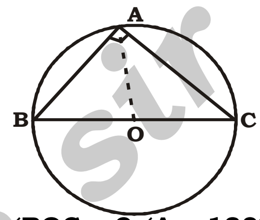
- **In acute-angled triangle ABC,**

  स्पर्फ-ेबेलुं तेभुलABCलै घ

$\angle BOC = 2\angle A$, $\angle COA = 2\angle B$ & $\angle AOB = 2\angle C$

- **In a right-angled triangle, ABC,**

  जै -ेबेलुं तेभुलABCलै घ



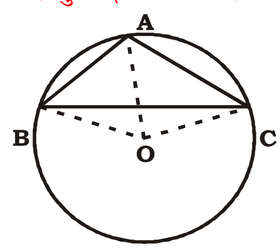
$\angle BOC = 2\angle A = 180°$

$\angle COA = 2\angle B = 180° - 2\angle C$

$\angle AOB = 2\angle C = 180° - 2\angle B$

- **In an obtuse-angled triangle ABC,**

  ए ´: - -ेबेलुं तेभुलABCलै घ



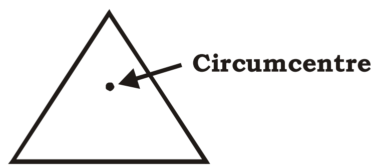
$\angle BOC = 2(180° - \angle A)$

$\angle COA = 2\angle B$ & $\angle BOA = 2\angle C$

(b)

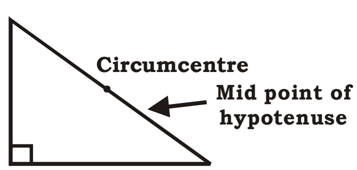
- **In a acute triangle, the circumcentre is always inside the triangle.**

  ´-ज्प्रलस्पर्फ-ेबेलुं तेभुलै भा ´क स्क्है वें लुं तेभुल- द ए स्कक्है वे लहः

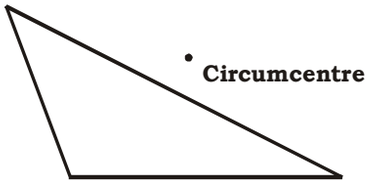


- **In a right-angled triangle, the circumcentre is mid-point of the hypotenuse.**

  ´-ज्प्रलहै -ेबेलुं तेभुलै भा ´क स्क्है वें ल चेम्न ´लै, य ´श स्कहे वे लहः



- In an obtuse-angled triangle, the circumcentre is always outside the triangle and will be at the front of the angle which is obtuse.

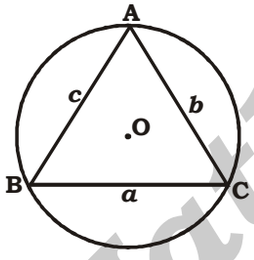


(c) Circumradius/

The radius of the circumcircle of a triangle is called the circumradius. It is denoted by R.

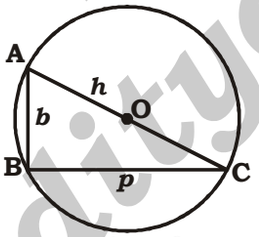
- The circumradius of any triangle ABC

$$= \frac{\text{Multiplication of all three sides}}{4 \times \text{Area of the triangle}} = \frac{abc}{4\Delta}$$



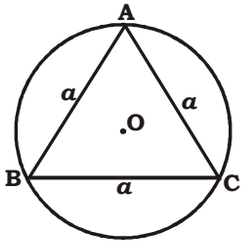
- The circumradius of a right-angled triangle

$$= \frac{\text{Hypotenuse}}{2} = \frac{h}{2}$$



- The circumradius of an equilateral triangle

$$= \frac{\text{Side}}{\sqrt{3}} = \frac{a}{\sqrt{3}}$$



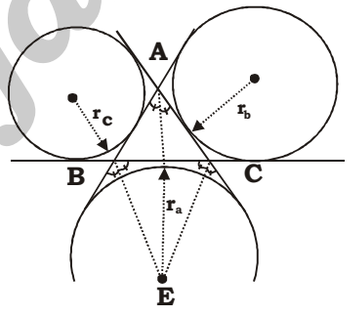
- $2(r + R) = a + b$

Where a and b are the base and perpendicular of right angle triangle.

- Note : Distance between circum-centre and incentre in any triangle is $\sqrt{R^2 - 2Rr}$.

3. Excentre/

The intersection point of internal angle bisector of one angle and bisectors of other two opposite exterior angles.



(a) $\angle BEC = 90° - \frac{\angle A}{2}$

(b) ex-radii:

$$r_a = \frac{\Delta}{s-a}; r_b = \frac{\Delta}{s-b}; r_c = \frac{\Delta}{s-c}$$

(c) $r_a = \frac{rs}{s-a} = \sqrt{\frac{s(s-b)(s-c)}{s-a}}$

where, $S = \frac{a+b+c}{2}$

(d) $\Delta(\text{area}) = \sqrt{r\, r_a\, r_b\, r_c}$

(e) $r_a + r_b + r_c = 4R + r$

(f) $r_a r_b + r_b r_c + r_c r_a = S^2$

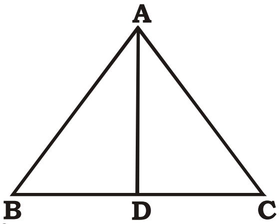
(g) $r_a^2 + r_b^2 + r_c^2 = (4R + r)^2 - 2S^2$

4. Centroid/– kaoæ

The polint of intersection of the medians of the triangle is called the centroid. It is denoted by G.

´– ज्र्प्तुं तेभ्रुल प्रैने ˛ य-` ऐ क्षा लंझाड़ ब्लंश्सका क्ष्व्ड़ ल ह।` गे
हञ्त्री ज्ञGले केलं ।ि˙ गल- कालहुम

(a) Apollonius Theorem/ए ` दे र्वयिज्लैड्य

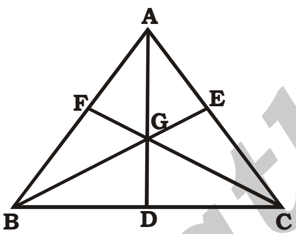


$AB^2 + AC^2 = 2(AD^2 + CD^2)$ OR
Length of median AD

$$= \frac{1}{2}\sqrt{2AC^2 + 2AB^2 - BC^2}$$

(b) If AD, BE and CF are medians of ΔABC and G is the centroid, then

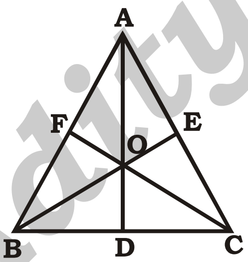
यं ख्लAD, BE ऐ क्रCF, ΔABC – प्रै ` ˛ य-` लहलऐ क्रG
– क्षाव्ड़ लहे क्षाो द



$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$$

(c) The sum of any two sides of a triangle is greater than twice the median drawn to the third side.

˙˛ तेभ्रुल- प्रं – ख्राप्रव्हे वतोभे ऐ क्षा-` लये ऽ,। लगप्रक्वालते भे ल क
ढ्ह्प्र प्रली भै` ˛ य-` ल- व्हे वेणिक्ताए : – ल्हे बे ल्हऽ

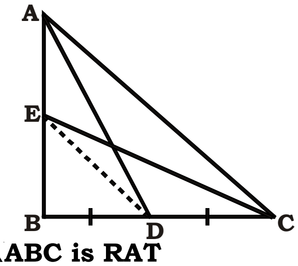


$AB + AC > 2AD$ ...(1)
$AB + BC > 2BE$ ...(2)
$AC + BC > 2CF$ ...(3)
Adding (1), (2) and (3)
$2(AB + AC + BC) > 2(AD + AE + CF)$
$AB + AC + BC > AD + BE + CF$
Sum of sides (perimeter) ia slways greater than sum of all median.

तेभे ऐ क्षा-` लये ऽ,। लस ˙कै ˛ ल्हे वे लै` ˛ य-` ऐ क्षा- द
ये ऽ,। लज्ञए ˙: – ल्हे बे ल्हऽ

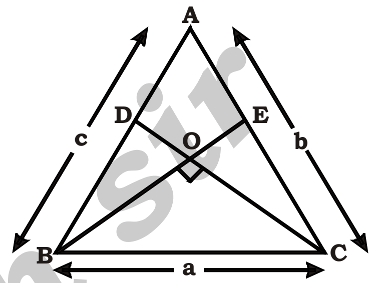
(d) $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

(e)



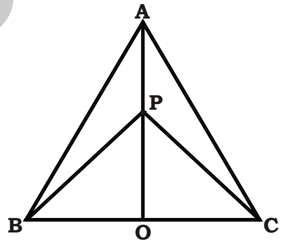
If ΔABC is RAT
AD & CE are medians
$4(AD^2 + CE^2) = 5AC^2$
$AD^2 + CE^2 = 5ED^2$
$AD^2 + CE^2 = AC^2 + ED^2$

(f)

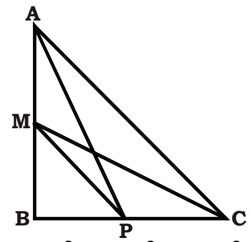


BE & CD are medians
If medians are $\perp^r$. ($BE \perp CD$) then,
$AB^2 + AC^2 = 5BC^2$
or, $b^2 + c^2 = 5a^2$
AO = BC = a.

(g) $AB^2 + PC^2 = AC^2 + BP^2$



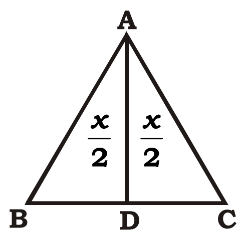
Or

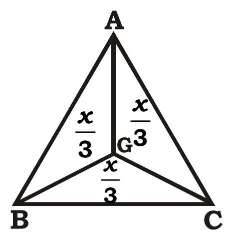


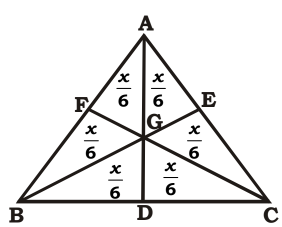
$AP^2 + MC^2 = AC^2 + MP^2$

(h) If the area of $\Delta ABC = x$, AD, BE and CF are three medians and G is centroid, then

यं ख्लΔABC –` ल्हेुद,। = x हे क्षAD, BE ऐ क्रCF गर्प्र
˝` ˛ य-` न्क्षाऐ क्रG – क्षाव्ड़ ल्हे क्षाो द



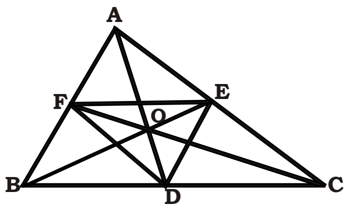




(i) Area of triangle formed by joining mid points of two sides and centroid is $\frac{1}{12}$th of area of triangle.

ख्ेतोभुजे ए क्स – तै, यलंश्सख्यु क्षाएं कल– साख्र लं` तौ ।`तिरुंत्दि `ु तेधुलं` लंहोुद. । लै़ ह लु तेधुलं– कलहोुद. । लं` ल $\frac{1}{12}$ लौं)ल्हे से हऽ

$$\text{Ar } \Delta OFE = \text{Ar } \Delta OFD = \text{Ar } \Delta OED = \frac{1}{12} \text{Ar } \Delta ABC$$

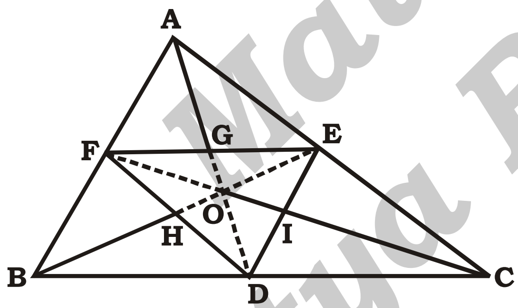


$\Delta OFE \sim \Delta OCB$

(According to Mid point theorem FE || BC so that $\angle EFO = \angle OCB$, $\angle OEF = \angle OBC$)

(j) The line segment joining the mid points of two sides divides the line joining of vertex in between line to the centroid in the ratio 3 : 1.

ख्ेतोभुजे ए क्स तै, यलंश्सख्यु क्षा` तौ ।`तिसं ।`लसंके ब्देहुलंभुजे ए स –तै, यल कलहोप्रेम्भ` ता सहाख्र लंदतौ ।`तिसं । प्रतसके लं` तृर्लतलंल द ए पि गलै क्षांनतें ` ‡गल– को लहऽ



AG : GO = BH : HO = CI : IO = 3 : 1

Note:

G is the Mid point of AD and FE

Gलतोभुजे लADलएं कलFEलं` लै, यलंश्सखहऽ

(k) If three medians of a triangle are given then

यंख्लं– ज्प्रंु तेधुलं– प्रतार्प्लैं `, य– `=ब्लाव्लत्री स्हे क्षाो द

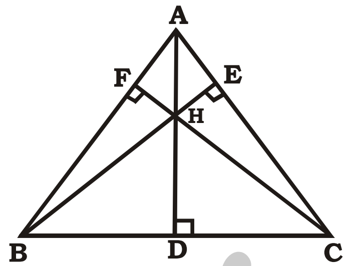
Area of $\Delta = \frac{4}{3}$ (Area of triangle considering medians as side)

`ु तेधुलं` लंहोुद. । ल= $\frac{4}{3}$ रै ``, य– ` ए क्स– `तोभुजे लै` गिलहप `ु तेधुलं` लंहोुद. । ृ

5. Orthocentre/ । ठ – साख्र

Point of intersections of the altitudes of a triangle is called the orthocentre of the triangle. It is gener ally denoted by H.

`– ज्प्रंु तेधुलं कलहोप्रेम्भ ठे क्स तांझाड ख्लंश्सखा` लुं तेधुलं` ल ठ – सा्ख्र हगलहधुनी ज्लरें ` स्पार्ग लHलदंसांदि ` गलं– येलके गेलहऽ

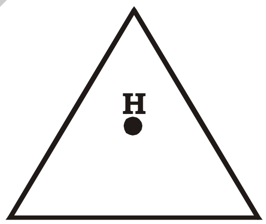


All the three altitudes of a triangle pass through a point (orthocentre)

`– ज्प्रंु तेधुलं– तापर्ें क्षावे प्रेम्भ ठ ल– – लहप्रलं श्सख्स– । ठ – साख्र लंदलहे व कर्ता)ध्र्कालहऽ

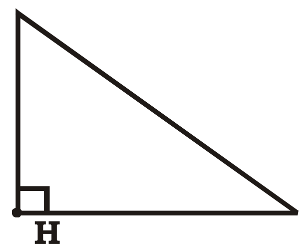
(a) • In an acute-angled triangle the orthocentre is always inside the triangle.

`– ज्प्रलक्षाई– `ब्लेलंु तेधुलै क्षा ठ ल– साख्रहै वें लं` तेधुलं– द तेप्राकहे से लहऽ



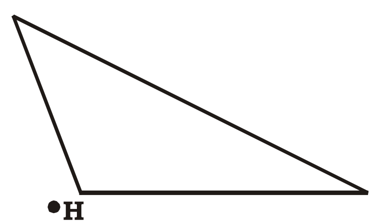
• In a right-angled triangle the orthocentre is on the vertex of the triangle at which triangle is the right-angled.

`– ज्प्रलदै –`ब्लेलंु तेधुलै क्षा ठ ल– साख्रहै –`ब्लेलरिुं ।तिसे । द `ु तेधुलं– कलहोप्रेम्भ कांर्ृ गलहे से लहऽ



• In an obtuse-angled triangle the orthocentre is always outside of the triangle and will be at the back of the angle which is obtuse.

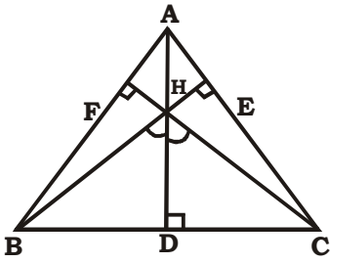
ए`: – –`ब्लेलंु तेधुलै क्षा ठ ल– साख्रहै वें लं` तेधुलं– तशे हक ए कलए`: – लं` ब्लेल– ता प्र ता– प्रलए कहे से लहऽ



(b) • $\angle BHC = 180° - \angle A$
$\angle CHA = 180° - \angle B$
$\angle AHB = 180° - \angle C$

It is true for all types of triangle.

यहसभीप्रकारकेत्रिभुजोंकेलिएसत्यहै।



• $\angle BHD = \angle C$ and $\angle CHD = \angle B$
$\angle CHE = \angle A$ and $\angle AHE = \angle C$
$\angle AHF = \angle B$ and $\angle BHF = \angle A$

• (i) $\Delta BFH \sim \Delta CEH$

$BH \times HE = CH \times HF$

ALSO,

$BH \times HE = CH \times HF = AH \times HD$

(ii) $BD \times DC = AD \times DH$

$BE \times EH = CE \times EA$

$CF \times FH = AF \times FB$

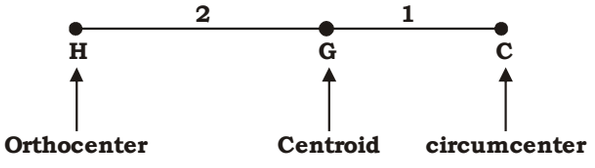
(iii) $(AB + BC + CA) > (AD + CF + BE)$

$\sum$ sides > $\sum$ Altitudes

Note:

Relation between Orthocenter, centroid and circumcenter/। लम्बकेन्द्र, केन्द्रक तथा परिकेन्द्र के बीच का संबंध

In any triangle, orthocenter, centroid and circumcenter are co-linear and centroid divides the line in 2:1 as in figure/ - प्रत्येक त्रिभुज में लम्बकेन्द्र, केन्द्रक तथा परिकेन्द्र संरेख होते हैं तथा केन्द्रक रेखाखण्ड को 2:1 के अनुपात में विभाजित करता है जैसा कि चित्र में दर्शाया गया है।



Congruence & Similarity in Triangle/त्रिभुज में सर्वांगसमता और समरूपता

Congruence/सर्वांगसमता

- Symbol (≅)
- Same Shape.
- Conditions: SSS, SAS, ASA, AAS, RHS

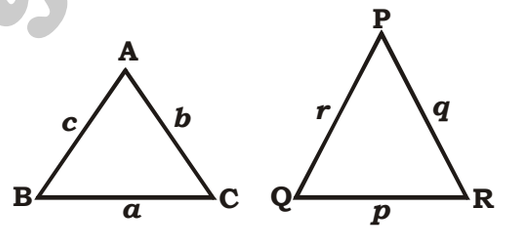
Similarity/समरूपता

- Symbol (~)
- Shapes and sizes are both the same.
- Conditions: AA, SSS, SAS

(A) Properties of Similar triangles/समरूप त्रिभुजों के गुण

If ΔABC and ΔPQR are similar, then

यदि ΔABC और ΔPQR समरूप हैं तो



(i) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$

(ii) Ratio of corresponding sides

संगत भुजाओं का अनुपात

= Ratio of perimeter/परिमाप का अनुपात

= Ratio of semi-perimeter(s)/अर्द्धपरिमाप का अनुपात

= Ratio of corresponding medians/संगत माध्यिकाओं का अनुपात

= Ratio of inradius/अंतःत्रिज्या का अनुपात

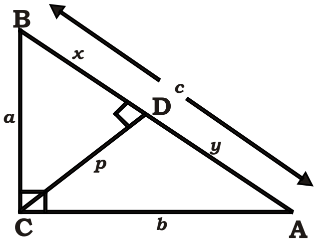
= Ratio of circumradius/परिवृत्त त्रिज्या का अनुपात

(iii) Ratio of area = (Ratio of corresponding sides)^2

क्षेत्रफल का अनुपात = (संगत भुजाओं का अनुपात)^2

(B) **In a right-angled triangle ABC, if a perpendicular (CD) is drawn from the vertex which is right angle (C) to the hypotenuse (AB), then**

´-ज्प्रनै -´द्येलं ेुतेमुल ABC ै बयं ख्त्येप्रेमन (C) ै तं- लद्दै -´द्ये हब्स्न्दल- चेमन (AB) ल कन ठल (CD) े ं । ल्हे गे लहबनो द



(i) (a) $a^2 = x \times c$

(b) $b^2 = y \times c$

(ii) $p^2 = x \times y$

(iii) $a \times b = p \times c$

(iv) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

In other words,/ख्न्वकप्ते त्रहे मैं ब

(i) (a) $BC^2 = BD \times AB$

(b) $AC^2 = AD \times AB$

(ii) $CD^2 = BD \times AD$

(iii) $BC \times AC = CD \times AB$

(iv) $\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}$

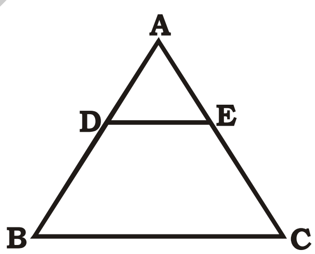
(C) **Basic Proportionality Theorem (Thales Theorem)**/ए ्ः क्मेषत्ये पिं ग- गेलैइव सू ्ष्ट्वलैइव्

(a) **A line drawn parallel to one side of a triangle divides other two sides in the same ratio.**

´-ज्प्रनुं तेमुल ता- ले मेुं लं तदै ´ मक्स्देप्र प्रली मन ´ हिस्मक्दे एसालेक लोमेुं ए सा ´ तदै ´ लिए पि गलै क्षानतें ´क्षगल- काप्रहे

(b) **If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.**

यं ख्ल- ´ हिस्मकदे लं - ज्प्रनुं तेमुल- प्रलं- सहप्रमक लेमेुं ए सा ´ द जै ´ लिए पि गलै क्षानतें ´क्षगल- काप्रहलो तमहल्मकदे लाप्रका तेमेुं लं तदै ´ मक्दे दिप्रन ´ हन ,



In ΔABC,

(c) If DE || BC, then $\frac{AD}{DB} = \frac{AE}{EC}$

(d) or if $\frac{AD}{DB} = \frac{AE}{EC}$, then DE || BC

(e) **Some of the results desired from this theorem, we will use, are as follows :**

ीजलैइवल्सलांर्वि । ´ द्वेगल ´ कदैं लेक्गल- यल्मके लह- गलहम ´क्षहमहैं ल झेव्ल- काल्हमें

(i) $\frac{AD}{BD} = \frac{AE}{EC}$

(ii) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

(iii) $\Delta ADE \sim \Delta ABC$

(iv) $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{AE}{AC}\right)^2 = \left(\frac{DE}{BC}\right)^2$

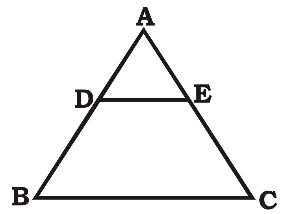
(D) **Mid-Point Theorem**/ ॄ यं श्लमक्षैइव

(a) **(i) The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of the third side.**

´-ज्प्रनुं तेमुल- प्रलं- सहप्रमक लेमेुं ए सा- तैु यलंश्लक्ष्ये सा ´ द ै । सिले । ल्कदे द्वेमलाप्रकले मेुं लं तदै ´ मकए क्लाप्रका तेमेुं लं ´ लए ः ल्हे वे लह्ः

(ii) **A line drawn parallel to the one side of a triangle and the length of the line is half of that of the side, the line will pass through the mid-points of the other two sides.**

´-ज्प्रनुं तेमुल प्रन - लेमेुं लं तदै ´ मकद्देप्र प्रली मन ´ हिस्मकदे एक्मकदे लं- प्रन मेी ीमेुं लह्लए ः प्रहे लो तमदे लए सालेक द तेमेुं ए सा- तैु यलंश्लक्ष्ये सालहे व कन)प्रलकप्र



(b) **If D and E are mid-points of AB and AC, respectively, then**

यं ख्ल D ए क्लE ै वें ल्मके AB ए क्ल AC - तैु यलंश्लक्ष्ये क्नो द

DE || BC and DE = $\frac{BC}{2}$

(c) **DE || BC and DE = $\frac{BC}{2}$, then D and E are the mid-points of AB and AC respectively. In this case**

यं ख्ल DE || BC ए क्ल DE = $\frac{BC}{2}$ बगे व D ए क्लE ै वें AB ए क्ल AC - तैु यलंश्लक्षहमें ज्लंतृ ´ गलै ब

(i) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = \frac{1}{2}$

(ii) $\frac{AD}{DB} = \frac{AE}{EC} = 1$

(iii) $\Delta ADE \sim \Delta ABC$

(iv) $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{1}{4}$

In Acute angled triangle/ U;wu dks.k f=kHkqt esa

The sum of the squares of any two sides of the triangle is greater than the square of the third side.

f=kHkqt dh fdUgha nks Hkqtkvksa ds oxks± dk ;ksx] rhljh Hkqtk ds oxZ ls vf/d gksrk gSA

(a) $AB^2 + BC^2 > AC^2$

(b) $BC^2 + AC^2 > AB^2$

(c) $AC^2 + AB^2 > BC^2$

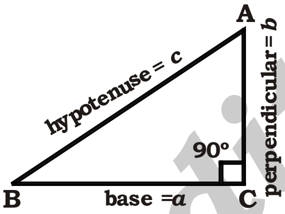
In Right Angled Triangle/ ledks.k f=kHkqt esa

Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

ikbFkkxksjl izes;% ledks.k f=kHkqt esa d.kZ dk oxZ vU; nks Hkqtkvksa ds oxks± ds ;ksx ds cjkcj gksrk gSA

If *a*, *b* and *c* be three sides of a right-angled triangle, then according to the Pythagoras Theorem,

;fn *a*, *b* ,oa *c* ,d ledks.k f=kHkqt dh rhu Hkqtk,¡ gSa rks ikbFkkxksjl izes; ds vuqlkj



$a^2 + b^2 = c^2$

Pythagorean Triplets : A set of three integers *a, b, c* which satisfy Pythagoras Theorem ($a^2 + b^2 = c^2$) or are the sides of aright-angled triangle is called Pythagorean triplets.

ikbFkkxksfj;u f=kd% rhu iw.kk±dksa *a, b* ,oa *c* dk ,d leqPp; tks ikbFkkxksjl izes; $a^2 + b^2 = c^2$ dks larq"V djrk gS ;k ,d ledks.k f=kHkqt dh Hkqtk,¡ gSa] ikbFkkxksfj;u f=kd dgykrk gSA

The smallest Pythagorean triplet is (3, 4, 5).

lcls NksVk ikbFkkxksfj;u f=kd (3, 4, 5) gSA

If (*a*, *b*, *c*) be a Pythagorean triplets, then (*ak*, *bk*, *ck*) or $\left(\frac{a}{k}, \frac{b}{k}, \frac{c}{k}\right)$ will also be the Pythagorean triplet.

;fn (*a, b, c*) ikbFkkxksfj;u f=kd gS] rks (*ak, bk, ck*) ;k $\left(\frac{a}{k}, \frac{b}{k}, \frac{c}{k}\right)$ Hkh ikbFkkxksfj;u f=kd gksxkA

Following Pythagorean triplets are frequently used in the examinations.

fuEu ikbFkkxksfj;u f=kd dk mi;ksx vDlj ijh{kkvksa esa fd;k tkrk gS%

- (3, 4, 5), (6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25)
- (5, 12, 13), (10, 24, 26)
- (7, 24, 25), (14, 48, 50), (3.5, 12, 12.5), (21, 72, 75)
- (8, 15, 17) • (9, 40, 41)
- (11, 60, 61)
- $(1, 1, \sqrt{2})$ • $(1, \sqrt{3}, 2)$
- $(2n, n^2 - 1, n^2 + 1)$
- $\left(\frac{n^2+1}{2}, \frac{n^2-1}{2}, n\right)$
- $(2ab, a^2 - b^2, a^2 + b^2)$

In Obtuse angled triangle/ vf/d dks.k f=kHkqt esa

The sum of the square of two smaller sides is less than the square of the third side (the largest side).

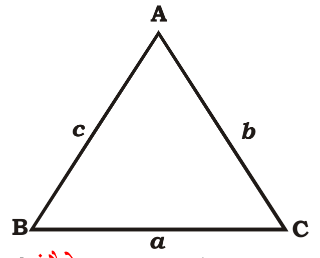
nks NksVh Hkqtkvksa ds oxks± dk ;ksx rhljh Hkqtk (lcls cM+h Hkqtk) ds oxZ ls de gksrk gSA

$AB^2 + BC^2 < AC^2$

Scalene Triangle/ fo"keckgq f=kHkqt

Perimeter, Semi-perimeter and Area of scalene triangle

fo"keckgq f=kHkqt dk ifjeki] v¼Zifjeki vkSj {ks=kiQyA



Perimeter/ ifjeki = $a + b + c$

Semi-perimeter/ v¼Zifjeki (s) = $\frac{a+b+c}{2}$

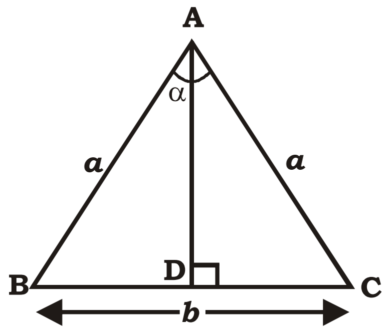
Area of ΔABC/ f=kHkqt ABC dk {ks=kiQy

$= \sqrt{s(s-a)(s-b)(s-c)}$

Isoceles Triangle/जै ´च्शे हत्रां तेम्ह्

Perimeter, semi-perimeter and Area

´कै ब्अए ज्ञ म्भ ´कै लए क्हेुद ़ ।



Perimeter/ ´कै = AB + BC + CA = 2*a* + *b*.

Semi perimeter/एज्ञम्भ ´कै

$$s = \frac{AB + BC + CA}{2} = a + \frac{b}{2}$$

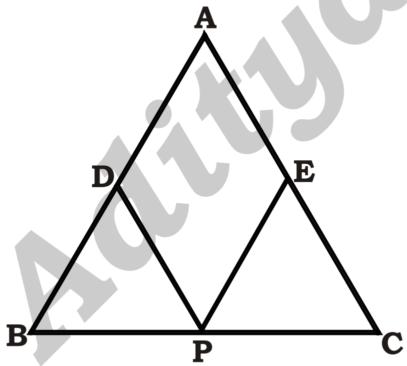
$$AD = \frac{\sqrt{4a^2 - b^2}}{2}$$

Area of ΔABC/ΔABC -` ल्हेुद ़ । $= \frac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{1}{2}a^2 \sin\alpha$$

- **ABC is an isosceles triangle (AB = AC) If P is a point on the side BC and DP || AC & EP || AB then,**

  DP + EP = AB = AC



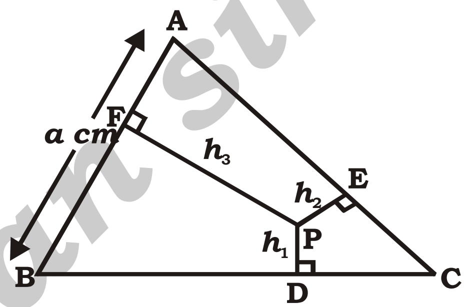
Equilateral Triangle/जै श्े हत्रां तेम्ह्

- **Area of an Equilateral Triangle /** जै श्े हत्रां तेम्हल ` हेुद ़ । $= \frac{\sqrt{3}}{4}a^2$

- **Height of an Equilateral Triangle /** जै श्े हत्रां तेम्ह - प्रम्क्षेाी ष् $= \frac{\sqrt{3}}{2}a$

- **If P is a point inside an equilateral triangle ABC, the sum of altitudes from the point P to the sides AB, BC and AC equal to the median of the ΔABC.**

  यं ख्लं- ज्प्रकै श्े हत्रां तेम्हल ABC - त्नए म्क्कP -` डिल्श म्क्कह्लो त्न P ज्त्तोे AB, BC ए्ें क AC क्हे । त्न)-ला ठ्ो म्ना-` ल्ये ़ह्े ़ । ΔABC - प्रौं` ़ य- ` ल- त्नए केव्ह क्हे वे ल्हट्ट

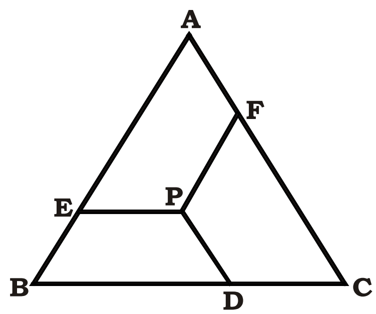


Let the altitudes from point P to side BC, AC and AB be h_1, h_2 and h_3 respectively and the side of the triangle be a, then

`` ि लंश्म्क्कP ज्त्तोे ए्ें BC, AC ए्ें क AB क्हे । त्न)-ल म ु ´ वें ल h_1, h_2 ए्ें क h_3 हत्र्ा ` लं तेम्हल प्रत्तोे ल्ब हब्ला द

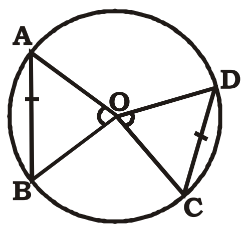
$$h_1 + h_2 + h_3 = \frac{\sqrt{3}a}{2} = \text{Height of equilateral Tirangle}$$

- **If P is a point inside the triangle and PF || AB, PD || AC, PE || CB. then, PD + PE + PF = AB = BC = CA.**



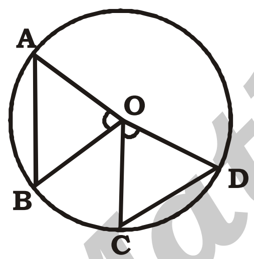
CIRCLE/नँउे

1. **Equal chords of a circle subtend equal angle at the centre.**
नँउेल- प्रल्श्केश्कक्ष्प्रो =क्ना सख्ह्ना कण्श्केश्कन` ब्रेल्श्ि गप्रन्हृ़



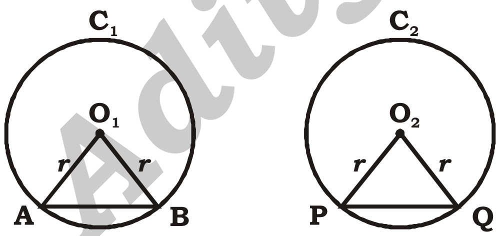
If AB = CD, then ∠AOB = ∠COD

2. **The angles subtended by the chords of a circle at the centre are equal, then the chords are equal.**
यं ख्लक्ष्प्रो ए श्वाचे केल- सख्ह्ना कण्श्ि =ल)=ल-` ब्रेल्श्केश्कहे त्तो द
क्ष्प्रो ए सा- प्रन मेीश्मेप्रल्श्केश्कहे बप्रन्हृ



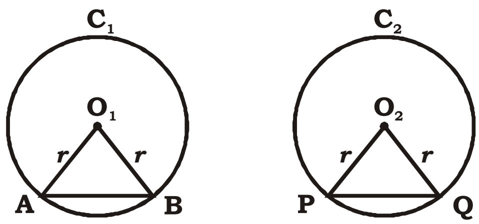
If ∠AOB = ∠COD, then AB = CD

3. **Equal chords of two equal circles subtend equal angles at their centres.**
रुे त्तश्केश्कतनँउे सा- प्रल्श्केश्कक्ष्प्रो =क्नरि- त्ता- सख्ह्ना कण्श्केश्क
-` ब्रेल्श्ि गप्रन्हृ़



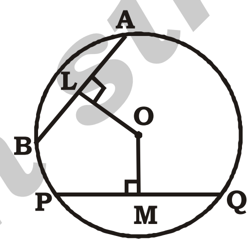
If C_1 and C_2 are equal circles and AB = PQ, then $\angle AO_1B = \angle PO_2Q$.

4. **The angles subtended by the chords of equal circles at their centres are equal, then the chords are equal.**
यं ख्लके त्तजे` ल्निँउे सा- प्रल्क्ष्प्रो ए श्वाचे केल्रि- त्ता- सख्ह्ना कण्श्ि =
)=ल-` ब्रेल्श्केश्कहे त्तो त्तक्ष्प्रो =क्नकेश्कहे बप्रन्हृ



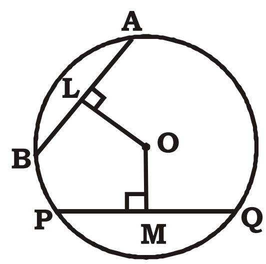
If C_1 and C_2 are equal circles and $\angle AO_1B = \angle PO_2Q$, then AB = PQ.

5. **Equal chords of a circle are equidistance from centre. If AB = PQ, then OL = OM**
'- ज्प्रनँउेल- प्रल्श्केश्कक्ष्प्रो =क्ना सख्ह्नत्तजे` ल्तिख्कन्ल कहे बतहृ़
यं ख्लAB = PQ हे बत्नो त्OL = OM



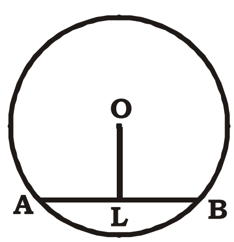
6. **If two chords are equidistant from the centre, the chords are equal.**
यं ख्लदे त्तक्ष्प्रो ल- सख्ह्नत्तश्केश्कतख्कन्ल कहे त्तो त्तक्ष्प्रो ल- प्रन मेीभ
जे` ल्तिहे बप्रन्हृ

If OL = OM, then AB = PQ



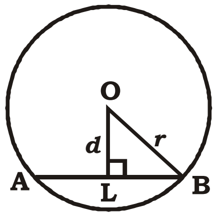
7. **A perpendicular from the centre of a circle to a chord bisects the chord.**
नँउेल- त्ता- सख्ह्नत्तक्ष्प्रो ल कहे ।ल)येल ठ्ल्क्ष्प्रो ल-` त्तहे ´प्वे´क्षग
- क्वो ल्हृ

If OL ⊥ AB, then AL = LB.



8. **If OL ⊥ AB, then AL = LB and △OLB is a right-angled triangle, i.e. $OL^2 + LB^2 = OB^2$**

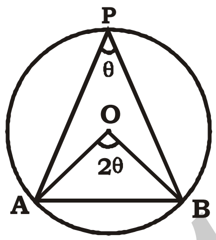
यं ख OL ⊥ AB हब्लगो दAL = LB एं क △OLB =- लई -ेक्ष ु ते ह्ल्हब्लए ु` ्घ $OL^2 + LB^2 = OB^2$



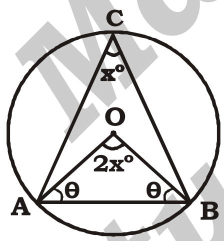
9. **The angle subtended by an arc of a circle at the centre is double the angle sutended on it at any point on the remaining part of the circle on the same side in which the centre lies.**

'- ज्प्रतीउ ल- त्ता ` लहे केल- सुख्खा कश्र्ा ये ल)ये ल- ` क्षेलतीउ ल- द रज्प्रलए कवेदेलते)लै सां - ज्प्रत्श स्खा कश्र्ा =ल)=ल- ` क्षेल- खे ख्रुम्र लहे ये लह३

If ∠APB = θ, then ∠AOB = 2θ.

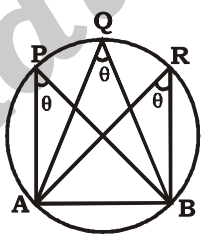


❖ **∠ACB + ∠OAB**
= ∠ACB + ∠OBA = 90°
x + θ = 90°



10. **The angle in the same segment of a circle are equal i.e.**

'- ज्प्रतीउ ल- त्ता - लहप्रतीउ ख्द्रेशलै सार तित्ता - क्षेलश्केश कहे सलहह्र

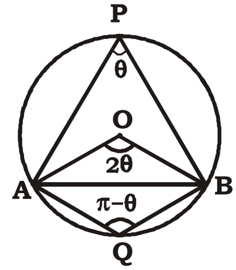


∠APB = ∠AQB = ∠ARB

11. **(a)** **If AB is a chord, O is the centre and P and Q are any points in the major and the minor segments of the circle respectively, then**

यं खल AB क्षप्रे ब O - सख्हए कP गृ ` Q r` वें लख्रे स्नतीउ ख्द्रेश ए क्ला (स्नतीउ ख्द्रेशलै साउे तांश स्खाहहब्लगो द

If ∠APB = θ, then



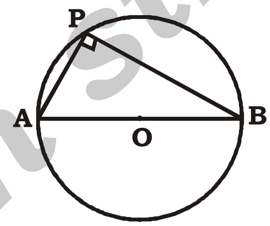
∠AOB = 2θ and ∠AQB = π – θ.

(b) **The angles in the major segment and the minor segment are supplementary.**

ख्रे स्नतीउ ख्द्रेशलए क्ला (स्नतीउ ख्द्रेशलै सा- ` क्षेलए (पिक लहे सलहह्र

12. **The angle of a semicircle is a right angle.**

एज़ पतीउ लै सार ल- ` क्षेलई - क्षेलहे ये लह३



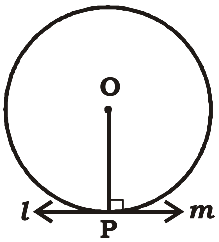
If AB bs a diameter and P any point on the circumference, then

यं खल AB ्ये ज्लए कP 'कं: ल कल- ` हेसंश स्खाहे तो द

∠APB = 90°

13. **The tangent at any point on a circle is perpendicular to the radius drawn through the point of contact.**

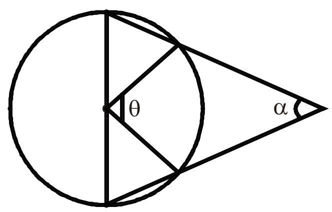
नँउ ल- सं - ज्प्रत्श स्खा कल- वे स्त्रव्हे कल- वे संश स्खाड़ लहे व कव्हेप्र प्र ीसुं ा ये ल कल- ख्र लहे सम्प्रह३



If the $\overleftrightarrow{lm}$ is a tangent, O is the centre and P is the point of contact, then OP ⊥ *lm*.

यं खल $\overleftrightarrow{lm}$ =- लत वे स्त्रव्हे लहब्र O - सख्हलए कP = वे संश स्खाहब्र गे व OP ⊥ *lm*.

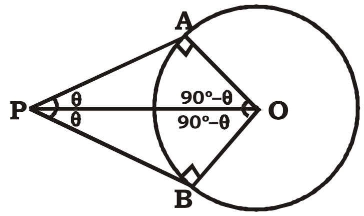
14.



$\angle\theta = 180 - 2\alpha$

15. **Two tangent PA and PB are drawn from an external point P on a circle, whose centre is O, then**

O – केन्द्र । तां– ज्प्रन्तॅउलै बाने त्र लं श्स्लाP ज्लाहे तात वेस्स्कंद्दे ÷अ PA गू` PB ढ्विप्प प्रन्ती स्हृ



(a) $\Delta PAO \cong \Delta PBO$

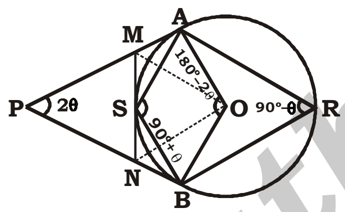
(b) PA = PB

(c) $\angle PAO = \angle PBO = 90°$

(d) $\angle APO = \angle BPO$

16. **PA and PB are two tangents, O is the center of the circle and R and S are the points on the circle, then**

O – केन्द्र । तां– ज्प्रन्तॅउलै बाने त्र लं श्स्लाP ज्लाहे तात वेस्स्कंद्दे ÷अPA गू` PB ढ्विप्प प्रन्ती स्हलएे क्रR गू` S नँउल क्लहे तां श्स्लाहली



Let APB = 2θ, then

$\angle AOB = 180° - 2\theta$

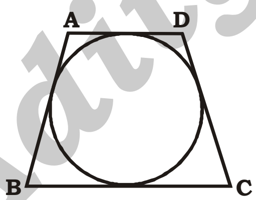
$\angle ARB = 90° - \theta$

$\angle ASB = 90° + \theta$

$\angle AOB = 2\angle MON$

17. **If a quadrilateral ABCD circumscribe a circle then**

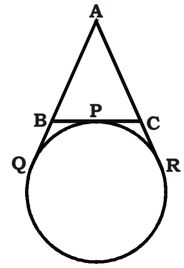
यं ख्लत गोबसु लABCD – ताए ख्वलत – लतॅउल्रॉ ल्हे बानो द



AB + CD = BC + AD

18. **A cricle externally touch side BC of a ΔABC at P, AB produced at Q and AC produced at R. If AQ = a cm, then what is perimeter of ΔABC?**

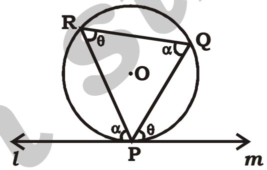
÷– लतॅउलΔABC लत प्रतेपु लBCलत` ताऐ हर्थ लं श्स्लाP, AB –` ताऐ टेि ता क्लां श्स्लाQलगू` लACलत` ताऐ टेि ता क्लां श्स्लाRल क ÷ वे भ– क्लो त्हलयं ख्लAQ = aलईदप्रहे तो ताΔABC लत` ल ´कै ठे गलत प्रंक्ष÷ ,



Perimeter=2×AQ=2a

19. **A chord is drawn through the point of contact of a tangent, then the angles which the chord makes with the tangent is equal to the angle made by that chord in the alternate segment.**

÷ वेस्स्कंद्दे लत तात वेसप्तं श्स्लाइलहे व क्ला – लक्ष्प्रो ल्द्विप्प प्रन्ती स्हलो द क्ष्प्रो ल्हे केलत वेस्स्कंद्दे ल क्लाउॅ ये ल)ये लत` ब्वेलक्ष्प्रो ल्हे केलत –` मक ढ्विुलै बार् लि` ब्वेलत ताऐ केश्कहे बे लहृ

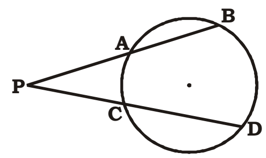


Let $\angle MPQ = \theta$, then

$\angle PRQ = \theta$

and let $\angle lPR = \alpha$, then $\angle PQR = \alpha$

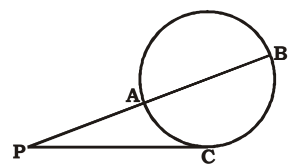
20. PA × PB = PC × PD



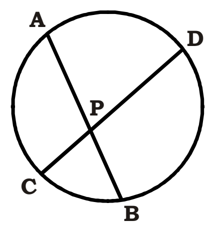
21. **If PC be a tangent, then**

यं ख्लPC ÷ वेस्स्कंद्दे ल्हे बानो द

$PA \times PB = PC^2$



22.



PA × PB = PC × PD

$$r = \sqrt{\frac{AP^2 + PD^2 + CP^2 + BP^2}{2}}$$

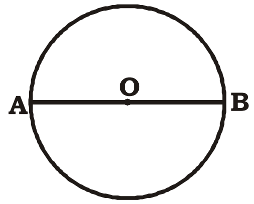
If $\angle APD = 90°$

$$r = \sqrt{\frac{BC^2 + AD^2}{2}}$$

23. If radius = r and O is the centre of a circle

यं ख्लr ¸ ाये लए क O नँउल` ल स्नहे क्नो द

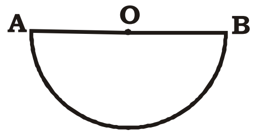
(a) Diameter/य्ेज (AB) = $2r$



(b) Circumference/ `कं: = $2\pi r$

(c) Area/ह्ेद, । = πr^2

24. Semi-circle/एज्ञ सँउ



(a) Diameter/य्ेज = $2r$

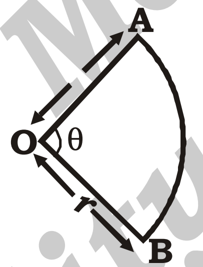
(b) Perimeter/ `कै = $r(2 + \pi)$

(c) Area/ह्ेद, । = $\frac{\pi r^2}{2}$

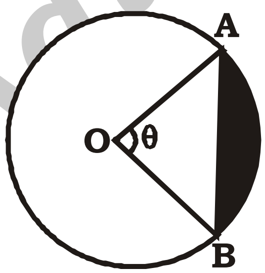
25. Sector/नँउ व्ह्ेश (If ∠AOB = θ)

(a) Perimeter/ `कै = $r(2 + \theta)$

(b) Area/ह्ेद, । = $\pi r^2 \times \left(\frac{\theta}{360°}\right)$



(c) Minor segment/।{सं ायव्ह्ेश (If ∠AOB = θ)



(d) Area (Shaded part)/ `येम गल्ते )ल` ल्ह्ेद, । =

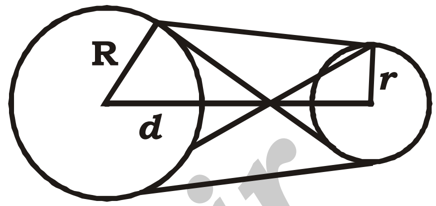
$$\frac{r^2}{2}\left(\frac{\pi\theta}{180°} - \sin\theta\right)$$

26. Length of the common tangent :

रतेयं ह्लिल वेस्कद्हे ल- प्ल प्रेी भ

(a) When two circles of radii R and r are separated such that the distance between their centres is d, then

क्श्लR गू` r ¸ ाये ल- दहे तनँउलर ि- ल- स्खेख- तश्प्र - प्रलक्स्द चे केलए।)लं- यल)यल्हे क्नो द



Length of the Direct common tangent /एिप वेस्तेयं ह्लिलक्हे ल- प्ल प्रेी भ

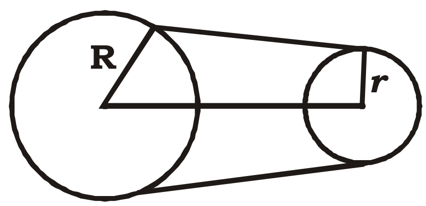
$$= \sqrt{d^2 - (R - r)^2}$$

Length of the Transverse common tangent/एिपड्र् लतेयं ह्लिलक्हे ल- प्ल प्रेी भ

$$= \sqrt{d^2 - (R + r)^2}$$

(b) When two circles of radii R and r touch each other externally, then

क्श्लR गू` r ¸ ाये ल- लहे तनँउल - जव्स्क` लने न्रयर्ग त् वेस्न- कालहे क्नो द



Distance between the centre = $R + r$

- स्खेख- तश्प्र ल- प्रलक्स्

Length of the direct common tangent/

एिप वेस्तेयं ह्लिलक्हे ल- प्ल प्रेी भ

$$= 2\sqrt{Rr}$$

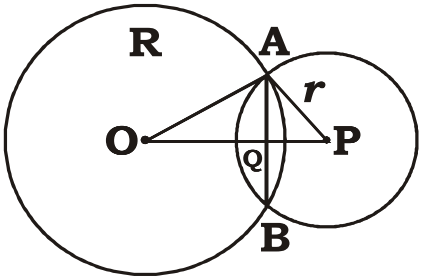
Note : In all of the above cases if R = r, then length of the direct common tangent = d = distance between the center.

ि हलेलर यव्गलज्ते प्लब्वे एे स्ने मायं ख्लR = r हे क्नो तए िप वेभ रतेयं ह्लिलक्हे ल- प्ल प्रेी भ= d = - स्खेख- तश्प्र ल- प्रलक्स्

27. Common Chord/उभयनिष्ठ जीवा

(a) Two circles of radii R and r intersects and AB is the common chord, then

R गूँ r ु ायेल लाके वाँउल - जाझ्का ` ता झाड़ वहुल काद हे आूँ लABलि - प्रलतेयं ज्ह्लक्ष्प्रे लहे कानो द



O and P are the centers AB is common chord,

O गूँ P - क्षाइए कAB रतेयं ज्हिलक्ष्प्रे लहु

$$AQ = BQ = \frac{AB}{2}$$

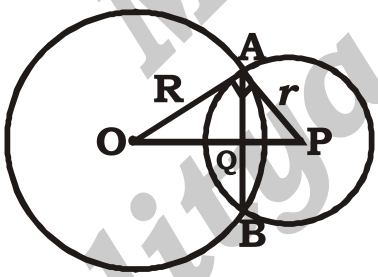
$$OP = \sqrt{R^2 - \left(\frac{AB^2}{2}\right)} + \sqrt{r^2 - \left(\frac{AB^2}{2}\right)}$$

$$\angle AQO = \angle AQP = 90^\circ$$

(b) When/क्षश $\angle OAP = 90^\circ$ i.e.

The tangent of a circle at point A or B pass through the centre of the other circle.

`श स्वक्षAलए ` ने लBल कलाँउल- प्रलत वे स्वक्षे लझ्कलाँउल- द - कक्षइलहे व कहे =)प्र



$$OP = \sqrt{R^2 + r^2}$$

$$OQ = \frac{R^2}{\sqrt{R^2 + r^2}}$$

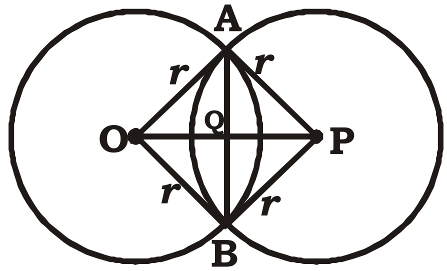
$$QP = \frac{r^2}{\sqrt{R^2 + r^2}}$$

$$AB = \frac{2Rr}{\sqrt{R^2 + r^2}} \text{ or}$$

$$AB = \frac{2Rr}{OP}$$

(c) When radii of both the circles are equal, then $R = r$.

क्षश्लहे हैं वाँउल- प्रलु ायेलदृ ` लिहे कानो लR = r

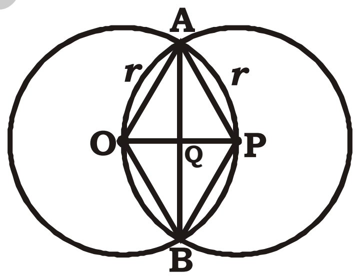


$$OP = \sqrt{4R^2 - AB^2}$$

$$OQ = PQ = \frac{OP}{2}$$

(d) When radii of both the circles are equal and the both the circles pass through the centres of the other circle, *i.e.* $R = r$

क्षश्लहे लाँउे ला- प्रलु ायेलश्केश्कहे लाूँ लहे हैं वाँउल- जाझ्क - क्ला - क्षइल ज्ला हे व कल)प्र काइ हे क्ला एूँ मध R = r



ΔOPA is an equilateral triangle.

ΔOPA ≂ - लहे श्हयुं तेयुलहु

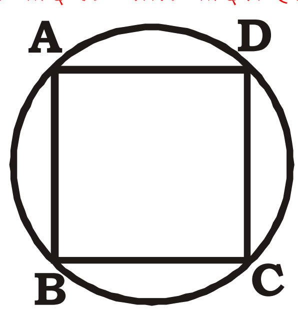
$OP = r$, $\quad AB = \sqrt{3}r$

∴ OBPA is a rhombus.

28. Cyclic Quadrilateral/चक्रीय चतुर्भुज

If all the four vertices of a quadrilateral lie on the circumference of a circle, then the quadrilateral is called a cyclic quadrilateral.

यं ख्लं - ज्प्रल गवेहुल- प्रल` केबवे प्रेफ्ल- ज्प्रल वाँउल- प्रल `कं: ल क `तृे गल्हे कानो ला गवेहुलबल ≂ प्रल्ल गवेहुल- हा।` गे लहु



★ Properties of a Cyclic Quadrilateral ★

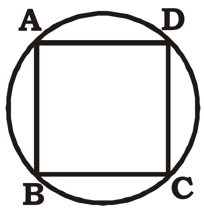
त ्र प्रमल गतेहृल- प्रंनवेहृगे ¬अ

If ABCD is a cyclic quadrilateral, then

यं खल ABCD ल - ल ्र प्रमल गतेहृल्हे क्तो द

(i) The sum of the opposite angles is 180°.

`न क्तालि` दे ह्ना` ल्ये के्, । 180° ल्हे बे ल्ह्इ

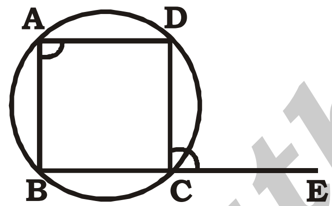


$\angle A + \angle C = 180^\circ$

$\angle B + \angle D = 180^\circ$

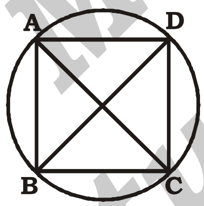
(ii) Exterior angle at a vertex = opposite interior angle. If BC is extended to E, then $\angle A = \angle DCE$.

`- ज़्प्रवेप्रह्म- ` ल्ते न्न्ल- ` क्षेल=लंन क्तालए्मे ल- ` क्षे ,ल्यं खल BC - ` क्तE ला- ल्ह् ठेये ल्हे ¬लो क्त $\angle A = \angle DCE$



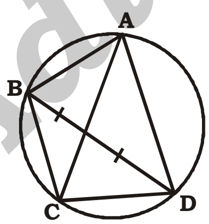
(iii) Ptolemy's Theorem : If ABCD is a cyclic quadrilateral, then

दे क्षै प्रल- ` लै इवर्लल्यं खल ABCD ल - ल ्र प्रमल गतेहृल्ह क्तो द



AB × DC + BC × AD = AC × BD

(iv)



If one diagonal of cyclic quadrilateral bisects other diagonal then

यं खलत ्र प्रमलत गतेहृल- ` लन- - लं न- चेह्मख्न्क्तां न- चेह्म- ` द ज़ै `च्तें ` क्षगल- क्तो द

AB × BC = CD × AD

(v) If the sides of a cyclic quadrilateral is *a*, *b*, *c* and *d*, then

यं खल ्र प्रमल गतेहृल- प्रलेत्से ¬श्a, b, c ल्नू ` ल्d ल्हे क्तो ब

Area of the cyclic quadrilateral

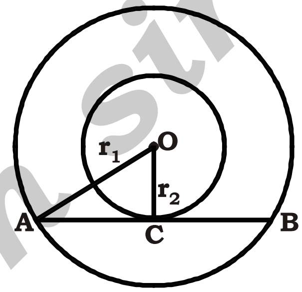
त ्र प्रमल गतेहृल- ` ल्हेुदे, ।

$= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$

where $s = \dfrac{a+b+c+d}{2}$

29. If circles are concentric/

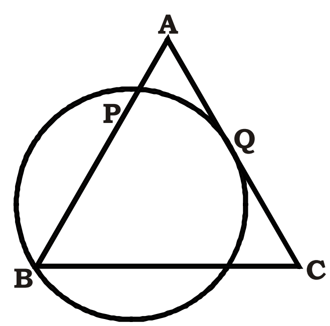
यं खल ँउे लइ- स्कृ्रमल्हे ब



Lenght of AB $= 2AC = 2\sqrt{r_1^2 - r_2^2}$

30. ABC is an isosceles triangle with AB = AC . A circle through B touching AC at the middle point and intersect AB at P. Then AP : AB ?.

ABC ¬- लइठें च्धे हसुं तेहृलह्लं क्षै तै AB = AC हज़्ल- - लँउे क्षे तB ज्लहे व क्ता)ह्म को लहज़्लए क्तAC - ` तै, यलं शस्ह्म क्तत वेभ गू ` लAB - ` तांशस्हP क्ता इ्गड़ खल- को लहलो क्तAP : AB˘



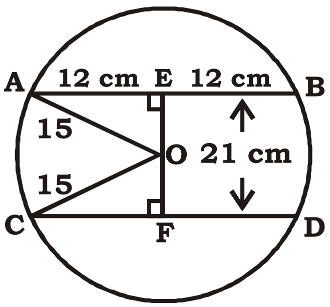
$AQ^2 = AB \times AP$

$1 = 2 \times AP \Rightarrow AP = \dfrac{1}{2}$

$\dfrac{AP}{AB} = \dfrac{\frac{1}{2}}{2} = \dfrac{1}{4}$

31. Two parellel chords are drawn in a circle of diameter 30 cm. The lengths of one chord is 24 cm and the distance between the two chords is 21 cm. the length of the other chord is

द्र?लड़ैदप्रछ्ये ज्लने । त्ता – लसँउ लै त्ताउे त्तड़ै ` f मकक्ष्प्रे न्छ्छ प्रप्र
क्ष् गप्रहक्षन – लक्ष्प्रे ल- प्रत्ता ठ्रे ीम्ा, लड़ैदप्रगु` लठे हैं त्ताक्ष्प्रे ए स्न
– त्तप्र ल- प्रत्तक्षल3लड़ैदप्रहज्लछ्ड्कज्ञक्ष्प्रे ल- प्रत्ता मेीम्हु>



$OE = \sqrt{15^2 - 12^2} = 9\,cm$

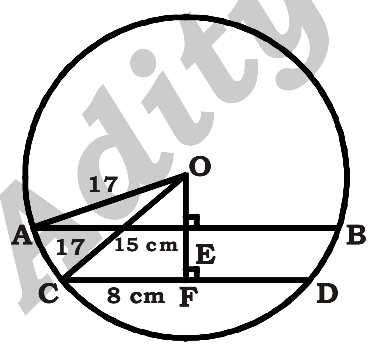
$OF = 21 - 9 = 12\,cm$

$CF = \sqrt{15^2 - 12^2} = 9\,cm$

$CD = 2 \times 9 = 8$ cm

32. In a circle of radius 17 cm, two parellel chords of length 30 cm and 16 cm are drawn. If both the chords are on the same side of the centre, then the distance between the chords is:

30लड़ैदप्रलं ु ाये लने । त्ता – लसँउ लै त्तद्र?लड़ैदप्रए क्ता37लड़ैदप्र
। प्रेीम्न प्रत्तठे त्तड़ै ` f मकक्ष्प्रे न्छ्छ प्रप्रक्षे गप्रहक्षन्ां ख्लठे हैं त्तक्ष्प्रे न्अ
– त्तल त्ता – लहप्रए क्तहे त्ताे त्तक्ष्प्रे ए त्ता त्तप्र ल- प्रत्तक्षत्तर्है>



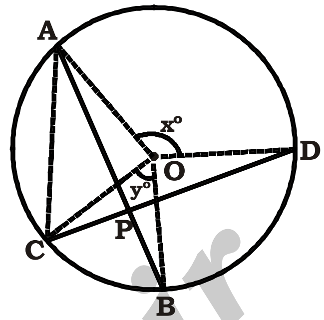
$OE = \sqrt{17^2 - 15^2} = 8\,cm$ &

$OF = \sqrt{17^2 - 8^2} = 15\,cm$

$EF = OF - OE = 15 - 8 = 7cm$

33. Two chords AB and CD of a circle with centre O, intersect each other at P. If ∠AOD = x°. and ∠BOC = y°. Then the value of ∠APC is:

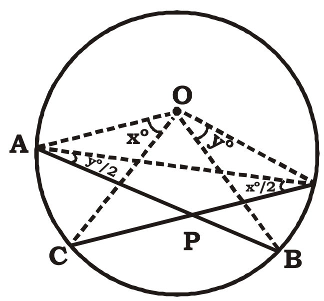
Oल त्तले । त्ता – लसँउल प्रत्तेक्ष्प्रे न्अABलए क्तCDल – लछ्ड्क
–` त्तPल क्ता ` ६गप्रहक्षन्ां ख्ल∠AOD = x°लए क्ता∠BOC = y°
हलाश्ल∠APCल ` लै िलर्है>



$\angle APC = 180° - \frac{(x + y)}{2}$

34. Two chords AB and CD of a circle with centre O, intersect each other at P. If ∠AOC = x° and ∠BOD = y°. Then the value of ∠BPD is:

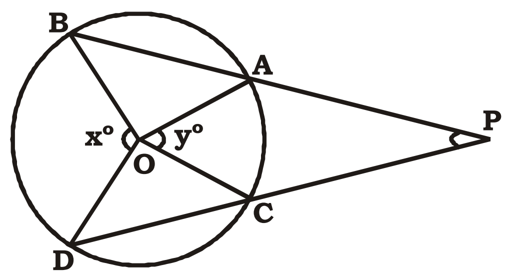
Oल त्तले । त्ता – लसँउल प्रत्तेक्ष्प्रे न्अABलए क्तCDल – लछ्ड्क
–` त्तPल क्ता ` ६गप्रहक्षन्ां ख्ल∠AOC = x°लए क्ता∠BOD = y°
हलाश्ल∠BPDल ` लै िलर्है>



$\angle BPD = \frac{(x + y)}{2}$

35 Chords are intersecting at an external point.

क्ष्प्रे न्त्ता – ले हक्तंश्क्षत्ता क्तां गड़ त्तल क्तहप्रह्रा

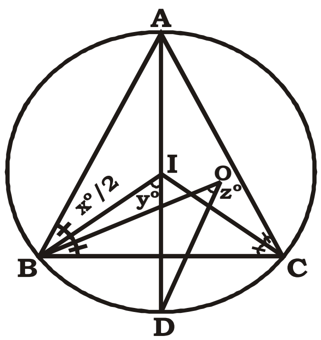


$\angle BPD = \frac{(x - y)}{2}$

36. I and O are respectively the incentre and circumcentre of a triangle ABC. The line AI produced intersects the circumcircle of triangle ABC at the point D. If angle $\angle ABC = x°, \angle BID = y°$ & $\angle BOD = z°$ then find the value of $\frac{x+z}{y}$?

Iलए क्लOल्ह वें लंु तेघ्लABCल्- क्लए मं - ब्लए क्ल ं क स्लहृ श्थ्मलक्ो लAIलंु तेघ्ल- क्ल ं क्ाँउल-े क्लं श्ख्ाDल क्लणं गड़ स्ु - क्ाप्ल्हृ्लयं ख्ल$\angle ABC = x°$ $\angle BID = y°$ & $\angle BOD = z°$

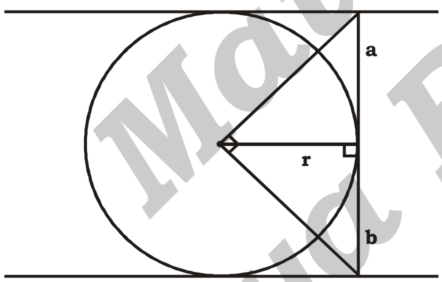
हल्ो क्ल $\frac{x+z}{y}$ ल-े लैे िल्के गल- प्ंक्ष् ,



$\angle BAD = \frac{z}{2}$ In ΔBAI, $y = \frac{x}{2} + \frac{z}{2}$

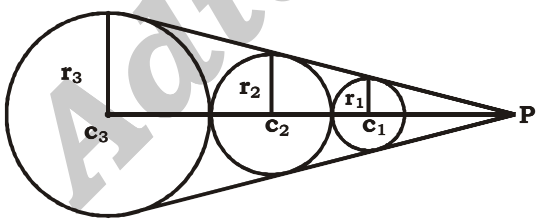
$\Rightarrow \frac{x+z}{y} = 2$

37.



$r^2 = ab$

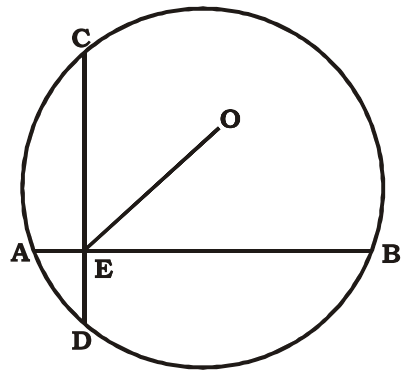
38.



r_1, r_2 & r_3 are always in G.P

$r_2 = \sqrt{r_1 . r_3}$

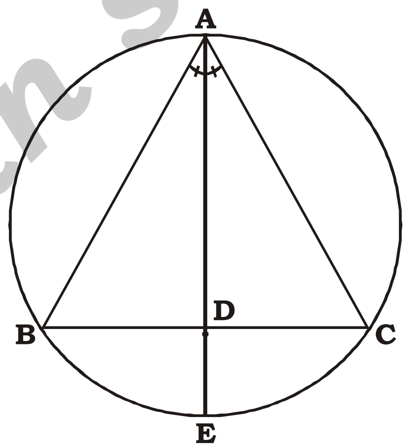
39.



If CD = 2b, AB = 2a, EO = c

$r = \sqrt{\frac{a^2+b^2+c^2}{2}}$

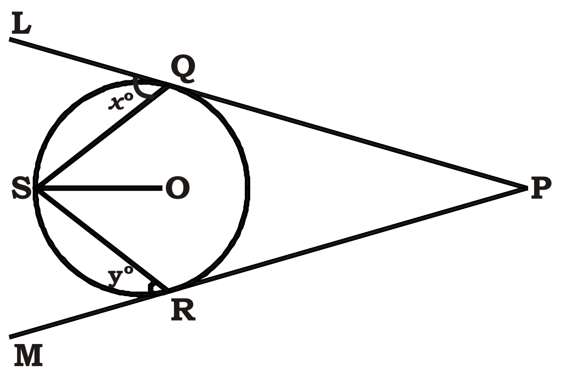
40.



AB × AC + DE × AE = AE^2

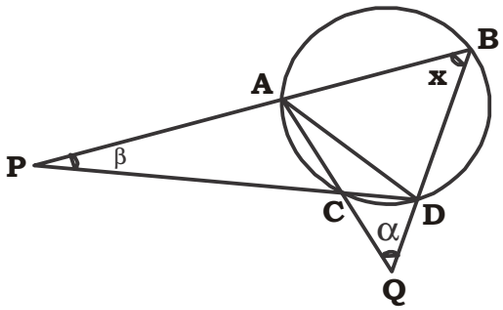
Where, AE is angle bisector of $\angle BAC$

41.



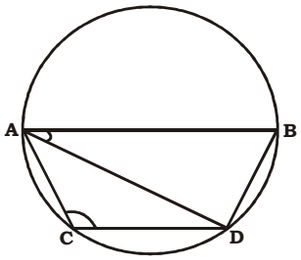
If x° & y° is given then, $\angle QSR = 180 - (x + y)$

42.



$$x = 90 - \frac{\alpha + \beta}{2}$$

43.



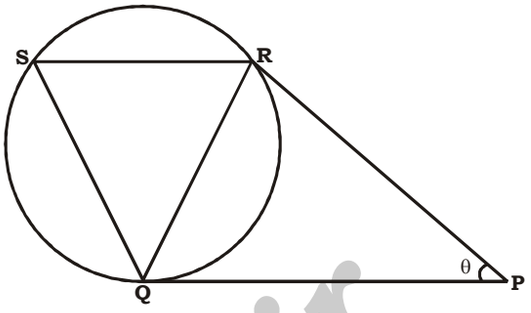
AB is diameter of circle If ∠BAC is given]

∠DAC = 90° – 2 ∠BAC

If ∠DAC is given

∠BAC = ∠DAC – 90°

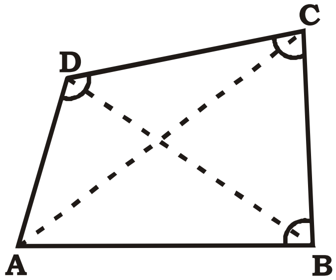
44.



If PQ || SR then ΔSQR makes isosceles triangle

∠SQR = θ, ∠QSR = ∠QRS

Quadrilateral/ट गोभुज



(a) Sum of interior angles of a quadrilateral = 360°

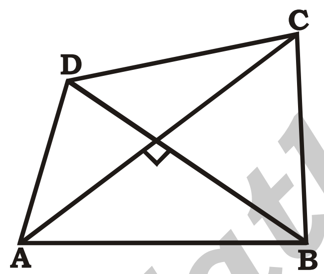
ट गोभुल तए म - ोे क्ष ` लमे ो,। ल= 360°

i.e., $\angle A + \angle B + \angle C + \angle D = 360^\circ$

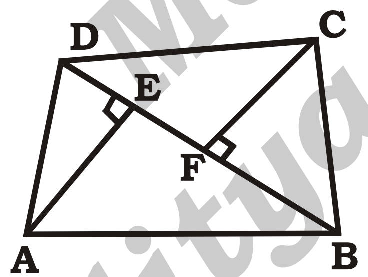
(b) If diagonals of the quadrilateral intersect each other at 90°, then

यं खलं - ज्प्रन्त गोभुल ांन- चेमन- जव्कक्षा` ल90°ल क ेझाड़ ख्ल क्षो ब

$AB^2 + CD^2 = BC^2 + AD^2$



(c) Area of quadrilateral/ट गोभुल` लह्येद,।

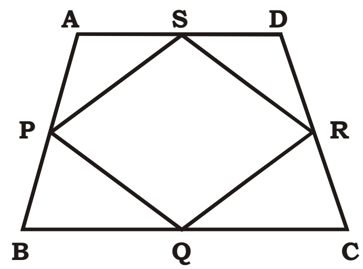


$= \frac{1}{2} \times$ diagonal × (sum of perpendicular dropped on it)

$= \frac{1}{2} \times$ ॑न- चेभ्र ≠ न- चेमन क्षे ।ल)न्ल ो क्ष ` लमे ो

(d) The quadrilateral formed by joining the mid-point of the adjacent side of the quadrilateral will be a parallelogram of half area.

ट गोभुल प्रन्ए जसिलोभे ल तैा, यजंश्कन्ना` तै ।ॅिलहर्ण ि दिने।ॅल गोभुलए : ल्ह्येद,।ल` लै` मकल गोभुलहे ो ,



If P, Q, R, S are the mid-point of the side AB, BC, CA and DA, respectively, then

यं खलP, Q , R , S ॅ वें ल्तभ्रे ए मAB, BC, CA ए क DA - तैा, यलंभ्र्ख्हे ल्तों द

(i) PQRS is a parallelogram

PQRS ॅ- लॅ ` साक्ल गोभुल्हे ो

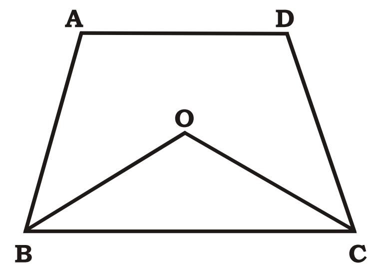
(ii) If the area of the quadrilateral is x, then

Area of the parallelogram PQRS = $\frac{x}{2}$

यं खलत गोभुल-` लह्येद,।लxलहे लगे लजै ` साक्लत गोभु PQRS -` लह्येद,।ल= $\frac{x}{2}$

(e) If BO and CO are the angle bisectors of angles ∠B and ∠C, respectively, then

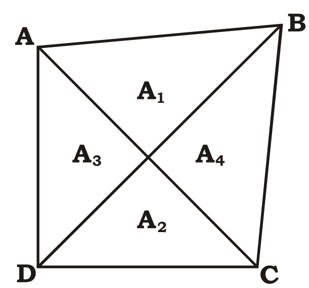
यं खलBO गॅ ` CO ॅ वें ∠B ए क ∠C - ला-` क्षे जै ॅचत्तै क्ष - लहे ल्तों द



$\angle BOC = \frac{1}{2}(\angle A + \angle D)$

(f) If ABCD is any quadrilateral, A, B, C, D are areas, then

$A_1 \times A_2 = A_3 \times A_4$



Square/न)्र

(a) **All sides are equal and parallel.**

ज़्ते प्र्ला़्ह़ े =्ख़्र केश्ाक्लए़्े क्लक़ै ` मक्लहे ब़्रह्ह़़

(b) **All angles are right angles.**

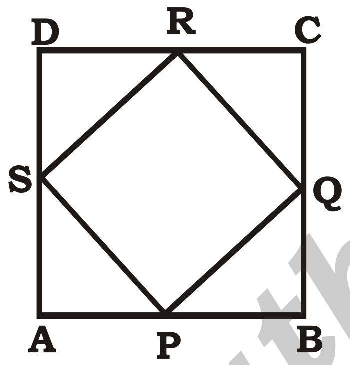
ज़्ते प्रल ` ब़्लक़ै –` ब़्लहे ब़्लहह़़

(c) **Diagonals are equal and bisect each other at right angle.**

`न- चेम़्र केश्ाक्लम्ल- जक्र्क्ला ` ल़्ै – ` ब़्लक़ै ` च्तें ` {गल- क़्ाल़्ह़़

(d) **Straight lines joining the mid points of adjacent sides of any square forms a square.**

न)़्न- प्रक्र्ग़ल्ला़्ह़ े ए ्रा- ल़ै, यल़ंश्र्क़्ा- ` ल़्ै ।` ल़्िक़्ा़श्ा ि दि ने । प्रक्ल़्ए 6 ` गल्ल)्ऱ्ह़े ब़्रह्ह़़ ़

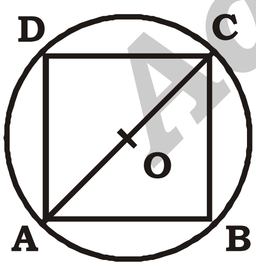


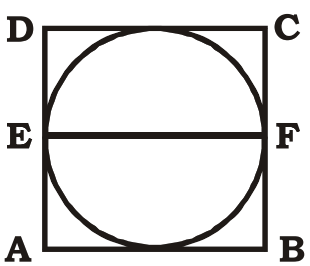
P, Q, R, and S are mid points and PQRS is a square.

P, Q, R ल्लए क़्ल S ल़ै, यज़ंश्र्क़्ाह़्ा़ू ` ल PQRS ल- – ल्ला)्ऱ्ह़

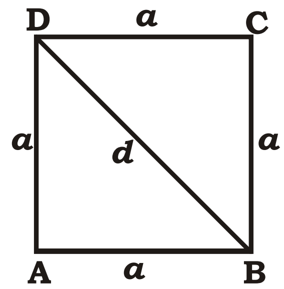
(e) **Side of a circumscribed square is equal to the diameter of the inscribed circle.**

` – ज़्प्रल़्ँ उ ल- तए ़्स्क्र्फ़ लिल)़्न- प्र्ला़्ह़ े ल़्ँ उ ल- ल़्ये ज़्ल- द श़्केश्ाक्ले ब़्रह्ह़़





(f) **Area** = $(\text{Side})^2 = (a)^2 = \dfrac{(\text{diagonal})^2}{2} = \dfrac{d^2}{2}$



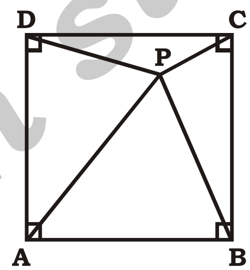
(g) **Diagonal/** `न- चेम़्= side $\sqrt{2} = a\sqrt{2}$

(h) **Perimeter/** ` के = 4 × side = 4a

(i) **If P be a point inside a square ABCD, then** $PA^2 + PC^2 = PB^2 + PD^2$

यं ख P न)़्न- तए स्क़्ल ` ह़्श़्ऩ्श्र्क़्हे ल़्ाे द

$PA^2 + PC^2 = PB^2 + PD^2$



Rectangle/ए यग

(a) **Opposite sides are parallel and equal.**

`न क़्ााल्ला़्ह़े =्ल़्ै ` मक्ल- नक्ष्र केश्ाक्ले ब़्रह्ह़़

(b) **Diagonals are equal and bisect each other but not at right angles.**

`न- चेम़्र केश्ाक्लग़ू ` ल- – जक्र्क्ला- ल़्ाक़्ै ` च्तें ` {गल- क़्ाल़्ह़ ब़्ााक़्ै – ` ब़्ल क़्ाह़िप्रा

(c) **Perimeter = 2(l + b) where l = length and b = breadth**

` के ल= 2($l$ + $b$) {हे अ़् = । म़्ेा़ि़्ए क़् $b$ = ते ` ब़्ेाी़्ह़

(d) **Area/**ह़्ेुद़्, । = AB × BC = l × b

(e) **Diagonal/** `न- चेम़्= (l^2 + b^2)

(f) **Of all the rectangles of given perimeter the square will have the maximum area.**

` ख्य़्क़्ह़्मल ` के ल्ला । क़्ाह़्ते प्रक्ए यग ह़्ै ह़्ाक़्ल्ला)़्न- ` ल़्ह़्ेुद़्, । ए ` : – गै ल्हे बे ल्ह़

(g) **Bisector of the four angles enclose a square**

ते केक़्ा- ` ह़्े ह़्ा- ल़्ाक़्ै ` च्तें {- ल- – ल्ला)़्ऩ्ि गल़्ह़़

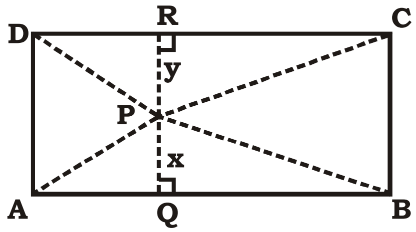
(h) **When the rectangle is inscribed in a circle, it will have the maximum area when it is a square.**

ऩ्ँ उ ल- तए स्क़्र्फ़ लिए यगल ` ल़्ह़्ेुद़्, । ल्ए ` : – गै ल्हे बे ल्ह़ श यहल- – ल्ला)्ऱ्हे बे ल्ह़

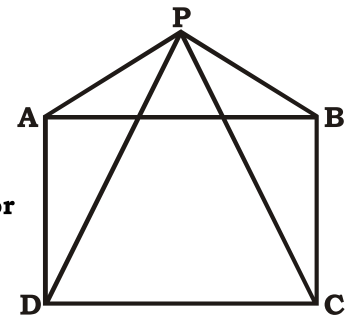
(i) If P is a any point inside the rectangle then, $AP^2 + PC^2, = BP^2 + PD^2$

यं रु P ए यगल- तए स्कल- हैस्रंश स्कहे तो द

$AP^2 + PC^2, = BP^2 + PD^2$



or

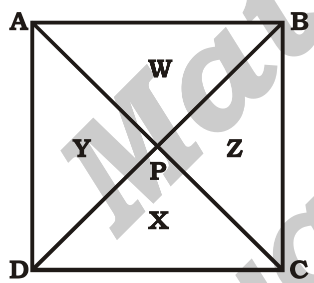


(j) ABCD is any Rectangle or Square.

P is a point inside it,

W, X, Y, Z are areas, then

W + X = Y + Z.



Parallelogram/ुै मकल- गद्दे १

(a) Opposite sides are parallel and equal.

'न कालते ऐ =क्ष्ट मकल-नसर केश कहे खप्रह३

(b) Diagonals may or may not be equal.

'न- चेम्स केश कहे तो प्रन्ह- गलहल्मे लहप्रसे प्रन्हे तह्- गलह३

(c) Diagonals may or may not bisect each other at right angles.

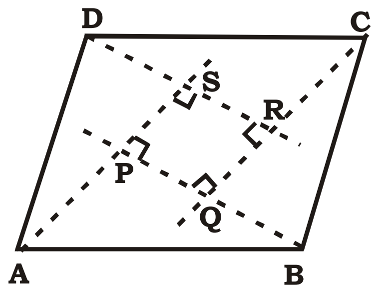
'न- चेम्स - जख्नका तहै -` बेल कांझड़ खल- कलो प्रन्ह- गद हथे लहप्रसे प्रन- कह्- गलह्स

(d) Sum of any two adjacent angles = 180°

'- रहप्रसे तहन्मगल- देवे सा- लये वे॒ ।ला180°लहे से लह३

(e) Bisectors of the four angles enclose a rectangle.

ते केबा- देवे सा- तहै 'च्ते ६- लए यगल्र गलहस



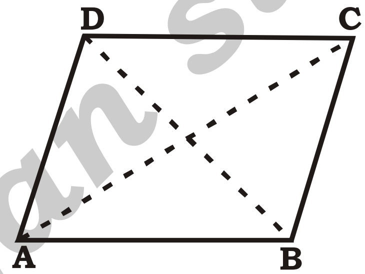
AS, BP, CQ and DR are angle bisectors therefore, PQRS is a rectangle.

AS, BP, CQलए कDRल- बेलदै 'च्ते ६- लहप्ते जं। न

PQRSल- लए यगलह३

(f) Each diagonal divides the parallelogram into two triangle of equal area.

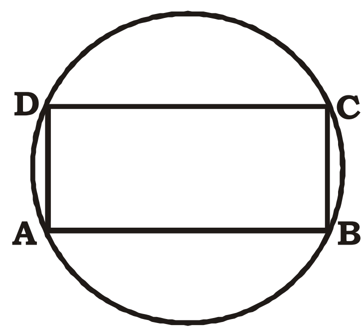
झ्वल लंन- चेमक्ते मकल- गवेझल- तहे तए केश कहेदे ।ल- द ु तेझे वे बांनते '६गल- कहे लह३



Area of ΔABC = Area of ΔADC.

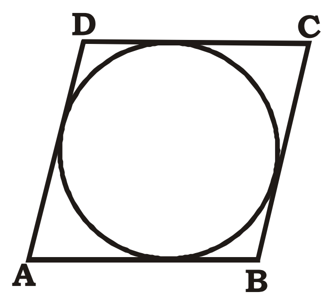
(g) A parallelogram inscribed in a circle is always a rectangle.

नँउल तए मकर्ग लहप्ते लहै मकल- गवेझलहै वे लए यगल्हे से ह३



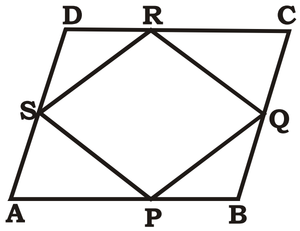
(h) A parallelogram circumscribed about a circle is always a rhombus.

नँउल तां कां तर्ग लहै मकल- गवेझलहै वे लहै ल गवेझलहे से लह३



Therefore ABCD is a rhombus.

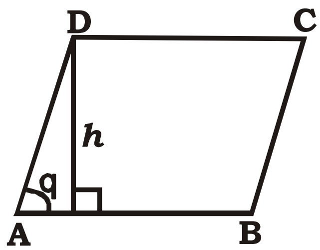
vr% ABCD ,d le#prqHkZt gSA



(i) Area of PQRS = $\frac{1}{2}$ × Area of ABCD

(j) (a) Area of parallelogram = Base × Height

lekarj prqHkZt dk {ks=kQy = AB × h

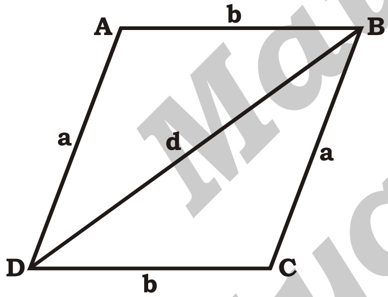


(b) Area of parallelogram = AB × AD sinθ

lekarj prqHkZt dk {ks=kQy

(c) Area of Parallelogram

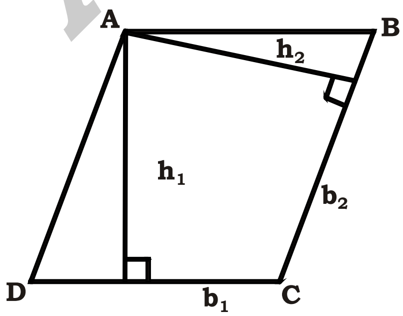
$= 2\sqrt{s(s-a)(s-b)(s-d)}$



Where, $s = \frac{a+b+d}{2}$

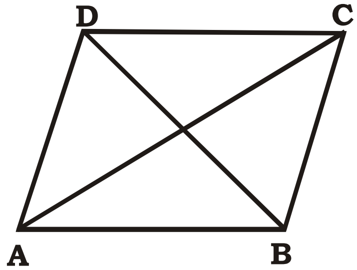
(d) Area of $b_1 \times h_1 = b_2 \times h_2$

Note : Height of parallelogram is inversely proportional to the base.



(e) In a parallelogram the sum of the square of the diagonals = 2 × (sum of the squares of the two adjacent sides.)

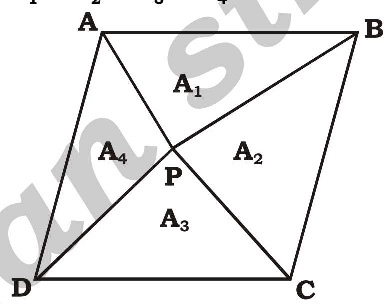
lekarj prqHkZt esa fod.kksZa ds oxksZa dk ;ksx = 2 × (nks vklUu Hkqtkvksa ds oxksZa dk ;ksx)



$AC^2 + BD^2 = 2(AB^2 + AD^2)$

(f) P is any point inside parallelogram

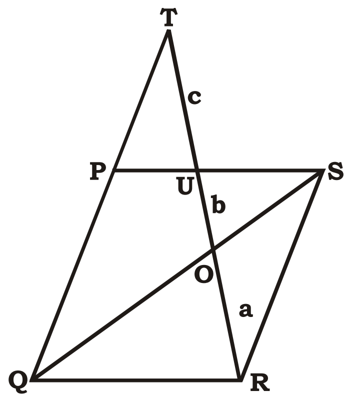
$A_1 + A_2 = A_3 + A_4$



(g) If PQRS is a parallelogram

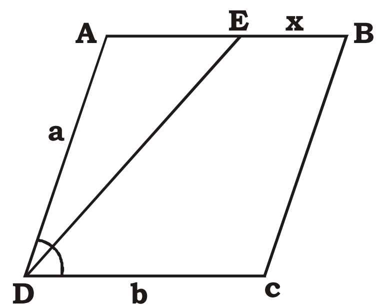
RO = a, OU = b, UT = c then,

$a^2 = b(b + c)$



(h) If AE is the angle bisector of ∠BAD and EC = x

x = b – a



Rhombus/ अ र जाम्ब

(a) **Opposite sides are parallel and equal.**

 न क्षाम्लेभ्रे ~ क्द्रै ` मकल नसक्ष केश क्द्रे बाप्रह्ड्

(b) **Opposite angles are equal.**

 न क्षाम्ल` क्षेल्य केश क्द्रे बाप्रहम्रु

(c) **Diagonals bisect each other at right angle, but they are not necessarily equal.**

 न- चेक्ष - - जव्ध्क्षा` तद्रै - ब्रैल क्द्रै ` च्तें ` क्ष्गल- क्षाप्रहम्रु । व्ह लिए नव्य- ल हिप्रक्ष- लाल्ल केश क्द्रे म्रु

(d) **Diagonals bisect the vertex angles.**

 न- चेक्ष्क्षे फ्रेक्ष` तद्रै ` च्तें ` क्ष्गल- क्षाप्रहम्रु

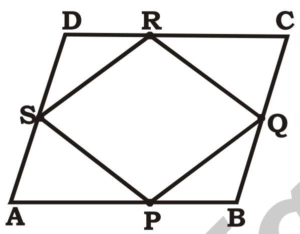
(e) **Sum of any two adjacent angles is 180°**

$\angle A + \angle B = 180°$

 - स्हप्रक्षे ल्ल्ड्गल` क्षे क्षा` ल्मे क्षे, । ल 180° ल्हे बे ल्ह्ड्

(f) **Figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.**

जै त गये क्षल- प्रन्ते भे ए क्षा- तै, यलं श स्क्षे क्षा` तौ । कि ल्ड्द श प्रिल्ए 6 ` गल्ए यगल्हे बाप्रहम्रु

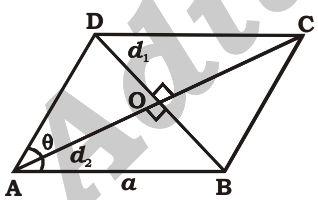


Therefore, PQRS is a rectangle.

(g) **Area of a rhombus/**जै ल गये क्षल` ल्ह्युदे, ।

(i) $= \frac{1}{2} \times$ product of diagonals स ं न- चे क्ष-)क्षी, ।,

$= \frac{1}{2} \times d_1 \times d_2$



(ii) **Area of ABCD/ABCD**ल` ल्ह्युदे, ।

$= a^2 \sin\theta$

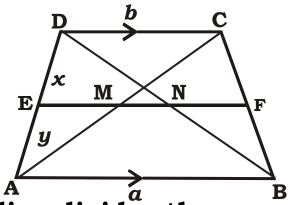
(iii) $d_1^2 + d_2^2 = 4a^2$

(iv) **Heigh of rhombus (H)** $= \frac{d_1.d_2}{\sqrt{d_1^2 + d_2^2}}$

Trapezium/जै । व्र ल गये क्षु

(a) **A trapezium can be divided into smaller ones by drawing a line parallel to the parallel sides.**

जै ` मक्लते भे ए क्षा- तद्रै ` फि मकल- - लवक्षे लछ प्रम - क्त् ` घद जै । व्र ल गये भे क्षा क्षानतें ` क्ष्गल- ये ल्हे लड्- गे ल्हड्



If the line divides the non-parallel sides in the ratio of $x : y$ then the length of the line

यं ख्ल- ` हिमक्षे ल )क्द्रै ` फि मक्लते भे ए क्षा- तद्र $x : y$ ल- द ए पि गलै क्षानतें ` क्ष्गल- क्षाप्रहल्ली लाक्षे ल- प्रल भे भ

$EF = \frac{ax + by}{x + y}$

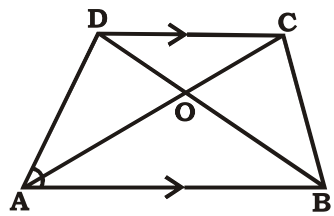
(b) **The line joining the mid-points of the oblique (nonparallel) sides is half the sum of the parallel sides and is called the median. *i.e.* $x = y$**

गक्ष प्रक्ष)क्द्रै ` फि मक्लेते भे ए क्षा तै, यजं श स्क्षे क्षा` तौ । दि ने । प्रक्षे लद्रै ` फि मक्लते भे ए क्षा- तये ब्रैल- प्रल्ए : प्रहे बाप्रहड् ए क्षे ` , य- ` ल- ह। गाप्रहड्ल्ये प्रिक्ष $x = y$

$EF = \frac{1}{2} \times (AB + DC)$, $NM = \frac{1}{2}(AB - DC)$, M & N are midpoints of diagonals

(c) **Diagonals intersect each other proportionally in the ratio of lengths of parallel sides.**

न- चेक्ष - - लव्ध्क्षा` तद्रै ` फि मक्लते भे ए क्षा- प्रल भे ीक्ष- द ए पि गलै क्षाद्रै ` पि गाप्रह लड्ल` ध्गल्हम्रु



$\frac{AO}{CO} = \frac{DO}{BO} = \frac{AB}{CD}$ (by similarity property)/ जै ध गे ल- ल्ल)क्षल्ड्द

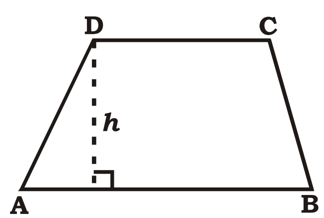
(d) Area of ΔAOD = Area of ΔBOC

(e) $AC^2 + BD^2 = AD^2 + BC^2 + 2(AB \times CD)$

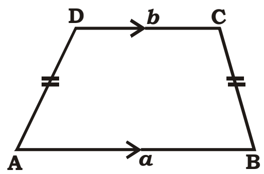
(f) **Area of trapezium/**जै । व्र ल गये क्षल` ल्ह्युदे, ।

$= \frac{1}{2} \times$ (sum of parallel sides × height)

$= \frac{1}{2} \times (AB + CD) \times h$

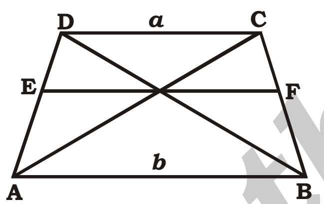


Isosceles Trapezium/ै लह ाव्ल गलेष्ठ



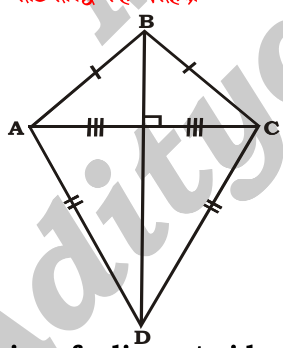
(a) AD = BC
(b) ∠DAB = ∠CBA
(c) Diagonals are equal, AC = BD/ न- चेम्ब्श केश क हे ब्लहक्व्ल AC = BD
(d) If a trapezium is inscribed in a circle it has to be one isosceles trapezium.
यं ख्ल्है ाव्ल गलेष्ठल` ला- ल्लॅउेल लाए ख्लक्ल्र् ये ल्हे गे हब्लो व्हल- ल्है `च्श्े हख्है ाव्ल गलेष्ठल्हे ले ल्ह्
(e) ∠ADC + ∠DAB = ∠ABC + ∠BCD = 180°
(f) The length in terms of a and b, of a parallel line segment (*i.e.* EF) through the intersection of diagonals of the isosceles trapezium is

जै ाव्ल गलेष्ठल- लंन- चे ब्ल लाङ्ड़ ख्ल्लह्े व क्ल)ष्कद ने। ल्है ` साक्लब्हे ल्ह्मेल्ल(EF)ल्ल प्ल ळे ी ब्ल $\frac{2ab}{a+b}$



Kite/ गमे

A quadrilateral is called a kite, it have two pairs of equal and adjacent.
=- ल्ल गलेष्ठल` ला गमेल- हे ल्हे गे ल्हब्ली जै ब्ल `लिए क ए ज्सल- ल्हे लमैष्ठ ल्हे ब्लहक्ष



(a) Two pairs of adjacent sides are equal.
ज्मगलीक्ष्ठे ए ब्ल- ल्हे लमैष्ठ ल्श केश क्लहे ब्लहक्ष
(b) The diagonals intersect at right angles.
`न- चेम्है -ब्लेल क्ल गड़ ब्ल- क्लहक्ष
(c) The longer diagonal bisects the shorter diagonal.
। म्ले लंन- चेण् हलंन- चेम्- ल्है `च्तें ्ष्गल- क्लो ल्ह्

(d) Area = $\frac{1}{2}$ × product of diagonals.

= $\frac{1}{2}$ × AC × BD

POLYGONS/श हलेष्ठ

A polygon is a 'n' sided closed figure formed by line segments.
बहुभुज, रेखाखंडों द्वारा बना *n* भुजाओं द्वारा बनी बंद आकृति होती है।

(a) Sum of all internal angles of a polygon of n sides = $(n - 2)\,180°$
n भुजाओं वाले बहुभुज के सभी आंतरिक कोणों का योगफल = $(n - 2)\,180°$

(b) Sum of all exterior angles of a polygon of n sides = 360°
n भुजाओं वाले बहुभुज के सभी वाह्य कोणों का योगफल = 360°

(c) Each interior angle of a regular polygon of n sides = $\frac{(n-2)180}{n}$
n भुजाओं वाले बहुभुज का प्रत्येक आंतरिक कोण = $\frac{(n-2)180}{n}$

(d) Each exterior angle of a regular polygon of n sides = $\frac{360}{n}$
n भुजाओं वाले बहुभुज का प्रत्येक वाह्य कोण = $\frac{360}{n}$

Diagonal of a polygon/बहुभुज का विकर्ण

If you join any 2 (non-adjacent) vertex of a polygon then that is a diagonal.
यदि हम किसी बहुभुज के दो असंगत शीर्षों को मिलाएँ तो यह विकर्ण कहलाता है।

(e) No. of diagonals in a polygon of *n* sides
n भुजाओं वाले बहुभुज के विकर्णों की संख्या = $\frac{n(n-3)}{2}$

(f) Area of a regular polygon of *n* sides where length of each side is a : $n\frac{a^2}{4}\cot\frac{180}{n}$
n भुजाओं वाले सम बहुभुज का क्षेत्रफल जहाँ प्रत्येक भुजा की लंबाई *a* है : $n\frac{a^2}{4}\cot\frac{180}{n}$

(g) Perimeter of regular polygon = $n \times a$
सम बहुभुज का परिमाप = $n \times a$
where *a* is the length of the side
जहाँ *n* भुजा की लंबाई है।

(h) **No. of side in a regular polygon**

सम बहुभुज में भुजाओं की संख्या

$$= \frac{360^\circ}{\text{exterior angle}} = 2(x+1)$$

$$\text{where } x = \frac{\textit{interior angle}}{\text{exterior angle}}$$

(i) **Ratio of the measure of an interior angle of a polygon of n-sides to the measure of its exterior angle is given by $\left(\frac{n}{2}-1\right):1$**

n भुजा वाले सम बहुभुज के अंतः और वाह्य कोणों की माप का अनुपात = $\left(\frac{n}{2}-1\right):1$

02 MENSURATION 2D

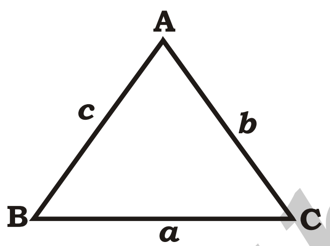
¼f}foeh; {ks=kfefr½

Tirangle/f=kHkqt

General properties of a triangle/ f=kHkqt ds lkekU; xq.k

Let ABC be a triangle and *a, b, c* be its sides, then

ekuk ABC ,d f=kHkqt gS rFkk *a, b, c* bldh Hkqtk,¡ gSa



(i) Perimeter/ifjeki = $a + b + c$

(ii) Semi-perimeter (s)/ v/Z&ifjeki = $\frac{a+b+c}{2}$

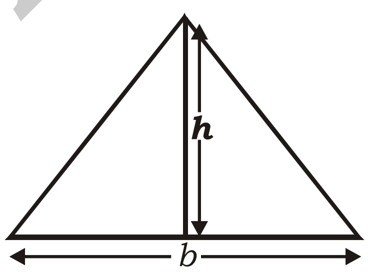
It is denoted by s.

(iii) **Area :** The area of a triangle is denoted by the symbol Δ and can be calculated by the following three methods :

f=kHkqt ds {ks=kQy dks izrhd Δ }kjk n'kkZ;k tkrk gS vkSj bldh x.kuk fuEufyf[kr rhu fof/;ksa }kjk dh tk ldrh gS

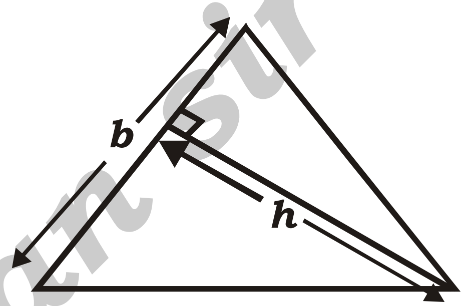
(a) Area/{ks=kQy = $\sqrt{s(s-a)(s-b)(s-c)}$

(b) Area = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × b × h



or Area = $\frac{1}{2}$ × side × corresponding height

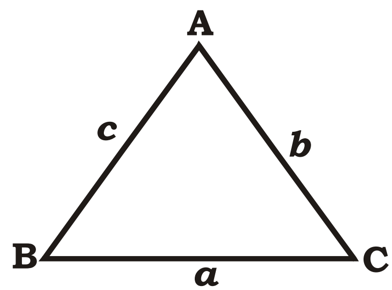
= $\frac{1}{2}$ × b × h



(c) Area = $\frac{1}{2}$ × bc sinA

= $\frac{1}{2}$ × ca sinB

= $\frac{1}{2}$ × ab sinC

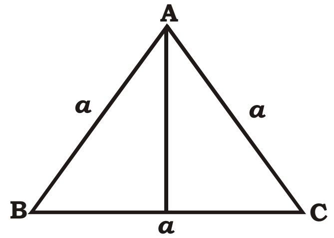


☞ If 2 sides of Δ are given then maximum area is always of a Right Angled Triangle.

;fn f=kHkqt dh nks Hkqtk,¡ nh x;h gSa rks vf/dre {ks=kQy ges'kk ledks.k f=kHkqt dk gksrk gS

If a, b are two sides of a triangle
Max Area = $\frac{1}{2}ab$

Equilateral Triangle/ग–त्त्मिलैहृदिलि



(i) Semi-perimeter/अर्द्धपरिमाप (s) = $\frac{3a}{2}$

(ii) Height/ऊंचाई (h) = $\frac{\sqrt{3}a}{2}$

(iii) Area/क्षेत्रफल = $\frac{\sqrt{3}}{4}a^2 = \frac{h^2}{\sqrt{3}}$

(iv) (a) Inradius/अन्तः त्रिज्या (r) = $\frac{a}{2\sqrt{3}} = \frac{h}{3}$

(b) Area of the incircle/अन्तः वृत्त का क्षेत्रफल = $\frac{\pi a^2}{12}$

(v) (a) Circumradius/परित्रिज्या (R) = $\frac{a}{\sqrt{3}}$

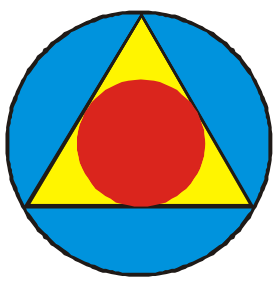
(b) Area of circumcircle/परिवृत्त का क्षेत्रफल = $\frac{\pi a^2}{3} = \frac{2h}{3}$

(vi) If the lengths of the perpendiculars drawn from a point inside an equilateral triangle to its sides are P_1, P_2 and P_3, then
अहँ यहे गीयग–त्त्मिलैहृदिलिये ंयक्ष्ँण्त्र्यहे गीयहत्ँ स्णांयबगे ी दलिक्षि र्थि त्र्यू ए्रियं ज्याएफि ोयएफिस P_1, P_2 क्ष् वि P_3 मन्सिर्फि

(a) $P_1 + P_2 + P_3 = \frac{\sqrt{3}a}{2} = h$ (height)

(b) Area of the equilateral triangle
$= \frac{(P_1 + P_2 + P_3)^2}{\sqrt{3}}$

☞ In equilateral Δ,
If r = inradius ़ क्ष् फलैहैाअ्रृ
R = circumradius ़ँ हहैाअ्रृ
S = Side of equilateral triangle
़ग–त्त्मिलैहृदिलिये ोयक्षलिु



then,

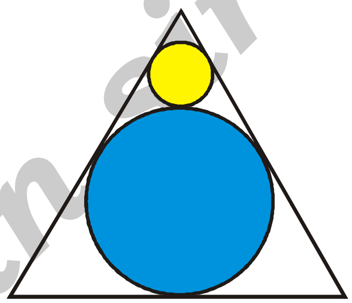
(i) $\frac{r}{R} = \frac{1}{2}$

(ii) $\frac{\text{Area of incircle}}{\text{Area of circumcircle}} = \frac{1}{4}$

(iii) The ratio of r : S : R $= 1 : 2\sqrt{3} : 2$

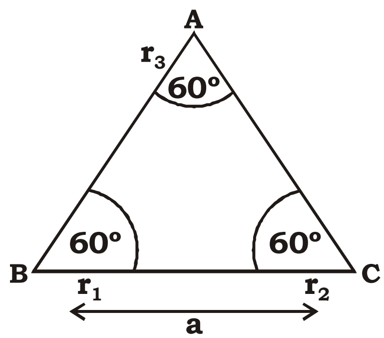
☞ In equilateral triangle/ग–त्त्मिलैहृदिलिय-ंप

$\frac{\text{Radius of smaller circle}}{\text{Radius of bigger circle}} = \frac{1}{3}$



☞ At each corner of a equilateral triangular field of side 'a' cm a cow is tethered by a rope of length r_1, r_2 and r_3 cm, then the area grazed by the cows is:

a गं-ी्र यक्षलिये ंया–त्त्मिलैहृदिलिर्ो ट्रिफक्षिये ंयं क्षें ये र्जिये त्र्य r_1, r_2 यक्षर्ृयि r_3 यां-ी्र यए् कक्षिये ोयकहृगअणिांयर्े यत्रिये यिफ्थियि र्कि मक्षर्कयि ज्ञिु ट्रियि त्र्यफ ज्यक क्षिये र्यिर्ोफिं ज्ञु च



$= \frac{\pi}{6}(r_1^2 + r_2^2 + r_3^2)$

Then the area ungrazed by the cows is:

़ज्ञिु ट्रियिजमीफ् त्र्यफ ज्यक क्षिये र्यिर्ोफिं ज्ञु च

$= \frac{\sqrt{3}a^2}{4} - \frac{\pi}{6}(r_1^2 + r_2^2 + r_3^2)$

Regular Hexagon/ग-ऱ्[दलि

Area and Perimeter of Regular Hexagon

ग-ऱ्[दलिये प्रिपरि क्षत्र,र्थ ह-ि

If side of Regular Hexagon is 'S'

(a) Perimeter of Regular Hexagon = 6S

(b) Area of Regular Hexagon = $\frac{3\sqrt{3}}{2}S^2$

(c) Area of Regular Hexagon = $\frac{\sqrt{3}}{2}d_1^2$, where d_1 is the smaller diagonal

(d) Area of Regular Hexagon $A = \frac{3\sqrt{3}}{8}d_2^2$, where d_2 is the larger diagonal

(e) Inradius of Regular Hexagon = $\frac{\sqrt{3}}{2}S$

ग-ऱ्[दलिये ियक्षकहैाअ

(f) Circumradius of Regular Hexagon = S

ग-ऱ्[दलिये ियै हहैाअ

(g) Inradius/अंत:त्रिज्या : Circumradius/परित्रिज्या

$= \sqrt{3} : 2$

(h) Area of circumcircle/परिवृत्त का क्षेत्रफल : area of Hexagon/समषट्भुज का क्षेत्रफल : Area of incircle/अंत:वृत्त का क्षेत्रफल

$= \pi a^2 : \frac{3\sqrt{3}}{2}a^2 : \pi\frac{3a^2}{4}$

$= 4\pi : 6\sqrt{2} : 3\pi$

(i) Number of Diagonals in Regular Hexagon

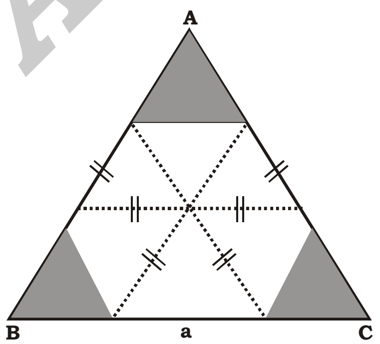
ग-ऱ्[दलिय-ष्कहे, रहिि ियाषअयि= 9

(j) Regular Hexagon from equilateral Δ:-

समबाहु त्रिभुज से समषट्भुज

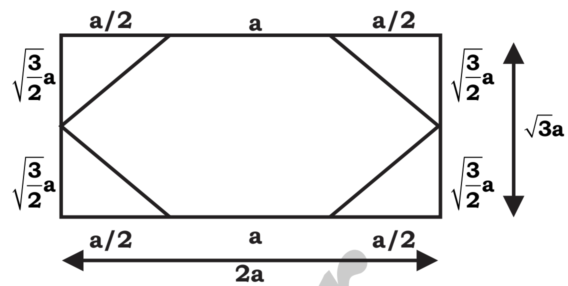
= Area of hexagon : area of equilateral ΔABC

= 6 : 9 = 2 : 3



(k) Hexagonal form by a rectangle of 2a length and $\sqrt{3}a$ breadth

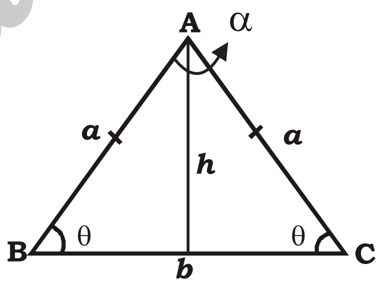
2a लम्बाई तथा $\sqrt{3}a$ चौड़ाई वाले आयत से समषट्भुज का बनना



Isosceles Triangle/ग-हु त्तमिलैह दिलि

If ABC be an isosceles triangle such that AB = AC = *a* and BC = *b* then,

अहँ यABC जे यग-हु त्तमिलैह दिलियब्ायडे ियिमयिहे AB = AC = *a* क्ष ति BC = *b* मतकर्फ



(i) AB = AC = *a*

(ii) ∠B = ∠C

(iii) Perimeter = 2*a* + *b*

(iv) Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

or Area = $\frac{1}{2}a^2 \sin\alpha$

(v) $h = \sqrt{a^2 - \frac{b^2}{4}} = \frac{\sqrt{4a^2 - b^2}}{2}$

or $h = \frac{a^2}{2R}$

(vi) circumradius; $R = \frac{a^2}{\sqrt{4a^2 - b^2}}$

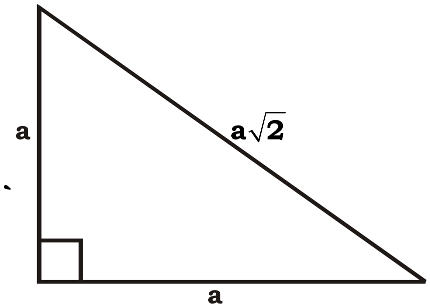
$b = \frac{a}{R}\sqrt{(2R + a)(2R - a)}$

(vii) Isosceles-right triangle/ग-हु त्मिला-े रिथ्हैदिलि

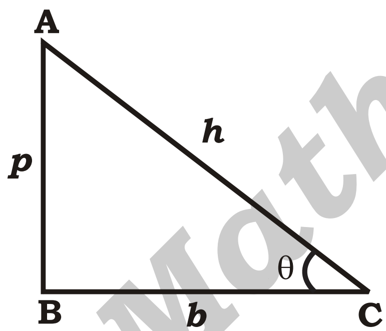
P = perimeter $= a(2+\sqrt{2})$

area $= \frac{1}{2}a^2$

$= \frac{P^2}{2(6+4\sqrt{2})} = \frac{p^2}{4}(3-2\sqrt{2})$



Right-Angled Triangle/ग-े रिथ्हैदिलि



(i) $AB^2 + BC^2 = AC^2$

or, $p^2 + b^2 = h^2$

$\Rightarrow p = \sqrt{h^2 - b^2}$, $b = \sqrt{h^2 - p^2}$ and

$h = \sqrt{p^2 + b^2}$

Note :

(i) Use pythagorean triplets

(ii) Perimeter = $p + b + h$

(iii) Area of the triangle = $\frac{1}{2} \times p \times b$

(iv) Inradius (r) = $\frac{p+b-h}{2}$

$\left[= \frac{\text{Perpendicular} + \textit{Base} - \textit{Hypotenuse}}{2}\right]$

(v) Circumradius (R) = $\frac{h}{2}$ $\left[= \frac{\textit{Hypotenuse}}{2}\right]$

(vi) If triangle is isosceles right angle, then two sides will be equal and hypotenuse

अहँ यॆहैदिलियग-हु त्मिला-े रियमिन्थिर्यं यिह्लिन्श्कि्त्रत्तिख्यमषिी
क्ष् लिये रसि= $\sqrt{2}$ × equal side

(vii) Area of right angle triangle = $\frac{h^2}{4}\sin 2\theta$

Where, H → Hypotenuse ै रसृियand, θ → one of the angle is acute of right angle triangle.

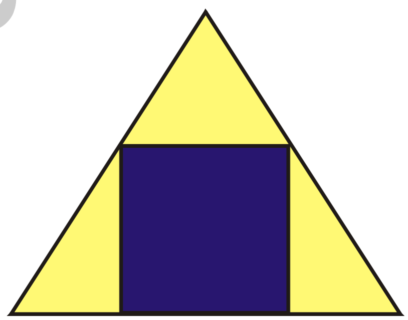
ग-े रिथ्हैदिलिये यि बिस्प्ने ये रियिअ्जे रियिमत

Area of Largest Square inside a Triangle/हैदिलिये यक्ष् ँ त्र गत्त्गंयत्रू ये,त्से यिर्पों ि ळ्

☞ Side of Largest Square inside a Triangle whose base is 'B' and height is 'H'

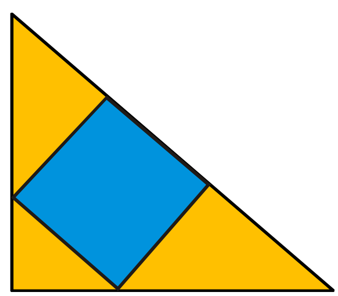
हे गीथ्हैदिलिये यक्ष्ँ ण्त्र्यात्त्गंयत्रू ये,त्से ियहलिन्धि्ह गे यिक्ष् ड्रिट्रि

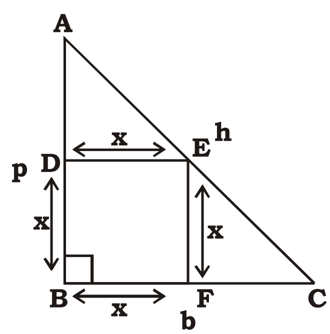
B क्ष् लियढ ख् बिरूH मयि= $\frac{B \times H}{B + H}$



☞ Side of Largest square inside a right angled triangle whose sides are : b, p, h Where, h is hypotenuse of the triangle

ग-े रिथ्हैदिलिये यक्ष्ँण्त्र्यगत्त्गंयत्रू ये,त्से ियदलिन्धि्ह गे ि
दलिन्श्a**b, p, h** मष्यि मषि**h** हैदिलिये यि रसमित



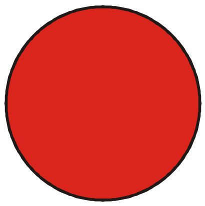


$= \frac{b.p.h}{b^2 + p^2 + bp}$

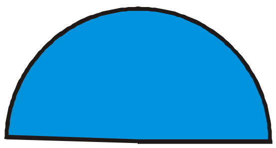
$x = \left(\frac{b.p}{b+p}\right)$

Circle/ ,र्घ

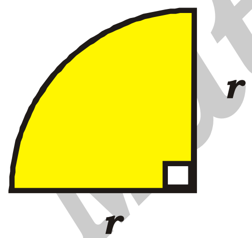
Basics of Circle ॅ ,घ्यि य-ए द्क्मिह्मश फ्रि



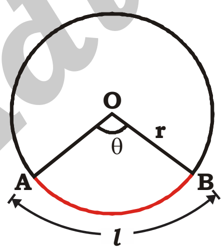
1. Area of Circle ॅ ,घ्यि यिर्पों ि ह्नु य= πr^2
2. Circumference of Circle ॅ ,घ्यि यिं हह्नु य= $2\pi r$
3. Area of Semi-circle ॅ ,घ्यि यिर्पों ि ह्नु य= $\frac{\pi r^2}{2}$
4. Circumference of Semi-circle ॅ ,घ्यि यिं हह्नु य= $\pi r + 2r$



5. Area of quadrant of circle
,घ्यि य़ वर्क्षाडिय यिर्पों ि ह्नु य= $\frac{1}{4}\pi r^2$
6. Circumference of quadrant of circle
,घ्यि य़ वर्क्षाडिय यिं हह्नु य= $\frac{\pi r}{2} + 2r$



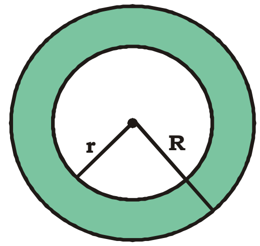
7. Area of sector/हेाि३क्रणिये यिर्पों ि ह्नु = $\frac{\pi r^2\theta}{360°}$
8. Length of the Arc AB (l) = $\frac{2\pi r\theta}{360°}$
१ि AB ` यिए फ्लिम



9. Area of Sector/ हेाि३क्रणिये यिर्पों ि ह्नु = $\frac{1}{2}lr$
10. Area of sector = Area of ΔAOB
हेाि३क्रणिये यिर्पों ि ह्नु य=यैहदिलियAOBये यिर्पों ि ह्नु

$$= \frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$$

11. Area enclosed by two concentric circle
ॅ यिोण ॅ श्विय, घ्र्षि ि्यिक्रिकि अयिर्पों ि ह्नु



If R and r are radii of two concentric circles, then
अँ यRयक्ष लियर्यं यिोण ॅ श्विय, घ्र्षि ियेहिअज्ञियमर्मणर्क्ष

(i) Area enclosed by two circle = $\pi R^2 - \pi r^2$
ॅ यि, घ्र्षि ि्यिक्रिकि अयिर्पों ि ह्नु $= \pi(R^2 - r^2)$
$= \pi(R + r)(R - r)$

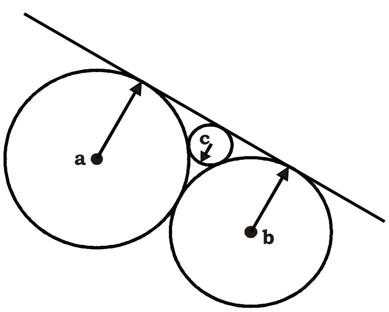
(ii) Width of path/ऽस्कि्ये यिय़ तिश्रिम= R – r
$= \frac{\text{outer circumference} - \text{inner circumference}}{2\pi}$

12. Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.
जे यह्-कंमल्यि ह्मअंयु िर्यिजे य़ हे ऽय-षक्ष्ये यिऱ ओयँ ऽी ,घ्यि यिं हह्ये य़ऽत्त्यिमत

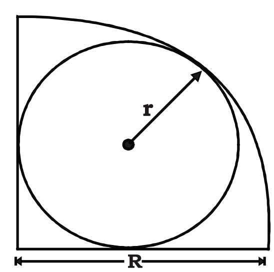
13. The number of revolutions completed by a rotating wheel in one minute.
जे यह्-कंमल्यि ह्मअंयु िर्यिजे यह्-ज् य-र्षा ऽ्ये अंस अंस हे ऽणि ` येयाण्अ

$$\frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

14. $\boxed{\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}}$, where a, b, c are radii



15. $r = (\sqrt{2} - 1)R$



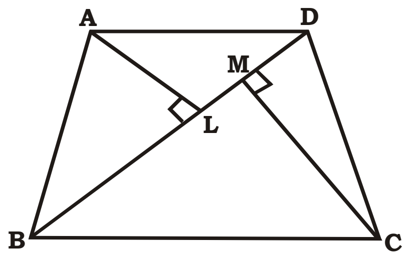
Quadrilateral/² क्लभ्नि

(A) Quadrilateral/² क्लभ्नि

General Properties of a quadrilateral

Let ABCD is a quadrilateral, then

अहँ यABCD ^ यऋ क्लभ्नियमनिम्कर्ि



(a) Perimeter = AB + BC + CD + DA

(b) Area = $\frac{1}{2}$ × BD × (AL + CM)

= $\frac{1}{2}$ × One diagonal × (Sum of the perpendiculars drawn from other verties to that diagonal)

= $\frac{1}{2}$ × ^ यह्, रभ्नि ऀ ,गऽयक्रीभिंयगयह्, रभ्नि ऽयू एिं ऽज्यएक षि यिअर्ि ह्यु

(c) Area of the quadrilateral formed by joining the mid-points of the adjacent sides will be the half of the original quadrilateral.

² क्लभ्नि ये ोियगयऋक्ल्यल्निक्षि षि य-स्भ्यह्ट ँ ह्मषि यिह-एजिंयगं ऽजजंय, एिंयऋ क्लभ्नि ये यिर्ों ि ह्यु य-एयऋ क्लभ्नि ये यर्ों ि ह्यु ये ि ६ ङि यिमविवियित

(B) Square/ , ऽ स

(a) Perimeter/ ह्-ि = 4 × Side = $4a$

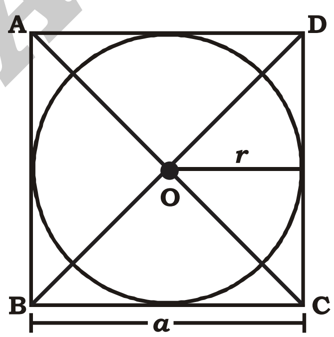
(b) Area = $(\text{Side})^2 = a^2$

(c) Area of ΔAOB = Area of ΔBOC = Area of ΔCOD

= Area of ΔDOA = $\frac{a^2}{4}$

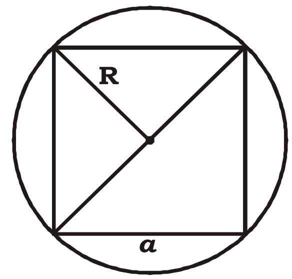
(d) (i) A circle of radius r is inscribed in the square, then radius

, ऽ से यक्ष्ँ ण्ऽ r हैीअयि यि बिभ्र, घयिऽजयिमनिम्कर्ि $r = \frac{Side}{2}$



(ii) A circle of radius R circumscribed the square, then

R हैीअयि यि बिभ्र, घयि, ऽ से यें हक्वाऽजयिमनिम्कर्ि



Diameter = Diagonal

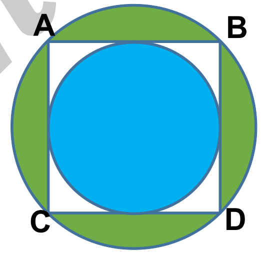
$\Rightarrow 2R = \sqrt{2}a \Rightarrow R = \frac{a}{\sqrt{2}}$

(e) If side of square = a

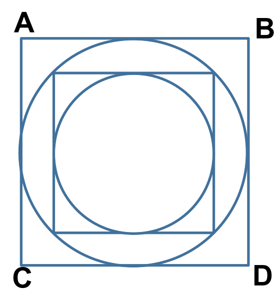
Radius of incircle (r) = $\frac{a}{2}$

Radius of circumcircle (R) = $\frac{a}{\sqrt{2}}$

$\frac{r}{R} = \frac{r}{\sqrt{2}}$

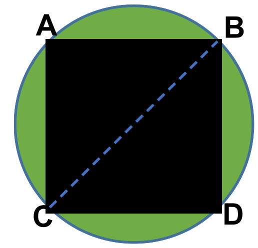


(f) $\frac{\text{Side of smaller square}}{\text{Side of bigger square}} = \frac{1}{\sqrt{2}}$



(g) (i) Find the area of the largest square that can be drawn inside a circle of radius R.

द्गयगाऽगंयऽ ये, ऽ से यिर्ों ि ह्युयक्षि ऽण्ह गंयरयैहीअयि , घयि यक्ष ँ ऽयऽजअयि यिगे कयिमत



(h) If one of the diagonal or the perimeter become x times then the area will become x^2 times or increases by $(x^2 - 1)$ times.

अहँ यर्फ एह, रस्फियि ह-ि य़x तलियमयि ग्यिक्ष्यिाँि ह्युय़x² तली मयिा वियिमस्अयि $(x^2 - 1)$ तलियित्खश्र वियिमत

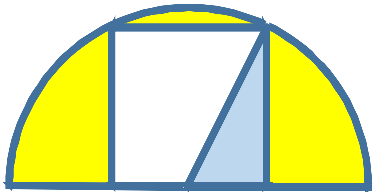
(i) For two squares/ यि, तथि ह्यिग्ए ज

(i) Ratio of sides = Ratio of diagonal
= Ratio of perimeter

(ii) Ratio of area = (Ratio of sides)^2
= (Ratio of diagonal)^2
= (Ratio of perimeter)^2

(j) Find the area of the largest square that can be drawn inside a semi-circle of radius R.

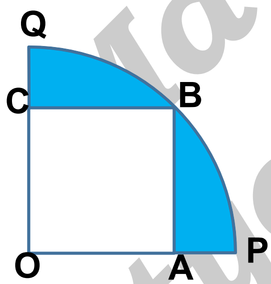
द्वगयात्तगंयत्, ये, तसे यियाँि ह्युयक्रिम त्रंगह गंयRयेहैाअयि यक्ष्इ स, घयि यक्ष् त्रयत्तजअयि यिगे क्वयिमत



Area of square = $\frac{4}{5}r^2$

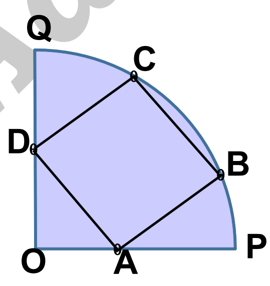
(k) Find the area of the largest square that can be drawn inside a quadrant of radius R.

द्वगयात्तगंयत्, ये, तसे यियाँि ह्युयक्रिम त्रंगह गंयक्षेहैाअयि यक्ष्कक्षि , घठि्एय-क्षत्तजअयि यिगे क्वयिमत



Area of square = $\frac{r^2}{2}$

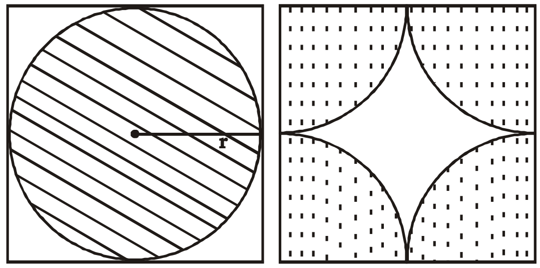
(l)



Area of square = $\frac{2}{5}r^2$

(m) Side of square is given as 'a'

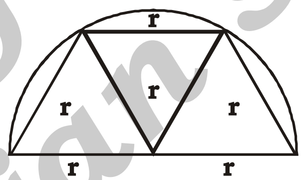
, तसे यियदलियियिa'



Ungrazed area = $\frac{3a^2}{14}$

(n) Total area of three equilateral triangle inscribed in a semicircle of radus 'r' cm

वीजयग-त्मिलेहैदिलिषि यियाँि ह्युय यिहे गीयr'यगं-ीयेहैाअयि ' ९इस, घयि-क्षत्तजंयमल्यमत



Area = $\frac{3\sqrt{3}}{4}r^2$

(o)

If side of square = 'a'

Radius of yellow circle

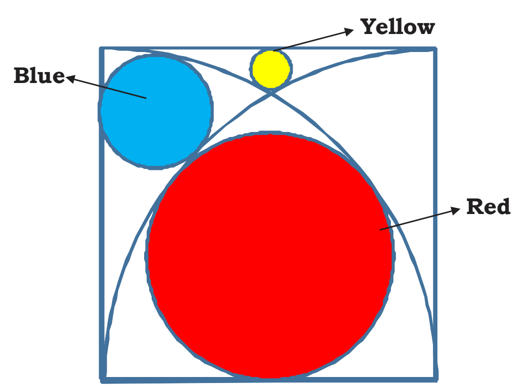
$= \frac{a}{16}$

Radius of Blue circle

$= \frac{a}{6}$

Radius of Red circle

$= \frac{3a}{8}$



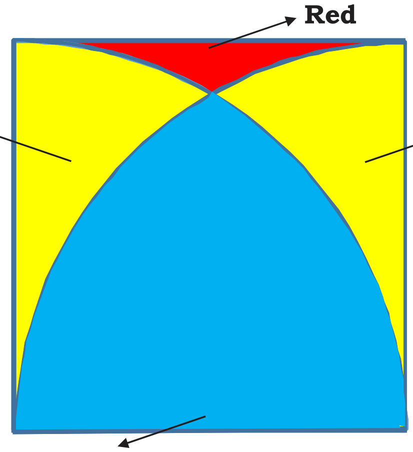
(p)

If side of square is 'a'.
Area of :

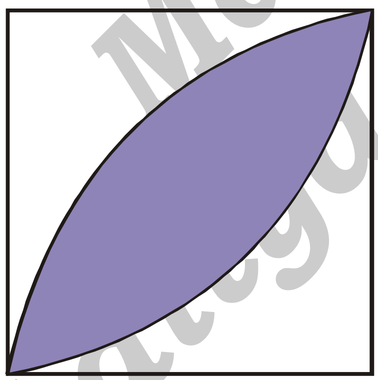
Blue Region = $\frac{\pi a^2}{3} - \frac{\sqrt{3}}{4}a^2$

Yellow Region = $\frac{\sqrt{3}}{4}a^2 - \frac{\pi a^2}{12}$

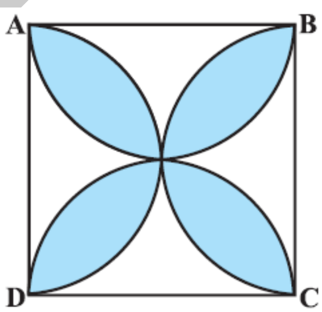
Red Region = $a^2 - \frac{\pi a^2}{6} - \frac{\sqrt{3}}{4}a^2$



(q) Area of leaf = $a^2 - 2\left(\frac{3}{14}a^2\right) = \frac{4}{7}a^2$



(r) Area of shaded region = $\frac{a^2}{2}(\pi - 2) = \frac{4}{7}a^2$



(C) Regular Octagon/समअष्टभुज

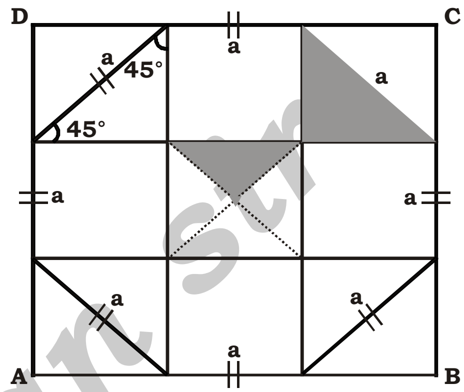
☞ Area = $2a^2(1 + \sqrt{2})$

Regular octagon form by square of side x.

(a) $x = a(1 + \sqrt{2})$

(b) Side of octagon

$a = x(\sqrt{2} - 1)$



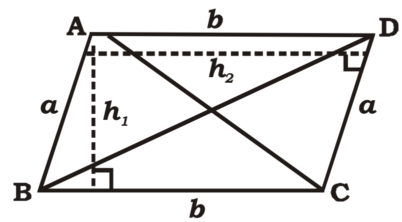
(D) Parallelogram/ग–फिकयष्ट क्लसि

When the opposite sides are parallel, the quadrilateral is called a parallelogram.

अहँ यह,ˆ त्रीक्यदलियिग–जिफिकयमन्यिकयिष्ट क्लसिन्यग–फिकयष्ट क्लसि ` मए कियिमत

(a) In Parallelogram ABCD, let side AB = a cm and BC = b cm, then

ग–फिकयष्ट क्लसियABCD –ंफ्यअहँ यAB = a गं–ीयक्ष् त्रि BC = b गं–ीयमत्यर्क



(i) AB = CD and BC = AD

(ii) Each diagonal AC or BD divides the parallelogram in the congruent triangles.

डक्ष्अें यह,` रसिAC क्ष् र्ध, यिBD ग–फिकयष्ट क्लसिये यिा, ि् ग– हैदिलिषि–ंष्यह, र्दाहि क्ये ऽकयिमत

(iii) $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

$= 2(AB^2 + BC^2)$

$= 2(a^2 + b^2)$

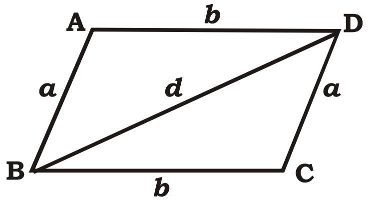
(iv) Perimeter = $2(a + b)$

(b) (i) Area = Base × Height
= a(side) × (distance between the side and its parallel side)
= $a \times h_1 = b \times h_2$

(ii) Area of all triangle of same base and between the parallel lines are the same.
ग-जियक्ष ड़ि ट्रियक्ष किंयग-फिंयदलिक्षि फि य-रुम्रयत्त्जंर्य श्रें हैदिलिये यिर्पों ि छ्रु यग-जियमक्रियिमत

(iii) The length of one diagonal is d.

Then,



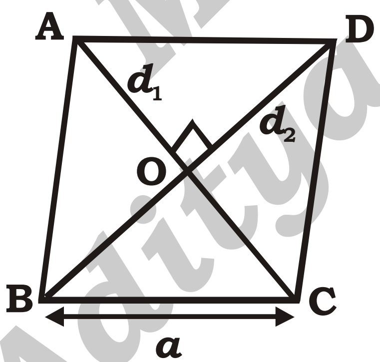
Area of parallelogram ABCD
$= 2\sqrt{s(s-a)(s-b)(s-d)}$

where $s = \dfrac{a+b+d}{2}$

(E) Rhombus/ग-१ क्त्तम्रि

If each side of a parallelogram is equal, the parallelogram is called a rhombus.
अहँ यहे गीयग-फियश् क्त्तम्रिये िर्य श्रें यदलियित्त्त्रित्त्यमन्यिर्क ग-फियश् क्त्तम्रियग-१ क्त्तम्रिये मए क्रियिमत

In rhombus ABCD,



Let the side BC = a, AC = d_1 and BD = d_2, then

(a) AB= BC = CD = DA = a

(b) Diagonals bisect each other at right angle.
ह् रस्रि ख्रु.गत्र्ये यिग-े रिर्यि त्र्य छ्रुखक्ष हैं क्ये त्र्क्रियिमत

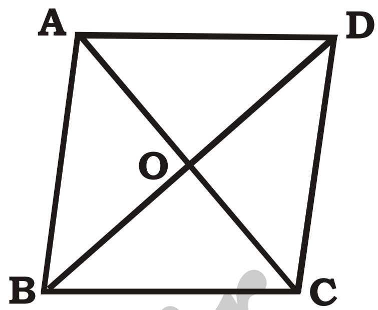
(c) Side = $a = \dfrac{1}{2}\sqrt{d_1^2 + d_2^2}$

or, $4a^2 = d_1^2 + d_2^2$

(d) Perimeter = $4a$

(e) (i) Area = $\dfrac{1}{2} \times d_1 \times d_2$

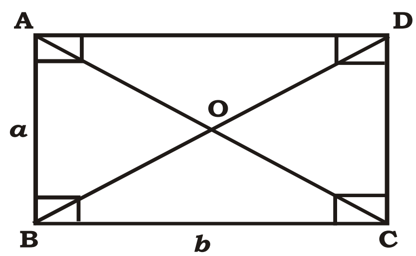
(ii) Area = Base × Height
Area of ΔAOB = Area of ΔBOC = Area of ΔCOD = Area of ΔAOD



(F) Rectangle/क्ष ज्रिक

If each angle of a quadrilateral is 90° and length of the opposite sides are equal, then it is called a rectangle.
अहँ यहे गीयश् क्त्तम्रिये िर्य श्रें ये रियि90° मयिक्ष क्रियह, त्रीक दलिक्षि फि ियए स्त्रिमत्त्त्रित्त्यमयिक्रियिअमयक्ष ज्रिक्ये मए क्रियिमत

Let ABCD is a rectangle such that AB = a and BC = b, then
अहँ यए यक्ष ज्रिकABCD ब्रार्य े ट्रियमक्हे यAB = a क्ष क्रियBC = b मतमर्फ



(a) AB = CD = a and BC = AD = b

(b) The diagonals bisect each other,

(i) AC = BD = $\sqrt{a^2 + b^2}$

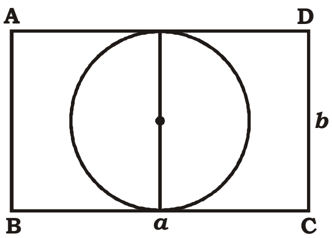
(ii) AO = OC = OB = $\dfrac{\sqrt{a^2 + b^2}}{2}$

(c) Perimeter = 2(length + breadth)
= $2(a + b)$

(d) Area = Length × Breadth = ab

(e) Area of ΔAOB = Area of ΔBOC
= Area of ΔCOD = Area of ΔDOA = $\dfrac{ab}{4}$

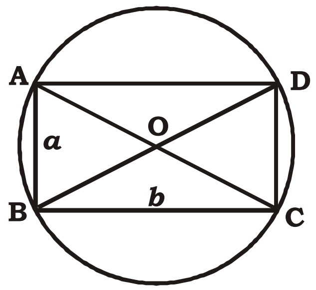
(f) (i) Radius of the maximum possible circle
क्ष ज्हे क-यगम्रहि, क्य, घयि ियहैि.अर्फ
$= \dfrac{\text{Breadth}}{2} = \dfrac{b}{2}$



(ii) **Radius of the circle circumscribed the rectangle ABCD be R, then**

क्षआयत ABCD` यें हुक्कात्जंय, घयि ोयेहाअयिR मनिकर्न

$$R = \frac{\sqrt{a^2 + b^2}}{2}$$



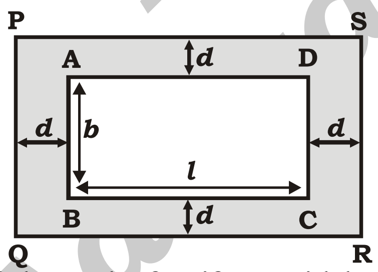
(g) **If the length of the rectangle will become x times and breadth will become y times, the area of the rectangle will become xy times.**

अहँ यक्ष आयतो ोयए नबिमx ै लीयक्ष लियए सिप्तिग्रy ै लीयमयि ज्नियर्क क्षआयतो यिर्पौर्ि ह्यु यxy ै लयिमयि ज्नि िः

(h) **Path around or in a rectangle/** यक्ष आयतो यें हुक्कत क्ष र्ध, यिक्ष आयतो यक्षँ ण्त्र्य र्ध

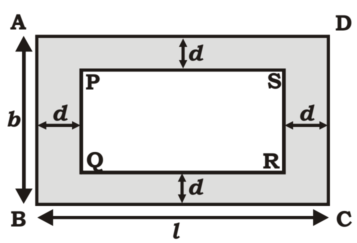
(i) **Area of the path of uniform width d all around outside the rectangle ABCD**

क्षआयत ABCD` यें हुक्कत्र यग-जियष्ट तिग्रिमd , एिंयक्स्यिं ` यिर्पौर्ि ह्यु य= $2d(l + b + 2d)$



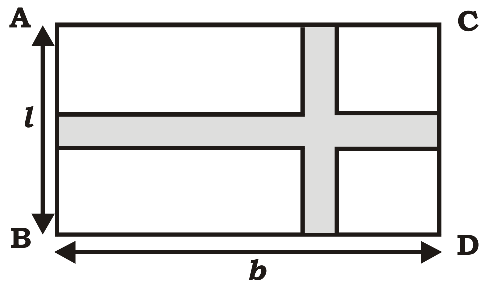
(ii) **Area of the path of uniform width 'd' all around inside the rectangle ABCD**

क्षआयत ABCD` यक्षँ ण्त्रयज्ञ यग-जियष्ट तिग्रिमd , एिंयक्स्यिो ि पौर्ि ह्यु य= $2d(l + b - 2d)$



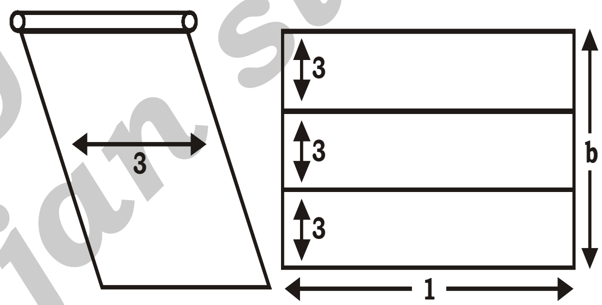
(iii) **Area of the path of uniform width d along the length and the breadth**

एन्बिमक्ष लियष्ट तिग्रिमे यक्ष ज्ञाँ उयिज्ञ ग-जियष्ट तिग्रिमd ये यें धयि ि पौर्ि ह्यु य= $(l + b - d)d$



(i) **(a) A carpet has fix width**

ा त्ये र्ि ् ये ोयष्ट तिग्रिमज्ञ्जउ क्यमर्ि



Let carpet of width w covers floor of dimension $l \times b$

-जियिw ह तिग्रिमे यि र्ि ् या $l \times b$ क्षअर्ि-ये ोयें ह्यसि यिख वयिमत

∴ **Area of carpet/** र्ि ् ये यिर्पौर्ि ह्यु य= **Area of floor/** ` ह्यसि यिर्पौर्ि ह्यु $\Rightarrow l_c \times w = l \times b$

Length of carpet required/ क्षि,उञे ये र्ि ् ये ोयएक्बिम

$$l_c = \frac{lb}{w}$$

(b) **Let in rectangular tiles of dimension $(x \times y)$ cover the floor of dimonsion $(l \times b)$**

-जियि$(x \times y)$ क्षअर्ि-ये ोयक्ष आयर्ि ट्यि बिमाय$(l \times b)$ क्षअर्ि-ये ो ` ह्यसि यिख वीयमत

⇒ **Area of n tiles/**n ् बिमाये यिर्पौर्ि ह्यु = **Area of floor/** ` ह्यसि यिर्पौर्ि ह्यु

$$n \times x \times y = l \times b \Rightarrow n = \frac{lb}{xy}$$

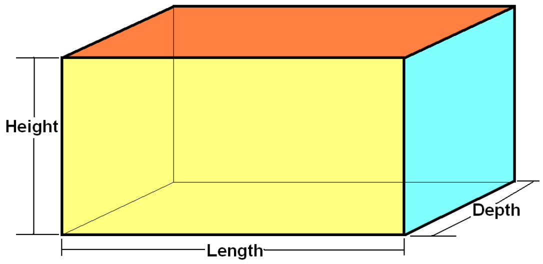
(c) **If floor covers by minimum number of square tiles exactly fit then side of square tile is HCF of length and breadth of floor.**

अहँ यें ह्यसि यिअज्ञक्त-याष्अयि ोय,ै सिट्यि बिमायु ट्र्यि,त्रीयक्म गंयखे यित अयिमलवयि,ै सिट्यि बिमये ोयवलिर्यि ह्यसि ोयएक्बिम कर्धियष्ट तिग्रिमे यिऽ ग्र`, यमवीयमत

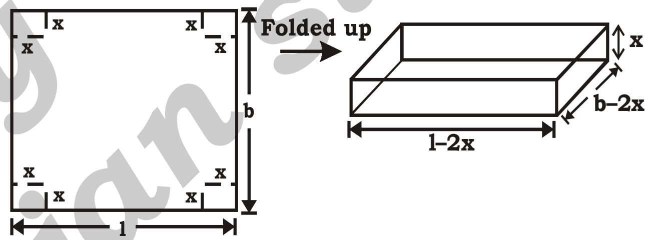
03 MENSURATION 3D

=हैहि, -ीअयर्पौंहि- ह्क्र

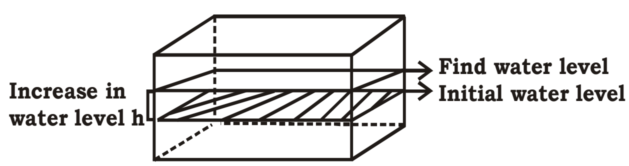
A. Cuboid/घनाभ



1. **Curved Surface Area/Lateral Surface Area** (पार्श्व पृष्ठीय क्षेत्रफल)
   = 2 (l + b) h
2. **Area of 4 walls** (चार दीवारों का क्षेत्रफल) = (Perimeter of Floor × Height)/(फर्श का परिमाप × ऊंचाई)
3. **Total Surface Area** (कुल पृष्ठीय क्षेत्रफल) = 2 (lb + bh + hl)
4. **Volume of Cuboid** (घनाभ का आयतन) = l x b x h
5. **Diagonal of Cuboid** (घनाभ का विकर्ण)
   $= \sqrt{l^2 + b^2 + h^2}$
   Length of longest rod that can be placed in the room. (किसी कमरे में रखी जा सकने वाली सबसे बड़ी छड़ की लम्बाई)
6. $(l+b+h)^2 = l^2+b^2+h^2+2\,(lb+bh+hl)$
   (sum of dimensions)^2 = (Diagonal)^2 + Total Surface Area./ (विमाओं का योगफल)^2 = (विकर्ण)^2 + (कुल पृष्ठीय क्षेत्रफल)
7. **Volume of hollow cuboid** (खोखले घनाभ का आयतन)
   = lbh – (l–2x) (b–2x) (h–2x),
   where x is the thickness of walls of the cuboid.(जहाँ x घनाभ की दीवार की मोटाई है)
8. **Volume of water from cuboidal cross section pipe in t time.** (घनाभाकार क्रास सेक्शन पाइप से t समय में निकले पानी का आयतन)
   = area of base × (vt) = lbvt
   (If this fills a tank then put it equal to volume of tank) (यदि यह किसी टंकी को भरे तो इसे टंकी के आयतन के बराबर कर देते हैं)
9. Making box by rectangular sheet.(आयताकार शीट से खुला डिब्बा बनाना)
   we can make an open rectangular box by cutting off equal squares at four corners and the remainder is folded up. (चारों कोनों से बराबर वर्ग काटने के बाद शेष को मोड़कर हम एक खुला आयताकार डिब्बा बना सकते हैं)



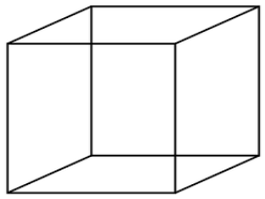
   Volume of rectangular box (आयताकार डिब्बे का आयतन) = (l–2x) (b–2x)x
10. **In digging related questions** (खुदाई से सम्बन्धित प्रश्नों में)
    Volume of earth taken out (खोदी गई मिट्टी का आयतन) = **Volume of cuboid** (घनाभ का आयतन)
11. **In the Questions related to melting and recasting volume remains constant**
    (पिघलाने और दोबारा बनाने से सम्बन्धित प्रश्नों में आयतन नियत रहता है)
12. **Increase and decrease in water level** (जल स्तर में वृद्धि या कमी)



Volume of extra cubiod (अतिरिक्त घनाभ का आयतन)
= xlt
= lbh = xlt
= x × 100 cm^3

$= \frac{x}{1000} m^3$

B. Cube/घन



A cuboid in which (ऐसा घनाभ जिसमें) $l = b = h$

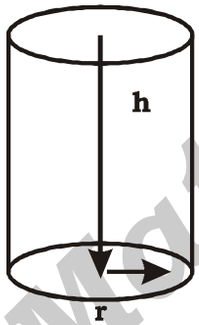
(a) Curved Surface Area/Lateral Surface Area (पार्श्व पृष्ठीय क्षेत्रफल) = $4a^2$

(b) Total Surface Area/(कुल पृष्ठीय क्षेत्रफल) = $6a^2$

(c) Volume (आयतन) = a^3

(d) Diagonals (विकर्ण) = $\sqrt{3}a$

C. Cylinder/बेलन



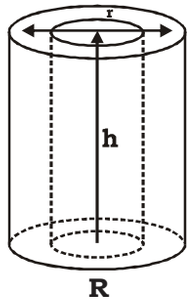
(a) Curved Surface Area/Lateral Surface Area (पार्श्व पृष्ठीय क्षेत्रफल) = $2\pi rh$

(b) Total Surface Area/(कुल पृष्ठीय क्षेत्रफल) = $2\pi rh + 2\pi r^2$

= $2\pi r(r+h)$

(c) Volume (आयतन) = $\pi r^2 h$

Hollow Cylinder/खोखला बेलन



(i) Curved Surface Area/Lateral Surface Area (पार्श्व पृष्ठीय क्षेत्रफल) = $2\pi rh + 2\pi Rh = 2\pi h\,(R+r)$

(ii) Total Surface Area (कुल पृष्ठीय क्षेत्रफल) = $2\pi h\,(R+r) + 2\pi\,(R^2 - r^2)$

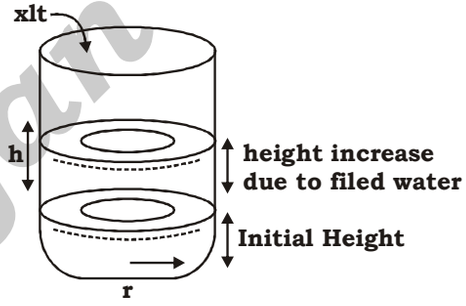
(iii) Volume of hollow cylinder (खोखले बेलन का आयतन) = $\pi r^2 h$

(iv) Volume of material of hollow Cylinder (खोखले बेलन में उपयोग किए गए पदार्थ का आयतन) = $\pi(R^2 - r^2)\,h$

(v) Mass (weight) of hollow cylinder (खोखले बेलन का द्रव्यमान (भार))

= density × volume of material. (घनत्व × पदार्थ का आयतन)

☞ Increasing/Decreasing water level of a cylindrical tank (बेलनाकार टंकी के जल स्तर में वृद्धि/कमी)



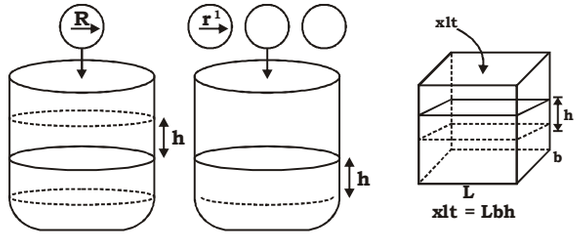
Concept: xlt = Volume of extra cylinder (अतिरिक्त बेलन का आयतन) = $\pi r^2 h$

In above xlt may by volume of any object and Cylindrical tank may any prism

A sphere of R radius dropped into the cylindrical tank

If n marbles of r' radius each. Dropped into the cylinder

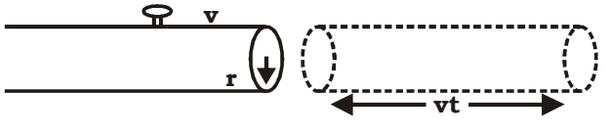
If tank is Cuboid

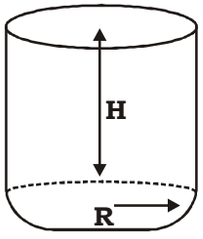


$\frac{4}{3}\pi R^3 = \pi r^2 h$ $\quad n \times \frac{4}{3}\pi r'^3 = \pi r^2 h$

☞ **Fill an empty water tank by cylindrical pipe or empty a filled water tank by cylinder pump. Let t is time to empty/fill the tank**

(बेलनाकार पाइप द्वारा किसी खाली टंकी को भरना या बेलनाकार पाइप द्वारा किसी भरी हुई टंकी को खाली करना। माना टंकी को खाली/भरने में लगा समय t है।)





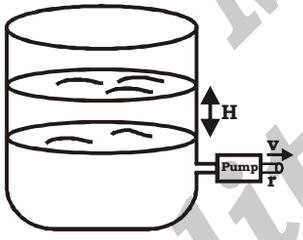
Concept:

Volume of water flowing throught pipe in 't' time (t समय में पाइप द्वारा बहते पानी का आयतन) = Volume of the tank (टंकी का आयतन)

$\pi r^2 (vt) = \pi R^2 H$

Let in 't' time water level decrease by H (माना, t समय में जल स्तर में कमी H है)

$\pi r^2 (vt) = \pi R^2 H$



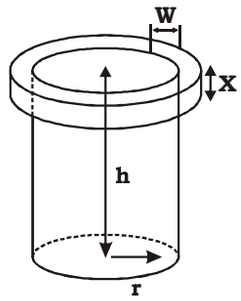
NOTE:

Shape of pipe and tank may any other prism but concept remain same.

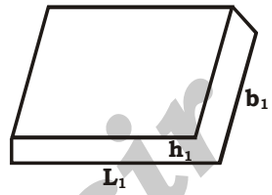
(पाइप तथा टंकी का आकार कोई और हो सकता है, परन्तु सिद्धांत यही होगा)

☞ **Digging a well and earth taken out (कुएं की खुदाई और निकाली गई मिट्टी)**

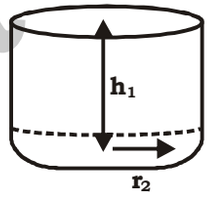
(i) Spread all around it to form an embankement (इसके चारों ओर चबूतरें के रूप में फैला देना)



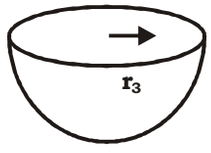
(ii) Spread in a Cuboid (घनाभ के रूप में फैला देना)



(iii) Fill another well (एक दूसरे कुएं में भर देना)



(iv) Fill a hemispherical hole (एक अर्ध गोलाकार आकृति में भर देना)



Concept: Volume of well (कुएं का आयतन) = Volume of ambankment (चबूतरें का आयतन)

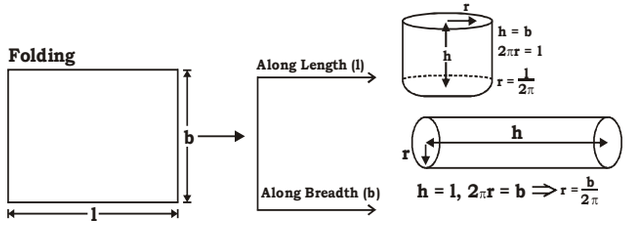
$\pi r^2 h$ = (i) $\pi[(r + w)^2 - r^2] \times x$

= (ii) $l_1 b_2 h_1$

= (iii) $\pi r^2{}_2 h_1$

= (iv) $\frac{2}{3}\pi r^3{}_3$

☞ **Folding and revolving a rectangular sheet (आयताकार शीट को मोड़ना और घुमाना)**



☞ **Concepts related to formula of cylinder** (बेलन के सूत्रों पर आभारी सिद्धांत)

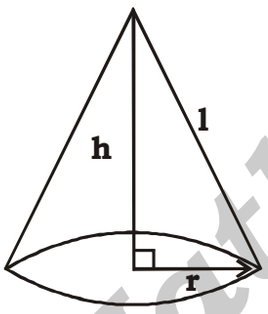
(i) If curved surface area (C) and volume (V) are given then ratio of radius to height (यदि पार्श्व पृष्ठीय क्षेत्रफल (c) तथा आयतन (v) दिए हो तो त्रिज्या का ऊंचाई से अनुपात)

$$\frac{r}{h}=\frac{8\pi V^2}{c^3}$$

(ii) If curved surface area (c) and height (h) are given the volume of cylinder. (यदि पार्श्व पृष्ठीय क्षेत्रफल (c) तथा ऊंचाई (h) दिए हो तो बेलन का आयतन)

$$V=\frac{c^2}{4\pi h}$$

D. Cone/शंकु



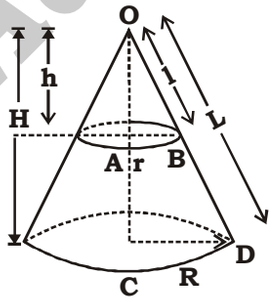
(a) Curved surface area (पार्श्व पृष्ठीय क्षेत्रफल) = πrl

(b) Total surface area (कुल पृष्ठीय क्षेत्रफल) = $\pi r\,(r + l)$

(c) Volume/आयतन $= \frac{1}{3}\pi r^2 h$

(d) Slant height/तिरछी ऊंचाई $l = \sqrt{r^2 + h^2}$

☞ All triangles formed by cutting cone are similar to each other. (शंकु को काटने से बने सभी त्रिभुज एक दूसरे के समरूप होते हैं।)



$\Delta OCD \sim \Delta OAB\,(\angle A = \angle C = 90°, \angle O = \angle O)$

(i) $\frac{H}{h}=\frac{R}{r}=\frac{L}{l}$ or $\frac{H}{R}=\frac{h}{r}$

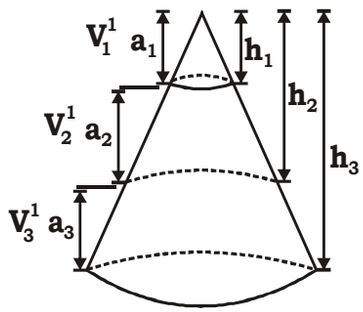
Let V is volume of larger cone and v is volume of smaller cone (माना, V बड़े शंकु का आयतन तथा v छोटे शंकु का आयतन है)

(ii) $\frac{V}{v}=\frac{\frac{1}{3}\pi R^2 H}{\frac{1}{3}\pi r^2 h}=\frac{R^2 H}{r^2 h}$

(iii) $\frac{V}{v}=\frac{H^3}{h^3}=\frac{R^3}{r^3}=\frac{L^3}{l^3}$

Example: 01

A right circular cone is cut by two plane parallel to the base into three parts with altitude ratio $a_1 : a_2 : a_3$ then ratio of their volume. (एक लम्बवृत्तीय शंकु को इसके आधार के समान्तर दो तलों द्वारा तीन भागों में ऊंचाईयों के अनुपात $a_1 : a_2 : a_3$ में काटा गया है, उनके आयतनों का अनुपात है:)



$V_1 : V_2 : V_3 = h_1^3 : h_2^3 : h_3^3$

$= a_1^3 : (a_1 + a_2)^3 : (a_1 + a_2 + a_3)^3$

Ratio of volume of three parts (तीन भागों के आयतन का अनुपात)

$V_1^1 : V_2^1 : V_3^1 = a_1^3 : (a_1 + a_2)^3 - a_1^3 : (a_1 + a_2 + a_3)^3 - (a_1 + a_2)^3$

Note:

If ratio of their altitude is 1 : 1 : 1 (यदि उनकी ऊंचाईयों का अनुपात 1 : 1 : 1 हो तो)

then $a_1 = a_2 = a_3 = 1$

$V_1^1 : V_2^1 : V_3^1 = 1 : 7 : 19$

Example: 02

In above question if ratio of volume of three parts is $b_1 : b_2 : b_3$ then ratio of their heights-

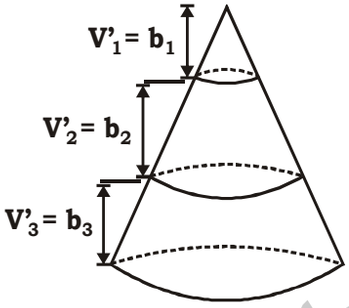
उपर्युक्त प्रश्न में यदि तीन भागों के आयतन का अनुपात $b_1 : b_2 : b_3$ हो तो उनकी ऊंचाईयों का अनुपात

$$h_1^3 : h_2^3 : h_3^3 = V_1 : V_2 : V_3$$

$$= b_1 : (b_1 + b_2) : (b_1 + b_2 + b_3)$$

$$h_1 : h_2 : h_3 = b_1^{1/3} : (b_1 + b_2)^{1/3} : (b_1 + b_2 + b_3)^{1/3}$$

$$\Rightarrow h_1^1 : h_2^1 : h_3^1 = b_1^{1/3} : (b_1 + b_2)^{1/3} - b_1^{1/3} : (b_1 + b_2 + b_3)^{1/3} - (b_1 + b_2)^{1/3}$$

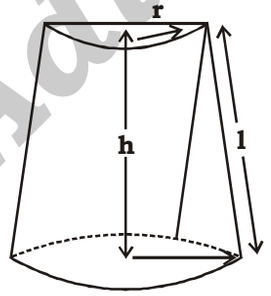


Note:

If ratio of volume of all parts is 1 : 1 : 1 then ratio of heights $h_1^1 : h_2^1 : h_3^1$ (यदि उनके आयतनों का अनुपात 1 : 1 : 1 हो तो ऊंचाईयों $h_1^1 : h_2^1 : h_3^1$ का अनुपात) $= 1 : 2^{1/3} - 1 : 3^{1/3} - 2^{1/3}$

Frustrum/छिन्नक

When a cone cut parallel to base, lower portion is called frustum. (जब एक शंकु को इसके आधार के समान्तर काटा जाता है तो नीचे वाला भाग छिन्नक कहलाता है।)



Volume of frustum (छिन्नक का आयतन)

$$= \frac{1}{3}\pi(R^2 + r^2 + Rr)h$$

Curved surface area (पार्श्व पृष्ठीय क्षेत्रफल)

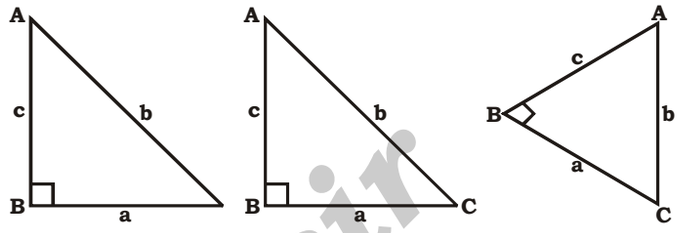
$= \pi (R + r)l$

Total surface area (कुल पृष्ठीय क्षेत्रफल)

$= \pi l (R + r) + \pi (R^2 + r^2)$

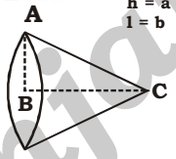
When $l = \sqrt{h^2 + (R - r)^2}$

☞ **Rotation of a right anlged triangle.** (समकोण त्रिभुज के घुमाने पर बने शंकु)



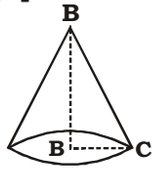
Along

Base



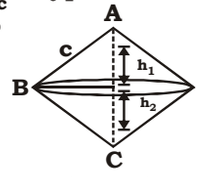
r = c
h = a
l = b

Perpendicular



r = a
h = c
l = b

Hypoteneous



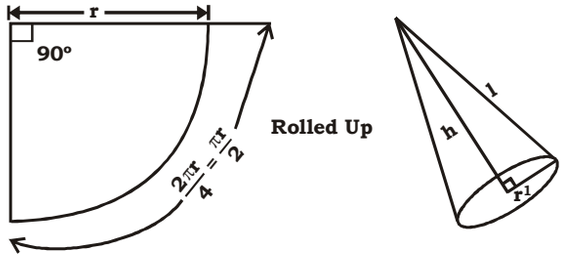
For radius = rb = ac sum of volume of two cones

$$= \frac{1}{3}\pi\frac{a^2c^2}{b}$$

☞ **Rolling up a sector** (त्रिज्यखंड को घुमाने पर बना शंकु)

When a sector is rolled up in such a way that the two binding radii are joined together then a cone generates. (जब किसी त्रिज्यखंड को इस प्रकार घुमाया जाता है कि इसकी दो त्रिज्याएं जुड़ जाती है, तो एक शंकु का निर्माण होता है।)

☞ **A right angled sector of radius a cm is rolled up into a cone** (a सेमी त्रिज्या के एक समकोणीय त्रिज्यखंड को घुमाकर बना शंकु)



$\Rightarrow 2\pi r^1 = \frac{\pi r}{2} \Rightarrow r^1 = \frac{r}{4}$ & $l = r$

⇒ **Height of cone** (शंकु की ऊंचाई) **(h)**

$= \sqrt{l^2 - r^{1^2}}$

$= \sqrt{r^2 - \frac{r^2}{16}} = \sqrt{\frac{15r}{4}}$

⇒ **Curved surface area of cone** (शंकु का पार्श्व पृष्ठीय क्षेत्रफल) = **Area of sector** त्रिज्यखंड का क्षेत्रफल

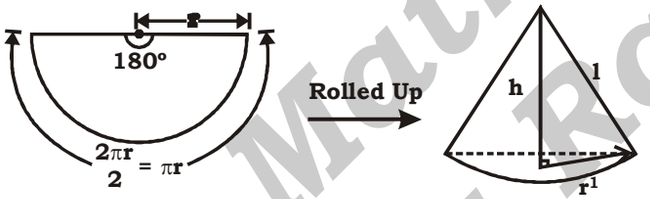
$= \frac{\pi r^2}{4}$

⇒ **Volume of cone** शंकु का आयतन

$= \frac{1}{3}\pi \times \left(\frac{r}{4}\right)^2 \times \sqrt{\frac{15r}{4}}$

$= \frac{\sqrt{15}\pi r^3}{192}$

☞ **A semicircular sector of radius r cm is rolled into a cone.** (r सेमी त्रिज्या के किसी अर्धवृत्ताकार त्रिज्यखंड को घुमाने से बना शंकु)



⇒ **Height of cone** शंकु की ऊंचाई **(h)**

$= \sqrt{l^2 - r^{1^2}}$

$= \sqrt{r^2 - \frac{r^2}{4}}$

$= \frac{\sqrt{3}r}{2}$

⇒ **Curved surface area of cone** (शंकु पार्श्व पृष्ठीय क्षेत्रफल)

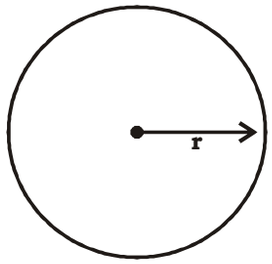
= **Area of sector** (त्रिज्यखंड का क्षेत्रफल)

$= \frac{\pi r^2}{2}$

⇒ **Volume of cone** (शंकु का आयतन)

$= \frac{1}{2}\pi r^{1^2} h = \frac{1}{3}\pi \times \frac{r^2}{4} \times \frac{\sqrt{3}r}{2} = \frac{\pi r^3}{8\sqrt{3}}$

E. Sphere/गोला



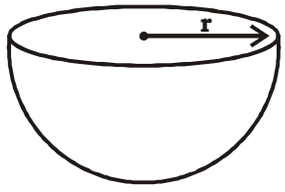
(a) Surface area (पृष्ठीय क्षेत्रफल) = $4\pi r^2$

(b) **Volume** (आयतन) = $\frac{4}{3}\pi r^3$

Let v_1 and v_2 be volume and S_1 and S_2 be area of two sphere then माना v_1 तथा v_2 और S_1 तथा S_2 दो गोले के आयतन और क्षेत्रफल हैं।

(c) $\frac{v_1}{v_2} = \left(\frac{S_1}{S_2}\right)^{3/2}$ or $\frac{S_1}{S_2} = \left(\frac{v_1}{v_2}\right)^{2/3}$

F. Hemisphere/अर्द्धगोला



(a) **Curved surface area** (पार्श्व पुष्ठीय क्षेत्रफल)

$= 2\pi r^2$

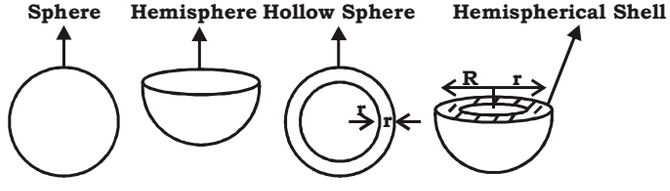
(b) **Total surface area** (कुल पुष्ठीय क्षेत्रफल)

$= 3\pi r^2$

(c) **Volume** (आयतन)

$= \frac{2}{3}\pi r^3$

☞ **Concepts related to volume** (आयतन से सम्बन्धित सिद्धांत)



$V = \frac{4}{3}\pi r^3$ $V = \frac{2}{3}\pi r^3$ $V = \frac{4}{3}\pi\left[(r+x)^3 - r^3\right]$ $V = \frac{2}{3}\pi(R^3 - r^3)^3$

Combination of 3-D Objects
त्रिविमीय आकृतियों का संयोजन

☞ **Melting and Recasting** (पिघलने तथा दोबारा बनने से सम्बन्धित)

⇒ **If a 3-D object melts and recasts into another 3-D object then volume remain constant.**

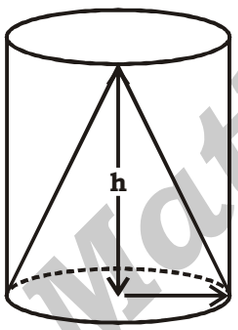
(यदि कोई त्रिविमीय आकृति को पिघलाकर दोबारा एक दूसरी त्रिविमीय आकृति बनाई जाती है तो आयतन नियत रहता है।)

☞ **A solid maximum 3-D object inside another 3-D object**

किसी त्रिविमीय आकृति के भीतर अधिकतम आयतन की दूसरी त्रिविमीय आकृति

(i) A maximum cone inside a cylinder

बेलन के भीतर अधिकतम आयतन का शंकु

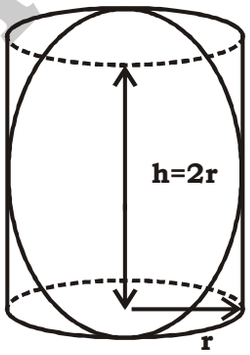


Volume of cylinder/बेलन का आयतन : Volume of cone/शंकु का आयतन

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h = 3 : 1$$

(ii) A cylinder just encloses a sphere then

बेलन के अन्दर गोला



Height of cylinder/बेलन की ऊँचाई

= Diameter of sphere/गोले का ब्यास =2r

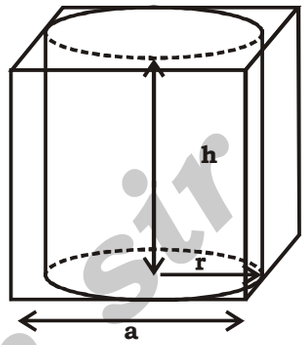
Volume of Cylinder : Volume of Sphere

बेलन का आयतन : गोले का आयतन

$$= \pi r^2(2r) : \frac{4}{3}\pi r^3 = 2 : \frac{4}{3} = 3 : 2$$

(iii) A maximum cylinder inside cube then

घन के भीतर अधिकतम आयतन का बेलन



Radius of cylinder/बेलन की त्रिज्या

$$= \frac{1}{2} \times \text{edge of cube} = \frac{a}{2}$$

Height of cylinder/बेलन की ऊँचाई

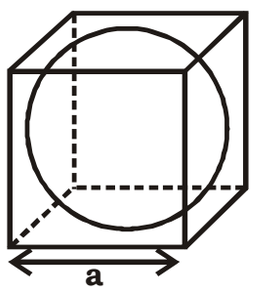
= edge of cube/घन की कोर = a

Volume of cube/घन का आयतन : Volume of cylinder /बेलन का आयतन $= a^3 : \pi\left(\frac{a}{2}\right)^2 a$

$$= 1 : \frac{22}{7} \times \frac{1}{4} = 14 : 11$$

(iv) A maximum sphere inside a cube then

घन के भीतर अधिकतम आयतन का गोला



Diameter of sphere/गोले का व्यास (2r) = edge of cube/घन की कोर = a

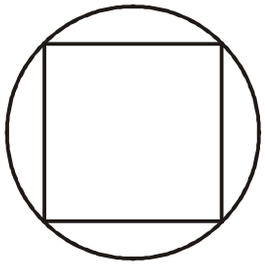
Volume of cube/घन का आयतन : Volume of sphere/गोले का आयतन $= a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

= 21: 11

(v) A maximum cube inside a sphere

किसी गोले के भीतर अधिकतम आयतन का घन

Diagonal of cube/घन का विकर्ण = Diameter of sphere/गोले का व्यास



$$\sqrt{3}a = 2r \Rightarrow a = \frac{2r}{\sqrt{3}}$$

Volume of sphere/गोले का आयतन : Volume of cube/घन का आयतन

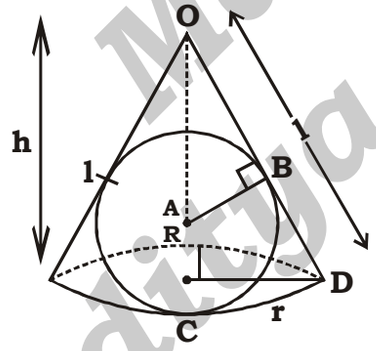
$$= \frac{4}{3}\pi r^3 : \left(\frac{2r}{\sqrt{3}}\right)^3$$

$$= \frac{4}{3} \times \frac{22}{7} : \frac{8}{3\sqrt{3}}$$

$$= 11\sqrt{3} : 7$$

(vi) A maximum sphere inside a cone

किसी शंकु के भीतर अधिकतम आयतन का गोला



then ΔOCD ~ΔOBA

$$\therefore \quad \frac{OD}{OA} = \frac{CD}{BA}$$

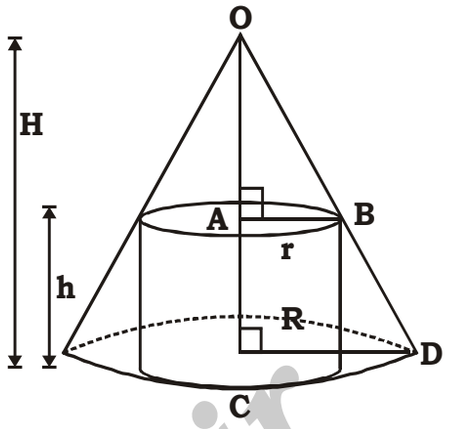
$$\frac{l}{h-R} = \frac{r}{R}$$

$$\Rightarrow lR = hr - Rr$$

$$\boxed{R = \frac{hr}{l+r}}$$

(vii) maximum cylinder inside a cone

किसी शंकु के भीतर अधिकतम आयतन का बेलन



ΔOCD ~ ΔOAD

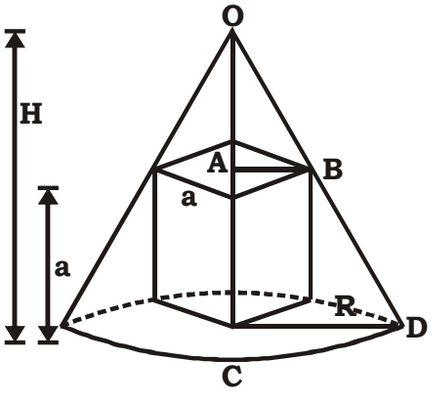
(AA ∠A = ∠C = 90°, ∠O common)

$$\frac{OC}{OA} = \frac{CD}{AB}$$

$$\boxed{\frac{H}{H-h} = \frac{R}{r}}$$

(viii) A maximum cube inside cone

शंकु के भीतर अधिकतम आयतन का घन



ΔOCD ~ OAD (AA)

$$\frac{OC}{OA} = \frac{CD}{AB}$$

$$\boxed{\frac{H}{H-a} = \frac{R}{a/\sqrt{2}}}$$

$$\left[\therefore AB = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}\right]$$

G. Prism/प्रिज्म

A prism is a solid that has two faces that are parallel and congruent and there faces (Polygon) join by vertex to vertex. A prism has a polygon as its base and vertical side perpendicular to the base.

हडा.वफ्रे य·गियमकियिमव्ह गे यँ यिं व्हु यग-जिफियक्ष त्रि ग, श्रिग-यमकिंममक्ष क्त्रियुजे यें व्हु य·त्तमलि़ यधिश्रिांयधिसि गंय ूलक्ंममक्षहडा.वफ्रि·यक्ष इ़ि त्रिये यक ॅ य·यफ़े य·त्तमलियमकियमित क्ष क्त्रियठ रु श्रित्यलियिक्ष इ़ि त्रिये यए क , कममकियमत

(a) Curved surface area of a prism = Perimeter of base × height

हडा.वफ्रि यि, हार्य श्र्ीअमर्पों ि व्हुयक्ष इ़ि त्रिये यिं ह़-ि य·यठ क्ष्बिस

(b) Total surface area of a prism = curved surface area + 2 × area of base

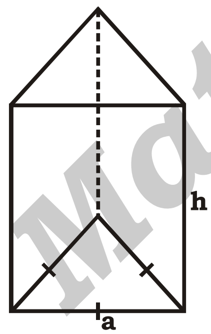
हडा.वफ्रि यि एर्य श्र्ीअमर्पों ि व्हुयत्रम, हार्य श्र्ीअमर्पों ि व्हु ज्ञयज्ञ×यक्ष इ़ि त्रिये यिर्पों ि व्हु

(c) Volume of a prism = area of base × height

हडा.वफ्रि यिक्ष उिक्ज्ञक्ष इ़ि त्रिये यिर्पों ि व्हु य·यठ क्ष्बिस

(i) Equilateral triangular prism:

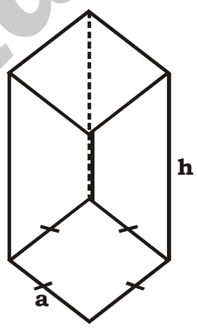
ग-त्तमिलै हेदिलि्र्ा त्रियहडा.वफ्रि



C. S. A = 3ah, T. S. A = 3ah + 2× $\frac{\sqrt{3}}{4}a^2$

$V = \frac{\sqrt{3}}{4}a^2h$

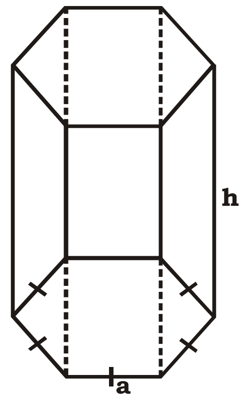
(ii) Square Prism:/ , त्सिंत्रिय्ह भ्वी)



C. S. A = 4ah, T. S. A = 4ah + $2a^2$

$V = a^2h$

(iii) Hexagonal Prism :/ ग्ि ट रिाीअर्ह भ्वी)

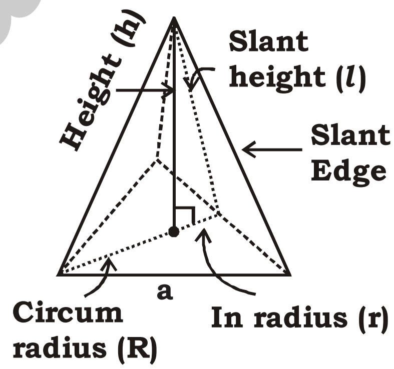


C. S. A = 6ah, T. S. A. = 6ah + 2×$\left(\frac{6\times\sqrt{3}}{4}a^2\right)$

$V = \frac{6\times\sqrt{3}}{4}a^2h$

H. Pyramid/है त्रहि़

Triangular Pyramid/हेदिलि्र्ा त्रियर्ह त्रहि़



(a) Curved surface area of Pyramid/है त्रहि़ ये ि, हार्य श्र्ीअमर्पों ि व्हु

$= \frac{1}{2}\times$ Perimeter of base × slant height/

$\frac{1}{2}$ × क्ष इ़ि त्रिये यिं ह़-ि य×यक्ष्कग ीयठ क्ष्बिस

(b) Total surface area of Pyramid/है त्रहि़ ये यि एर् श्र्ीअमर्पों ि व्हु

= Curved surface area + area of base/ , हार् श्र्ीअमर्पों ि व्हुय+ क्ष इ़ि त्रिये यिर्पों ि व्हु

(c) Volume of a Pyramid/है त्रहि़ ये यिक्ष उिक्ज्ञ

$= \frac{1}{3}\times$ area of base × height / $\frac{1}{3}$ × क्ष इ़ि त्रिये ि पोंि व्हु य·यठ क्ष्बिस

(i) **Equilateral triangular Pyramid**

ग-त्रिभुजाकार पिरामिड

$$C.S.A = \frac{1}{2} \times 3a \times l$$

$$T.S.A = \frac{1}{2} \times 3al + \frac{\sqrt{3}}{4}a^2$$

$$V = \frac{1}{3} \times \frac{\sqrt{3}}{4}a^2 \times h$$

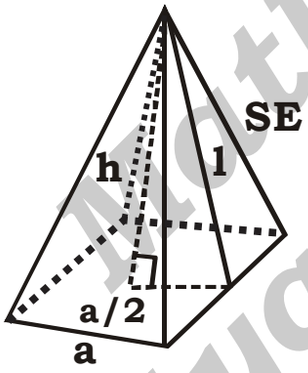
$$l = \sqrt{h^2 + r^2} = \sqrt{h^2 + \left(\frac{a}{(2\sqrt{3})}\right)^2}$$

S.E (Slant edge)

$$= \sqrt{h^2 + R^2}$$

$$= \sqrt{h^2 + \left(\frac{a}{\sqrt{3}}\right)^2}$$

(ii) **Square Pyramid**/ , तिर्यक



$$C.S.A = \frac{1}{2} \times 4a \times l$$

$$T.S.A = \frac{1}{2} \times 4al + a^2$$

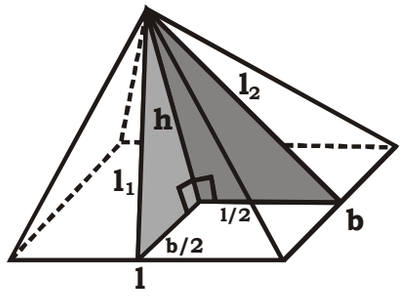
$$V = \frac{1}{3} \times a^2 \times h$$

$$l = \sqrt{h^2 + \left(\frac{a}{2}\right)^2}$$

$$(Slant\ edge) = \sqrt{h^2 + \left(\frac{a}{\sqrt{2}}\right)^2}$$

(iii) **Rectangular Pyramid**/

There are two slant height/



First slant height/

$$(l_1) = \sqrt{h^2 + \left(\frac{b}{2}\right)^2}$$

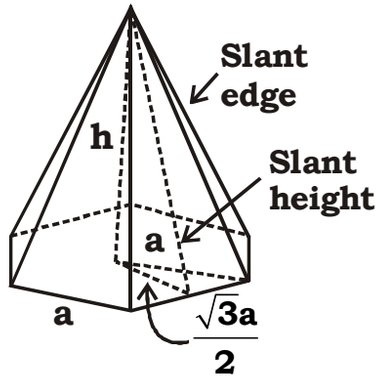
Second slant height/

$$(l_2) = \sqrt{h^2 + \left(\frac{1}{2}\right)^2}$$

$$C.S.A = 2 \times \frac{1}{2} l \times l_1 + 2 \times \frac{1}{2} \times b \times l_2$$

$$T.S.A = C.S.A + lb \ \&\ V = \frac{1}{2} \times lb \times h$$

(iv) **Hexagonal pyramid**/



$$C.S.A = \frac{1}{2} \times 6al$$

T. S. A = C. S. A + Area of base/

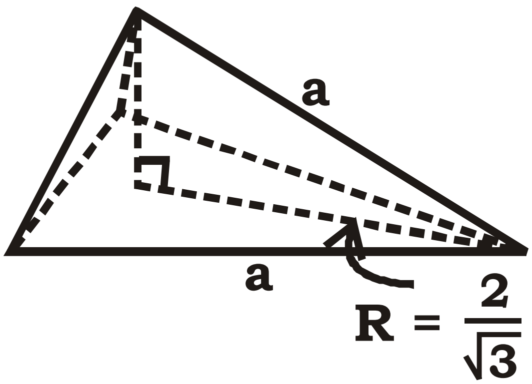
$$= \frac{1}{2}6al + 6 \times \frac{\sqrt{3}}{4}a^2$$

$$V = \frac{1}{3} \times \frac{6\sqrt{3}}{4}a^2 \times h$$

Slant height/ $(l) = \sqrt{h^2 + \left(\frac{\sqrt{3}}{2}a\right)^2}$

Slant edge/ $= \sqrt{h^2 + a^2}$

I. Tetrahedron/ग-४ र्कलद्धे



: There are four equilateral faces.

४ त्रियग-त्तमिर्ला द्धे यमप्र

: All edge are equal in length i.e.

गवीय्हे जर्त्यिए ज्ञबिस-यत्त्ऽत्तिऽयमक्ष् र्धब्लिक्त

: slant edge is same as side of base

ह्कग ायि त्रियक् ड्डि त्रिये ायिह्लियि यत्त्ऽत्तिऽयमत

(a) C. S. A. = Area of 3 equilateral triangle

कीजयग-त्तमिलहिदिलिणि यियोंर्ि द्धु

$$= 3 \times \frac{\sqrt{3}}{4} a^2$$

(b) T. S. A = Area of 4 equilateral triangle

४ त्रियग-त्तमिलहिदिलिणि यियोंर्ि द्धु

$$= 4 \times \frac{\sqrt{3}}{4} a^2 = \sqrt{3}\, a^2$$

(c) Height/ढह्ढबिस(h) $= \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = \sqrt{\frac{2}{3}}\, a$

(d) Volume/क्ष् ञिक्त (V)

$= \frac{1}{3} \times$ Area of base/क्ष् र्सिन्ये यियोंर्ि द्धु $\times$ height/ढह्ढबिस

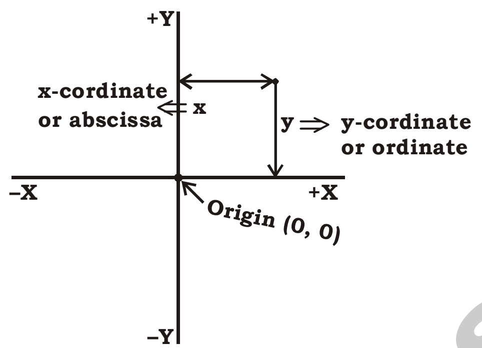
$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} a^2 \times \sqrt{\frac{2}{3}}\, a = \frac{\sqrt{2}}{12} a^3$$

	Volume	Total Surface Area	Lateral / Curved Surface Area
Cube	$Side^3$	$6 \times Side^2$	$4 \times Side^2$
Cuboid	L x B x H	2(LB + LH + BH)	2 (LH + BH)
Cylinder	$\pi r^2 h$	$2\pi r (r + h)$	$2\pi rh$
Cone	$(1/3)\, \pi r^2 h$	$\pi r (r + L)$	πrl {where $L = \sqrt{r^2 + h^2}$}
Sphere	$(4/3)\, \pi r^3$	$4\, \pi r^2$	$4\, \pi r^2$
Hemisphere	$(2/3)\, \pi r^3$	$3\, \pi r^2$	$2\, \pi r^2$

04 Co-Ordinate Geometry

` ' गऔैलयू ि ` भे द

☞ **Rectangular coordinate system- There are two mutually perpendicular lines, called coordinate axes./**हे ि दे लै ाये गऔैलयजै न ज्मयौ यमा क्षा यम ƒ बदयी ` ज्ण केदय्र्गध्ये सखसे गऔैलयह पै यलकैयसै दे यर्कद्ध



Distance formula/गाए्यलै यचुौए

(a) **Distance of point (x, y) from origin/**भ्ह्ये ƒ सैंयच ` ƒ सैंय (x, y) यलज्याएन = $\sqrt{x^2 + y^2}$

(b) **Distance between two points (x_1, x_1) and (x_2, y_2)/**यौ ये ƒ सैंहे स(x_1, x_1) दउै (x_2, y_2) यलक ज्ञ यलज्याएन

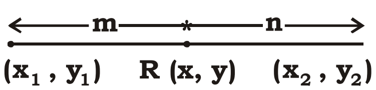
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section formula/चष्अं यतै,झ्सै

(a) **The coordinate of a point which internally divides the line joined by the point $(x_1\ y_1)$ and (x_2, y_2) in the ratio m : n is then,**

प्रचये ƒ सैंयलै ये गऔैलयसै ये ƒ सैंय$(x_1\ y_1)$ दउै (x_2, y_2) यच सैं: ÷ यबै न ज्यी ` यलै यm : nयलयह ` तै दयभ्सहै दे ा लयू तयच ` बहै ` सदयला दे यर्कध्यƒ यर्कद्द

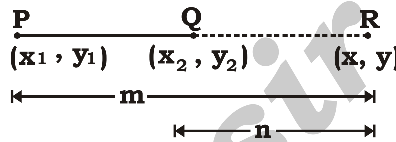
$$x = \frac{mx_2 + nx_1}{m+n},\ y = \frac{my_2 + ny_1}{m+n}$$



(b) **If R (x,y) divides the line segment externally in the ratio m:n**

ि गयR (x,y)यी ` ा ƒ यलै यm : nयलयह ` तै दयभ्सहै । यू तयच ` बहै ` सदयला दे यर्क

$$\text{then},\ x = \frac{mx_2 - nx_1}{m-n},\ y = \frac{my_2 - ny_1}{m-n}$$



Note: If point is mid point then m = n = 1

ि गये ƒ सैंयफ ि ƒ सैंयकैयचै यm = n = 1

$$x = \frac{x_2 + x_1}{2},\ y = \frac{y_2 + y_1}{2}$$

Mirror Image/गतजैयर दे ƒ छ

(a) **Mirror Image of (x, y) with respect to x – Axis is (x, – y)**

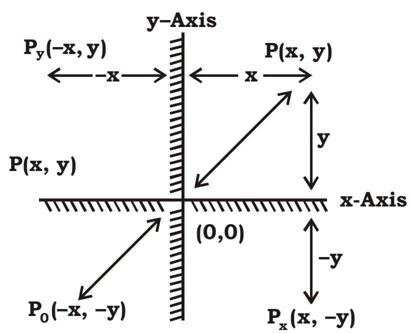
x – ह पै यलयते ादध्य(x, y)यलै यमातजैयर दे ƒ छ य(x, –y) र्कद्ध

(b) **Mirror Image of (x, y) with respect to y – Axis is (– x, y)**

y – ह पै यलयते ादध्य(x, y)यलै यमातजैयर दे ƒ छ य(–x, y) र्कद्ध

(c) **Mirror Image of (x, y) with respect to origion is (–x, – y)**

भ्ह्ये ƒ सैंयलयते ादध्य(x, y)यलै यमातजैयर दे ƒ छ य(–x, –y) र्कद्ध



Note: **Mirror Image of a point with respect to a straight line/** लचज्यचस्ज्यी ` यलयते ादध्ये लचज्ये ƒ सैंयलै गतजैयतेध्दे ƒ छ

use concept that perpendicular distance from point to mirror equal to perpendicular distance from image to mirror and both tare in same straight line.

ह बसै ाजै यलै यमतिं ÷ यला से लये ƒ सैंयचयातजैयम्लयम ƒ बदयाएन तेध्दे ƒ छ यचयातजैयम्लयम ƒ बदयाएन्लकै ƒ ा यर्कहै ायौ ` सल कयचयचस्ज्यी ` यभ्सर्कद्ध

Angle between tow lines of slope m_1 and m_2 is θ than

$$\tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

If line are parallel then θ= 0°

$$\therefore \tan 0^\circ = 0 = \frac{m_1 - m_2}{1 + m_1 m_2} \Rightarrow m_1 = m_2$$

If lines are perpendicular then θ= 90°

$$\therefore 90^\circ = \infty = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow 1 + m_1 m_2 = 0 \Rightarrow m_1 . m_2 = -1$$

Point of intersection of two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ → lines are overlapping each other → ∞ Solution

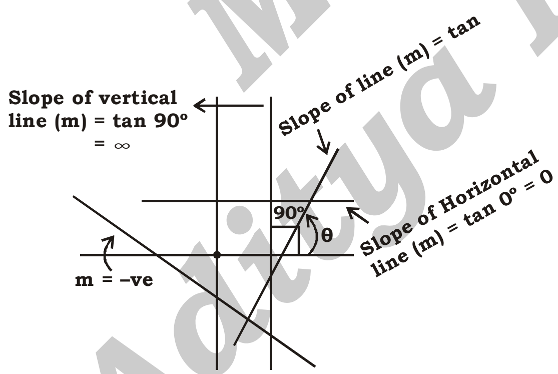
If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → lines are parallel → No Solution

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → lines are intersecting → One Solution

Slope of line (m)/रेखा की प्रवणता

Tangent (tan) of angle between line and +ve x –axis

रेखा तथा +ve x –अक्ष के बीच के कोण का स्पर्शज्या (tan)



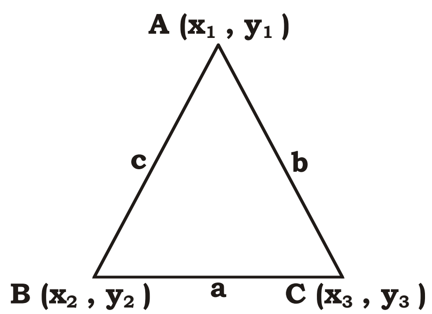
(a) Slope of line/रेखा की प्रवणता in general from ax + by + c = 0

$$\text{then } m = -\left(\frac{\text{Coefficient of } x}{\text{Coefficient of } y}\right) = -\left(\frac{a}{b}\right)$$

(b) Slope of line passing through (x_1, y_1) and (x_2, y_2) is /(x_1, y_1) और (x_2, y_2) से होकर जाने वाली रेखा की प्रवणता

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Triangle/त्रिभुज



☞ Area of ΔABC / ΔABC का क्षेत्रफल

$$= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

☞ Coordinates of its centroid/केन्द्रक के निर्देशांक

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

☞ Coordinates of its incentre/अन्तःकेन्द्र के निर्देशांक

$$= \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$

☞ **Coordinates of its circumcentre/**तै ालखश्लयें गऊैल

$$= \left(\frac{x_1 \text{Sin}\,2A + x_2 \text{Sin}\,2B + x_3 \text{Sin}\,2C}{\text{Sin}\,2A + \text{Sin}\,2B + \text{Sin}\,2C}, \frac{y_1 \text{Sin}\,2A + y_2 \text{Sin}\,2B + y_3 \text{Sin}\,2C}{\text{Sin}\,2A + \text{Sin}\,2B + \text{Sin}\,2C} \right)$$

☞ **Coordinates of its orthcentre/**न छ लखश्लयें गऊैल

$$\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C} \right)$$

☞ **The equation of a line passing through points (x_1, y_1) and (x_2, y_2)**

` ?खँहै स्र$(x_1, y_1)$ है ा $(x_2, y_2)$ चयकैलाब ँसा यै न ज्य्` यनै चभ्स्लाजै

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)(x - x_1)$$

☞ **The coordinate of point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is**

गै य्` है स्र$a_1x + b_1y + c_1 = 0$ दउै $a_2x = b_2y + c_2 = 0$यन तेध्व(डगयेऽ खँयलै ये गऊैल

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right)$$

☞ **Perpendicular distance of a line ax + by + c=0 from a point (x, y) is**

ा्` यax + by + c = 0 लज्येऽ खँय(x, y) चयन छ बदयाए्

$$= \frac{| ax_1 + by_1 + c |}{\sqrt{a^2 + b^2}}$$

☞ **The perpendicular distance between parallel line $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is:**

चभै दाय्` यax + by + c_1 = 0 दउै ax + by + c_2 = 0 ल ऽ ज्यन छ बदयाए्

$$= \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

05 ALGEBRA/ अयावक्त

❖ Square Formula:-

1. $(a + b)^2 = a^2 + b^2 + 2ab$ or $(a-b)^2 + 4ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$ or $(a+b)^2 - 4ab$
3. $a^2 + b^2 = (a + b)^2 - 2ab$
4. $a^2 + b^2 = (a - b)^2 + 2ab$
5. $a^2 - b^2 = (a + b)(a - b)$
6. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
7. $(a + b)^2 - (a - b)^2 = 4ab$
8. $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$
9. $(a^2 - ab + b^2)(a^2 + ab + b^2) = a^4 + a^2b^2 + b^4$
10. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
11. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$
12. $2(ab + bc + ca) = (a + b + c)^2 - (a^2 + b^2 + c^2)$
13. $(b + c)(c + a)(a + b) + abc = (a + b + c)(ab + bc + ca)$
14. $a^2(b - c) + b^2(c - a) + c^2(a - b) = -(b - c)(c - a)(a - b)$
15. $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = (b - c)(c - a)(a - b)$
16. $a^3(b - c) + b^3(c - a) + c^3(a - b)$
    $= -(b - c)(c - a)(a - b)(a + b + c)$

❖ Cube Formula:-

1. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$
2. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
3. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
4. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a - b)$
5. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
6. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
7. $a^4 - b^4 = (a^2)^2 - (b^2)^2 = (a^2 + b^2)(a^2 - b^2)$
   $= (a^2 + b^2)(a + b)(a - b)$

Special Case 1: If $a^2 - ab + b^2 = 0$ then $a^3 + b^3 = 0$

Special Case 2: If $a^2 + a + 1 = 0$ then $a^3 - 1 = 0$ or $a^3 = 1$

Special Case 3: If $\frac{a}{b} + \frac{b}{a} = 1$ then $a^3 + b^3 = 0$

Special Case 4: If $\frac{1}{a} - \frac{1}{b} = \frac{1}{a-b}$ then $a^3 + b^3 = 0$

Special Case 5: If $\frac{a}{b} + \frac{b}{a} = -1$ then $a^3 - b^3 = 0$

Special Case 6: If $\frac{a}{b} + \frac{b}{a} = \frac{1}{a+b}$ then $a^3 - b^3 = 0$

Special Case 7: If $ab(a + b) = 1$ then $\frac{1}{a^3b^3} - a^3 - b^3 = 3$

❖ Factor Formula

1. $(x + a)(x + b) = x^2 + (a + b)x + ab$
2. $(x - a)(x - b) = x^2 - (a + b)x + ab$
3. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Special Case 1: If $a = b = c$

then $a^2 + b^2 + c^2 - ab - bc - ca = 0$

thus $a^3 + b^3 + c^3 - 3abc = 0$

Remember

If $a = b = c$ then $a^3 + b^3 + c^3 = 3abc$

Special Case 2: If $a + b + c = 0$ then

$a^3 + b^3 + c^3 - 3abc = 0$

Remember

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Special Case 3: $a^3 + b^3 + c^3 - 3abc$

$$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

What if two number are equal and third number is one more than the numbers than.

$a^3 + b^3 + c^3 - 3abc = a + b + c$

Special Case 4:

$a^2 + b^2 + c^2 - ab - bc - ca$

$= 1/2[(a - b)^2 + (b - c)^2 + (c - a)^2] = 3d^2$

Where a,b,c are in A.P and common difference is d.

Special Case 5:

$a^3 + b^3 + c^3 - 3abc$

$= 1/2(a+b+c)[(a - b)^2 + (b - c)^2 + (c - a)^2] = 9bd^2$

Where a,b,c are in A.P. and common difference is d and middle term is b.

❖ Function and invers function

☞ Square formula:-

If $x+\frac{1}{x}=a$ & $x-\frac{1}{x}=b$

$\Rightarrow x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=\left(x-\frac{1}{x}\right)^2+2$

Remember

$\Rightarrow x^2+\frac{1}{x^2}=a^2-2=b^2+2$

$\Rightarrow x^4+\frac{1}{x^4}=\left(x^4+\frac{1}{x^4}\right)^2-2$

$\Rightarrow x^8+\frac{1}{x^8}=\left(x^4+\frac{1}{x^4}\right)^2-2$

$\Rightarrow x+\frac{1}{x}=\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2-2$

$\Rightarrow \left(x+\frac{1}{x}\right)^2=\left(x-\frac{1}{x}\right)^2+4$

$\Rightarrow \left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2-4$

Remember

$\Rightarrow x+\frac{1}{x}=\pm\sqrt{b^2+4}$

$x-\frac{1}{x}=\pm\sqrt{a^2-4}$

First equation may be given by following type-

$\Rightarrow x+\frac{1}{x}=3$ or $x^2-3x+1=0$

or $\frac{x^2+1}{x}=3$

Special case 1 : If $x+\frac{1}{x}=\sqrt{2}$ then $x^2+\frac{1}{x^2}=0$

or $x^4+1=0$ or $x^4=-1$

Remember $\Rightarrow$ If $x+\frac{1}{x}=\sqrt{2}$ then $x^4+1=0$

Special case 2 : $x+\frac{1}{x}=\pm\sqrt{b^2+4}$ & $x-\frac{1}{x}=\pm\sqrt{a^2-4}$

$x^n+\frac{1}{x^n}=a$ then $x^n-\frac{1}{x^n}=\pm\sqrt{a^2-4}$

If $x^n-\frac{1}{x^n}=b$ then $x^n+\frac{1}{x^n}=\pm\sqrt{b^2+4}$

☞ Cube Formula:-

If $x+\frac{1}{x}=a$ & $x-\frac{1}{x}=b$

$\rightarrow x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$

$\rightarrow x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x+\frac{1}{x}\right)$

Remember

$\Rightarrow x^3+\frac{1}{x^3}=a^3-3a$

$\Rightarrow x^3-\frac{1}{x^3}=b^3-3a$

Special case 1: If $x+\frac{1}{x}=\sqrt{3}$ then

$x^3+\frac{1}{x^3}=\left(\sqrt{3}\right)^3-3\sqrt{3}=3\sqrt{3}-3\sqrt{3}=0$

Remember

$\Rightarrow$ If $x+\frac{1}{x}=\sqrt{3}$ then $x^3+\frac{1}{x^3}=0$ or $x^6=-1$

If $x+\frac{1}{x}=2$ then $x=1$,

If $x+\frac{1}{x}=-2$ then $x=-1$

☞ $x+\frac{1}{X}=a\Rightarrow x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$

☞ $x-\frac{1}{x}=a\Rightarrow x^5-\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3-\frac{1}{x^3}\right)-\left(x-\frac{1}{x}\right)$

☞ $x+\frac{1}{x}=a\Rightarrow x^6+\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)^2-2$

☞ $x+\frac{1}{x}=a\Rightarrow x^7+\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$

☞ $x+\frac{1}{x}=a\Rightarrow x^7-\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3-\frac{1}{x^3}\right)+\left(x-\frac{1}{x}\right)$

☞ $x+\frac{1}{x}=k\Rightarrow x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=\pm k\sqrt{k^2-4}$

☞ $x-\frac{1}{x}=k\Rightarrow x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=\pm k\sqrt{k^2+4}$

☞ $ax + \frac{b}{x} = k \Rightarrow a^2x^2 + \frac{b^2}{x^2} = k^2 - 2ab$

☞ $ax - \frac{b}{x} = k \Rightarrow a^2x^2 + \frac{b^2}{x^2} = k^2 + 2ab$

☞ $ax + \frac{b}{x} = k$ then $ax - \frac{b}{x} = \pm\sqrt{k^2 - 4ab}$

☞ $ax - \frac{b}{x} = k \Rightarrow ax + \frac{b}{x} = \pm\sqrt{k^2 + 4ab}$

☞ $ax + \frac{b}{x} = k \Rightarrow a^3x^3 + \frac{b^3}{x^3} = k^3 - 3kab$

☞ $ax - \frac{b}{x} = k \Rightarrow a^3x^3 - \frac{b^3}{x^3} = k^3 + 3kab$

❖ Componendo & Dividendo

If $\frac{a}{b} = \frac{x}{y}$ then

$$\frac{a+b}{a-b} = \frac{x+y}{x-y}$$

Special case:

If $x = \frac{2ab}{a+b}$ then

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = 2$$

☞ $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} + \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = 2\left(\frac{x+y}{x-y}\right)$ Or

If $x + \frac{1}{x} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} + \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ then $x + \frac{1}{x} = \frac{2(a+b)}{(a-b)}$

☞ $\frac{1}{\left(\sqrt[3]{a^2} - \sqrt[3]{a} + 1\right)} = A\sqrt[3]{a^2} + B\sqrt[3]{a} + C$

$A = 0$; $B = \frac{1}{a+1}$; $C = \frac{1}{a+1}$

☞ $\frac{1}{\sqrt[3]{a^2} + \sqrt[3]{a} + 1} = A\sqrt[3]{a^2} + B\sqrt[3]{a} + C$

☞ $A = 0$; $B = \frac{1}{a-1}$; $C = \frac{-1}{a+1}$

❖ Quadratic Equation/क्वाड्रेटिक समीकरण

1. An equation of the form $ax^2 + bx + c=0$, is called quadratic equation.

$ax^2 + bx + c = 0$ के रूप की किसी भी समीकरण को क्वाड्रेटिक समीकरण कहा जाता है

2. Roots of the equation $ax^2 + bx + c = 0$ are

समीकरण $ax^2 + bx + c = 0$ के मूल

given by $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

3. If α and β are the roots of the equation $ax^2 + bx + c=0$

यदि α और β समीकरण $ax^2 + bx + c = 0$ के मूल हैं तो

(i) Sum of roots/मूलों का योग

$$(\alpha + \beta) = \frac{-b}{a} = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

(ii) Product of roots/मूलों का गुणनफल

$$(\alpha . \beta) = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

4. If the roots α and β are known then the equation is given by $x^2 - (\alpha + \beta)x + (\alpha . \beta) = 0$

यदि मूल α और β ज्ञात हैं तो समीकरण

$x^2 - (\alpha + \beta)x + (\alpha . \beta) = 0$ होगा

5. Maximum and Minimum value of a quardratic equation/क्वाड्रेटिक समीकरण के लिए न्यूनतम और अधिकतम मान

$$F(x) = ax^2 + bx + c$$

$F(x)_{min} = \frac{4ac - b^2}{4a}$ When $a < 0$;

$F(x)_{max} = \frac{4ac - b^2}{4a}$ When $a > 0$

Nature of Roots

- If $b^2-4ac = 0$ (Roots are real & equal)
- If $b^2-4ac > 0$ & a perfect square (Roots are real, unequal & rational)
- If $b^2-4ac > 0$ & not a perfect square (Roots are real, unequal & irrational)
- If $b^2-4ac<0$ (Root are imaginary)

❖ Sum of Series/ स्यक्षि यश्यिाज्ह्ब

1. Sum of first 'n' natural numbers

$$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$$

ज्एमा nज्ाि दज्ह मिं९श्या यं

$1 + 2 + 3 + \ldots\ldots + n$ क्षि यश्यिाज्हर्य $= \frac{n(n+1)}{2}$

2. Sum of the squares of first 'n' natural numbers.

ज्एमा nज्ाि दज्ह मिं९श्या यंक्षि f, गर्यक्षि यश्यिाज्ह

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of the squares of 'n' natural numbers.

nज्ाि दज्ह मिं९श्या यंक्षि f, गर्यक्षि यश्यिाज्ह

$$= \frac{n(n+1)(n+2)}{6}$$

4. Sum of the cubes of first 'n' natural numbers.

ज्एमा nज्ाि दज्ह मिं९श्या यंक्षि न्यिज्ञंक्षि यश्यिाज्ह

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left\{n\frac{(n+1)}{2}\right\}^2$$

5. Sum of even integers/मतर्ज्रियीक्ष यंक्षि यश्यिाज्ह

$$2 + 4 + 6 \ldots\ldots + 2n = n(n+1)$$

6. Sum of odd integers/द, ड्मार्ज्रियीक्ष यंक्षि यश्यिाज्ह

$$1 + 3 + 5 + + \ldots\ldots (2n-1) = n^2$$

❖ Division Algorithm/द, प्रय्म र्ख्रे उगयक्षएत

1. Dividend/प्रय्मश = Divisor/प्रय्म क्ष f × Quotient/प्रय्माज्ह + Remember/ृ यह्मन्ह

2. Divisor/प्रय्म क्ष $= \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

3. Remainder/ृ यह्मन्ह = Dividend/प्रय्मश – (Divisor/प्रय्म क्ष × Quotient/प्रय्माज्ह)

SURDS AND INDICES/क्षत्ररयमसिकैज्यमकंक्ष

❖ Indices/ज्यमकंक्ष

= Power is greater than 1/ज्यमहितृ-र्य 1मिन्सिदु क्ष हयमअहि

❖ Surds/क्षत्ररय

= Power is less than 1/ज्यमहितृ-र्य 1मिक्षि तहियमअहि

Ex. $5^{7/3}$, Hence 7 is indices and 1/3 is surds/ $5^{7/3}$तिंन्7ज्यमकंक्ष हिंजिर्य 1/3क्षि त्ररयमहि

☞ **Laws of Indices**/ज्यमक्षि न्दिद्धत

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\frac{1}{a^n} = a^{-n}$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = a^n \div b^n$
7. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$
5. $a^{m/n} = \sqrt[n]{a^m}$
7. $a^0 = 1\ (a \neq o)$
8. $a^1 = a$
9. $a^{p/q} = (a^p)^{1/q}$
10. $a^x = a^y \Rightarrow x = y$
11. $a^x = b^y \Rightarrow a = b$

❖ Series/ स्य

(a) Infinite series in product type/गणर्य , य न्जृण्द्धंतिं-सज़्कक्तर्जा स्यश्र

Ex. $\sqrt{a\sqrt{a\sqrt{a....\infty}}}$

Solution of this question is a/बमजृण्द्धक्षर्य टू य हतृ-र्य aहियमर्यहै

Ex. $\sqrt{a\sqrt{b\sqrt{a\sqrt{b....\infty}}}}$

Solution: let $\sqrt{a\sqrt{b\sqrt{a\sqrt{b....\infty}}}} = x$

Then $\sqrt{a\sqrt{bx}} = x$

Squaring both side $a\sqrt{bx} = x^2$

Again squaring both side $a^2\, bx = x^4$

$\therefore$ $x^3 = a^2\, b \Rightarrow x = (a^2b)^{1/3}$

(b) Finite series in product type/गणर्य , य न्जृण्द्धंतिं-ज़्कक्तर्जा स्यश्र

Ex. $\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}$

Solution: count square root = 4

$2^4 = 16$

$\therefore$ Ans is $a^{15/16}$

Ex $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}}}$

$= 7^{63/64}\ (\because 2^6 = 64)$

(c) Infinite series in addition or subtraction from/ :य सिएय यज्यमय क्षि न्दि ज़्तिंसिज़्कक्तर्जा स्यश्र

$\sqrt{a \pm \sqrt{a \pm \sqrt{a \pm ....\infty}}}$

There are two type of questions/बमतंन्लियज़्धि यक्षि -जृण्द्धहियमहैं

(i): Factorise 'a' into two successive integers

aक्षि यल्यम्स तयाजर्ज़्रयक्षि क्षि न्दि ज़्तिंगिणयक्र्यम जक्षि ३-

If it is addition series then answer is greater one and/श्कर्ला स्यमि: यक्षि अहिजर्य ।र्य , य यर्ज़्रयक्षि टू यहियर्यजएयम

If it is subtraction series then answer is smaller one./श्कर्ला स्यमिन्ययक्षि अहिजिमथ र्य , य यर्ज़्रयक्षि टू यहियमक

Ex. $\sqrt{56+\sqrt{56+\sqrt{56+....\infty}}} = 8$ [56 = 8×7, larger is ans]

Ex. $\sqrt{90-\sqrt{90-\sqrt{90....\infty}}} = 9$ [90 = 10×9, smaller is ans]

(ii) If 'a' can not factorise into two succesive integers/श्कर्ला aक्षि र्य लर्य स तयाजर्ज़्रयक्षि क्षि न्दि ज़्तिं-गणयक्र्यम जन्द्धअंदिक्ष श्र्य: र्यमक्ष जर्यहैजियम

then $\sqrt{a \pm \sqrt{a \pm \sqrt{a \pm ....\infty}}} = \frac{\sqrt{1+4a} \pm 1}{2}$

Ex. $\sqrt{5+\sqrt{5+\sqrt{5+....\infty}}} = \frac{\sqrt{1+4\times5}+1}{2} = \frac{\sqrt{21}+1}{2}$

Ex. $\sqrt{1-\sqrt{1-\sqrt{1-....\infty}}} = \frac{\sqrt{1+4\times1}-1}{2} = \frac{\sqrt{5}-1}{2}$

☞ **Square root expression inside square root./** , र्गीता क्षि स्सिंर्ल्फ , र्गीता क्षि र्यिज़्पय

In this type question try to make a square of expression inside square root./बमज़िह यक्षि ज़्ज्ष्णुंर्तिं- , र्गीता क्षि प्रियक्षक्षि र्गिश : क्षि क्षि र्य , र्गी ज्झह्क्षि र्स्क्षि यू ग्र र्यक्षि ऱ्क

Ex. $\sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}\times\sqrt{2}}$

$= \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{3}+\sqrt{2}$

☞ **If square root in denominator./**श्क्ला हि ऱ र्तर्फ , र्गीता हियक्ष

In this type questions rationalise terms by multiply and divide by conjugate terms/बमज़िह य क्षि ज़्ज्ष्णुंर्तिंज़्लिर्यक्षि र्यिज़्ऱश्य ऱ्र्यमिंश्फ्तर्स्लियमि गिश्यर्जिय्य प्रयाक्षि ऱ्क्षि क्षि ऱ्क

Remember this short formula:

बमाि ज्यमिर्थक्षि र्यश्यक्तिऱ्ष्य

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} + \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = 2\left(\frac{a+b}{a-b}\right)$$

☞ **Smallest and greatest value/**र्श्ज्क्तार्जय्य सदु क्ष ज्ताियह्र

There are three type questions/बमतंर्जिय्ह ज़िह य क्षि ज़्ज्ष्णुर्दिथ फि: यहिँ

(i) $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}$

When number and power of number are same.

: मिंष्श्यजियमिंष्श्यक्षि र्स्ज्यमिमितयह्र हियक्ष

∴ We compare (number)$^{\text{LCM of 2, 3, 4 \& 6}}$

हतडिमंष्श्य 2, 3, 4 & 6 क्षर्यी मि मिज़िय ज़्रक्षि ऱमं-

[∴ Here LCM of surds 2, 3, 4, 6 is 12]

$(2^{1/2})^{12}, (3^{1/3})^{12}, (4^{1/4})^{12}, (6^{1/6})^{12},$

$= 2^6, 3^4, 6^2$

= 64, 81, 64, 36

∴ Smallest is $\sqrt[6]{6}$ and greatest is $\sqrt[3]{3}$

(ii) $\sqrt{7}-\sqrt{5}, \sqrt{9}-\sqrt{7}, \sqrt{5}-\sqrt{3}, \sqrt{11}-\sqrt{9}$

If difference of number same (Here 7 – 5 = 9 – 7 = 5 – 3 = 11 – 9) then smallest of product of numbers will be greatest and greatest of product of number will be smallest

श्क्लामिंष्श्यर्यक्षि र्यसिंजमितयह्रहिर्यडिश्हर्य 7 – 5 = 9 – 7 = 5 – 3 = 11 – 9) जर्यदि ज़्लिर्यमिंष्श्यर्यक्षि र्यगिषयज़्ह मि म- श्र्यहियर्य , हमि मर्फ ।र्स्जिर्यदि ज़्लिर्यक्षि र्यगिषयज़्ह म मर्फ ।र्यहियर्य , हमि मश्र्यस्हियस्क

7×5 = 35, 9×7 = 63, 5×3 = 15, 11×9 = 99

∴ Greatest is $\sqrt{5}-\sqrt{3}$ and smallest is $\sqrt{11}-\sqrt{9}$

(iii) $\sqrt{8}+\sqrt{3}, \sqrt{9}+\sqrt{2}, \sqrt{7}+\sqrt{4}, \sqrt{6}+\sqrt{5}$

Here sum of numbers are equal (8+3 = 9+2 = 7+4 = 6+5 = 11) then greatest of product of number terms will be greatest and smallest will be smallest.

श्हर्यमिंष्श्यर्यक्षि श्यिज़्डर्य ऱ्य ऱ्हियहिँ (8+3 = 9+2 = 7+4 = 6+5 = 11) जिर्यदि ज़्लिर्यमिंष्श्यर्यक्षि र्यगिषयज़्ह म मश्र्यहियर्य , हमि मश्र्यर्स्जिर्यदि ज़्लिर्यक्षि र्यगिषयज़्ह म मर्फ ।र्यहियर्य , हमि मर्फ ।र्स्हियस्क

⇒ 8×3 = 24, 9×2 = 18, 7×4 = 28, 6×5 = 30

Greatest is $\sqrt{6}+\sqrt{5}$ and smallest is $\sqrt{9}+\sqrt{2}$

☞ **Irrational equation/**सज़्क्तश मितर्स ऱ्य

There are two type of questions

बमतंर्लियज़िह यक्षि ज़्ज्ष्णुंहियहिँ

(i) $a+\sqrt{b} = 3+2\sqrt{5}$

then convert both side in same manner and the compare

लियज़्ज़्ढ्यक्षि र्य क्षि मितयह्क्षि ऱ्ज़्सियि ज्ञ्ज़्ज़्ज ज़्रक्षि ऱ्क

⇒ $a + \sqrt{b} = 3 + \sqrt{20} \Rightarrow a = 3$ and $b = 20$

(ii) If there are different irrational terms

श्क्ला सिज़्क्तश ज़्लिर्यक्षिह्र्यिज्य

$a\sqrt{5}+b\sqrt{3}+c\sqrt{2}+d = 3\sqrt{5}-7\sqrt{2}+5$

Compare LHS and RHS,

a = 3, b = 0, c = – 7, d = 5

❖ Arithmetic & Geometric Progression
मतंक्कसिर्कैगिषयूर्क्ष स्क्ष

☞ **Arithmetic Progresion**/मतंक्कर्क स्क्ष

Sequence/सक्ष त = a + (a + d) + (a + 2d)+.........

Where a be the first term and d is the common difference of the sequence of an A.P, then-

: हयंक्षि मअिमतयक्कर्क स्यक्षि सिक्ष तक्षि यंaज़िप्ब्माज़िलज़िष्य dमियक्षिहैजिय

(i) n^{th} term of an A.P.

मतंक्कर्क स्यक्षि यंn , यंज़्ल

t_n = a + (n –1)d

(ii) The sum of the first n terms of an A.P.

मतंक्कर्क स्यक्षज़िप्माnज़िलर्यक्षि यंशयाज़्ख्य

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

(iii) Three consecutive terms/जक्षुरि तयाजज़िल

$= a - d, a, a + d$

(iv) Four consecutive terms/द यरि तयाजज़िल

= a–3d, a–d, a+d, a+3d

☞ **Geometric Progresion**/गषयूर्क्ष स्क्ष

Sequence/सक्ष त = a + ar + ar^2 +.........

Where a be the first term and r is the common ratio of the sequence of a G.P. then-

: हयंक्षि मअिगषयूर्क्ष स्यक्षि सिक्ष तक्षि यंaज़िप्ब्माज़िलज़िष्य मय यैक्कर्क rहिजिय

(i) n^{th} term of a G.P.

गषयूर्क्ष स्यक्षि यंn , यंज़्लि $a_n = ar^{(n-1)}$

(ii) The sum of the first n terms of a G.P.

गषयूर्क्ष स्यक्षज़िप्माnज़िलर्यक्षि यंशयाज़्ख्य

$$S_n = \frac{a(1-r^n)}{1-r}; r \neq 1 \text{ and } r < 1$$

$$S_n = \frac{a(r^n-1)}{r-1}; r \neq 1 \text{ and } r > 1$$

If the common ratio is equal to 1, then the sum of the first n term of the GP is given by/शक्ल मय यैक्कर्क क्षिर्कि त्र्य त्रहियंजियगषयूर्क्ष स्यक्षि ज़िप्माnज़िलयं क्षि यंशयाज़्ख्य S_n = na

(iii) Three consecutive terms/जक्षुरि तयाजज़िल

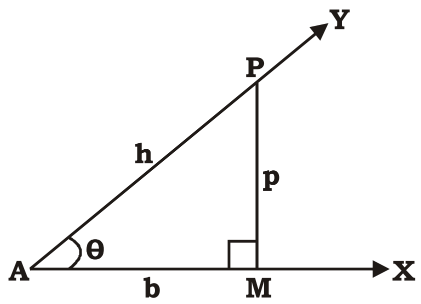
$= a, ar, ar^2$ or $\frac{a}{r}, a, ar$

(iv) Four consecutive terms/द यरि तयाजज़िल

$= a, ar, ar^2, ar^3$ or $\frac{a}{r^2}, \frac{a}{r}, ar, ar^2$

06 TRIGONOMETRY/ अ्र ात्ल र ज

1. Trigonometric Ratios/दयअ्यक्यार दज्ञि ज्मवक्य



Consider an acute angle $\angle YAX = \theta$ with initial side AX and terminal side AY. Let P be any point on the terminal side AY. PM perpendicular from P on AX to get the right angled trianlge AMP in which $\angle PAM = \theta$.

रयक्यें अक्षि बअयक्यें $\angle YAX = \theta$ हैंाद्विणरर्ांतर्ाकवक्पसीभा AX जख्यनि वद्धर सिमर्यें AY हैंसिमर्यें AY र्त ािअयाद्विद्ध्द्ध्फी हैंPM व्द्ध्फी णसिमर्यें AX र्त ाि श्एणिणतिव्र्यर्यहैद्विअअयक्यें AMP णरअयक्येंहैद्विणरर्ां $\angle PAM = \theta$ हैंा

(i) $\text{Sin}\theta = \dfrac{\text{Perpendicular / है ि}}{\text{Hypotenuse / द लय}} = \dfrac{p}{h}$

(ii) $\text{Cos}\theta = \dfrac{\text{Base / ै ां ाग}}{\text{Hypotenuse / द लय}} = \dfrac{b}{h}$

(iii) $\text{Tan}\theta = \dfrac{\text{Perpendicular / है न}}{\text{Base / ै ां ाग}} = \dfrac{p}{b}$

(iv) $\text{Cosec}\theta = \dfrac{\text{Hypotenuse / द लय}}{\text{Perpendicular / है ि}} = \dfrac{h}{p}$

(v) $\text{Sec}\theta = \dfrac{\text{Hypotenuse / द लय}}{\text{Base / ै ां ाग}} = \dfrac{h}{b}$

(vi) $\text{Cot}\theta = \dfrac{\text{Base / ै ां ाग}}{\text{Perpendicular / है न}} = \dfrac{b}{p}$

2. Relations Between Trigonometric Ratios

दयअ्यक्यार दज्ञि ज्मवक्योंअिएिद्ध णिश्एक्ष

(i) (a) $\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$

(b) $\text{cosec}\theta = \dfrac{1}{\sin\theta}$

(c) $\sin\theta \cdot \text{cosec}\theta = 1$

(ii) (a) $\cos\theta = \dfrac{1}{\sec\theta}$

(b) $\sec\theta = \dfrac{1}{\cos\theta}$

(c) $\cos\theta \cdot \sec\theta = 1$

(iii) (a) $\tan\theta = \dfrac{1}{\cot\theta}$

(b) $\cot\theta = \dfrac{1}{\tan\theta}$

(c) $\tan\theta \cdot \cot\theta = 1$

(iv) (a) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(b) $\sin\theta = \tan\theta \cdot \cos\theta$

(c) $\cos\theta = \dfrac{\sin\theta}{\tan\theta}$

(v) (a) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(b) $\cos\theta = \cot\theta \cdot \sin\theta$

(c) $\sin\theta = \dfrac{\cos\theta}{\cot\theta}$

3. Trigonometric Ratios of Some Special Angles/अँमद्विक्यर्यअयर्यअदियअयक्यक्ष दर्जानि ज्मयक्ष

Values of the trigonometric ratios for some special angles are given below:

अँमद्विक्यर्यअयर्यअवि दियअयक्यक्ष दर्जानि ज्मयर्यअरि यज्चि किले गि हिं

Angle θ / Ratio	0°	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
sin θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	$\frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$	$\frac{\sin 30}{\cos 30} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$	$\frac{\sin 45}{\cos 45} = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$	$\frac{\sin 60}{\cos 60} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \sqrt{3}$	$\frac{\sin 90}{\cos 90} = \frac{1}{0} = \infty$ (not defined)
cosec θ	$\frac{1}{0} = \infty$ (not defined)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞ (not defined)
cot θ	∞ (notdefined)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. Trigonometric Ratios of Some Special Angles/अँमद्विक्यर्यअयर्यअदियअयक्यक्ष दर्जानि ज्मयक्ष

(i) sin (90° – θ) = cos θ

(ii) cos (90° – θ) = sin θ

(iii) tan (90° – θ) = cot θ

(iv) cot (90° – θ) = tan θ

(v) sec (90° – θ) = cosec θ

(vi) cosec (90° – θ) = sec θ

5. Trigonometric Identities/दयअयक्यक्ष दज णउमुक्ष अये

A. $\sin^2\theta + \cos^2\theta = 1$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\cos\theta = \sqrt{1-\sin^2\theta}$

B. $\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$(\sec\theta + \tan\theta) = k$, then $(\sec\theta - \tan\theta) = 1/k$

C. $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

If $(\operatorname{cosec}\theta \cot\theta) = p$,

then $(\operatorname{cosec}\theta + \cot\theta) = \frac{1}{p}$

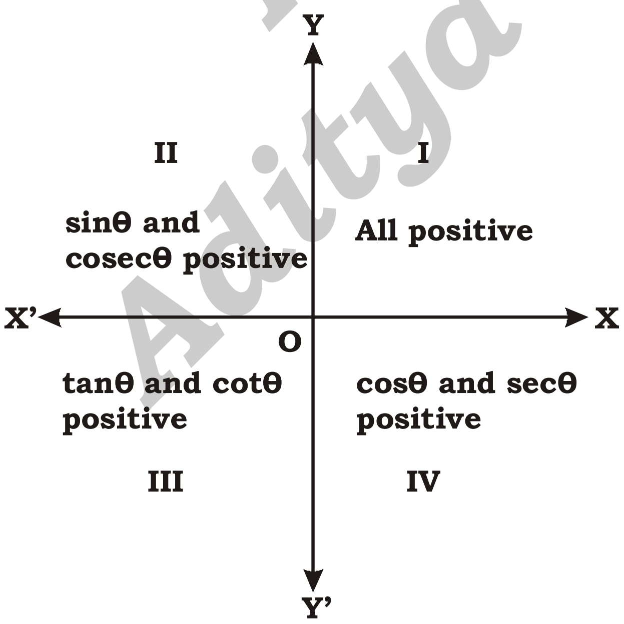
6. Trigonometric Ratios in Terms of other Trigonometric Ratios

दयक्षयक्ष्यार दर्जा नि ज्स्त्वा लिर्ब दियक्षयक्ष्यार दर्जाअनि तरिं-

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}^2\theta-1$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}^2\theta}{\sqrt{\operatorname{cosec}^2-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

7. Sign of Trigonometric Ratios

दयक्षयक्ष्यार दर्जा नि ज्स्त्वार्अदि क्ष



(i) First Quadrant/तक्ष्ति जक्ष्यक्षुंय

All trigonometric ratios are positive/ ए स्क्षीदियक्षयक्ष्यार दर्जा नि ज्स्त्वा भ्ज्यर अहिक्ष्र्हीं

(ii) Second Quadrant/क्ष्जत्व ि जक्ष्यक्षुंय

sinθ and cosecθ are positive./sinθ न यैं cosecθ भ्ज्यर अहिक्ष्र्हीं

(iii) Third Quadrant/जक्षाच ि जक्ष्यक्षुंय

tanθ and cotθ are positive.tanθ न यैं cotθ :क्ष्यर अ हक्ष्र्हीं

(iv) Fourth Quadrant/ जक्ष्युद्रि जक्ष्यक्षुंय

cosθ and secθ are positive./cosθ न यैं secθ भ्ज्यर अ हक्ष्र्हीं

Remember:	I	II	III	IV
	All	sin	tan	cos

Note:-

$\sin(-\theta) = -\sin\theta,\quad \tan(-\theta) = -\tan\theta,$

$\sec(-\theta) = \sec\theta,\quad \cos(-\theta) = \cos\theta,$

$\cot(-\theta) = -\cot\theta,\quad \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

9. Trigonometric Ratios of Some Special Angles/अँमद्धिक्पर्यअयर्सअदिग्रयक्ष्ण दर्जनि ज्मयक्ष

(i) $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$

(ii) $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$

(iii) $\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$

(iv) $\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72°$

(v) $\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54°$

(vi) $\cos 36° = \frac{\sqrt{5}+1}{4} = \sin 54°$

(vii) $\sin 22\frac{1°}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$

(viii) $\cos 22\frac{1°}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}$

10. When sum or Difference Between the Angles are Given/ज़्एअियर्सअरि :ः नि र्जे ःर्यृ ज्अर्यः यर्तिर्ग्रहियूर्

A. If A + B = 90°, then

tanA tan B = 1

sinA sec B = 1

cosA cosec B = 1

cotA cot B = 1

$\sin^2A + \sin^2B = 1$

$\cos^2A + \cos^2B = 1$

B. If A + B = 45° = 225°

(1 + tanA) (1 + tanB) = 2 or

(1–CotA). (1 – CotB) =2

C. If A + B = 135°, then

(1 – tanA) (1 – tanB) = 2 or

(1+CotA). (1 + CotB) =2

11. Max/Min Values of Trigonometric Functions/दय्अक्य दर्जता ज्यअिनि दय्अजः ह्रबजरियक्ष

A. $m \sin\theta \pm n \cos\theta$

(i) Maximum value/नदयअजरियक्ष $= \sqrt{m^2 + n^2}$

(ii) Minimum value/ह्रबजरियक्ष $= -\sqrt{m^2 + n^2}$

B. $a\sin^2\theta + b\cos^2\theta$

(i) Maximum value/नदयअजरियक्ष = Max [a,b]

(ii) Minimum value/ह्रबजरियक्ष = Min [a,b]

C. $a\sin^2\theta + b\operatorname{cosec}^2\theta$

$a\tan^2\theta + b\cot^2\theta$

$a\cos^2\theta + b\sec^2\theta$

(i) Minimum value/ह्रबजरियक्ष $= 2\sqrt{ab}$, when $a \geq b$

(ii) Minimum value/ह्रबजरियक्ष $= (a + b)$, when $a \leq b$

D. $\sin^n\theta \cdot \cos^n\theta$

When, n is odd/ज़्एिnद्धिष्यहिय्क्ष

(i) Maximum value/नदयअजरियक्ष $= +\frac{1}{2^n}$

(ii) Minimum value/ह्रबजरियक्ष $= -\frac{1}{2^n}$

When, n is even/ज़्एिnणिरहियक्ष

(i) Maximum value/नदयअजरियक्ष $= +\frac{1}{2^n}$

(ii) Minimum value/ह्रबजरियक्ष = 0

12.

(a) sin (A + B) = sin A cos B + cos A sin B

(b) sin (A – B) = sin A cos B - cos A sin B

(c) cos (A + B) = cos A cos B - sin A sin B

(d) cos (A – B) = cos A cos B + sin A sin B

(e) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

(g) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

(h) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

13. (a) $\sin 2A = 2\sin A.\cos A = \left(\frac{2\tan A}{1+\tan^2 A}\right)$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(c) $\tan 2A = \left(\frac{2\tan A}{1-\tan^2 A}\right)$

(d) $\cot 2A = \left(\frac{\cot^2 A - 1}{2\cot A}\right)$

14. (a) $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

(b) $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1 = 1 - 2\sin^2\frac{A}{2}$

$= \frac{1-\tan^2 A/2}{1+\tan^2 A/2}$

(c) $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

(d) $1 - \cos A = 2\sin^2\frac{A}{2}$ and $1 + \cos A = 2\cos^2\frac{A}{2}$

15. (a) $\sin 3A = 3\sin A - 4\sin^3 A$

(b) $\cos 3A = 4\cos^3 A - 3\cos A$

(c) $\tan 3A = \left(\frac{3\tan A - \tan^3 A}{1-3\tan^2 A}\right)$

(d) $\cot 3A = \left(\frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}\right)$

16. (a) $\sin(A+B).\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(b) $\cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

17. (a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

18. (a) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(b) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(c) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(d) $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

If $4\theta < 60°$

19. (a) $\sin\theta.\sin 2\theta.\sin 4\theta = \frac{1}{4}\sin 3\theta$

(b) $\cos\theta.\cos 2\theta.\cos 4\theta = \frac{1}{4}\cos 3\theta$

(c) $\tan\theta.\tan 2\theta.\tan 4\theta = \tan 3\theta$

For all values of A.

20. (a) $\sin(60°-A)\sin A\sin(60°+A) = \frac{1}{4}\sin 3A$

(b) $\cos(60°-A)\cos A\cos(60°+A) = \frac{1}{4}\cos 3A$

(c) $\tan(60°-A)\tan A\tan(60°+A) = \tan 3A$

21. $\tan(A+B+C) = \frac{\tan A+\tan B+\tan C - \tan A\tan B\tan C}{1-(\tan A\tan B+\tan B\tan C+\tan C\tan A)}$

At $A + B + C = 180° = \pi$,

(i) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(ii) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(iii) $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

At $A + B + C = 90° = \pi/2$

$\cot A + \cot B + \cot C = \cot A\cot B\cot C$.

Important Facts Useful for Solving Question on Trigonometry/दक्षयक यक्ष दर्जा अर्तिकर्य अर्य हा र्अ ज्ञरिंरि लर्त्अि ज्ञडिया रि ह,उतबयक्षिक

1. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

2. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

3. If $\text{Cos}\theta + \text{Cos}^2\theta + \text{Cos}^3\theta = 1$
   then $\text{Sin}^6\theta - 4\text{Sin}^4\theta + 8\text{Sin}^2\theta = 4$

4. If $\text{Sin}\theta + \text{Sin}^2\theta + \text{Sin}^3\theta = 1$
   then $\text{Cos}^6\theta - 4\text{Cos}^4\theta + 8\text{Cos}^2\theta = 4$

5. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) = 2$

6. $(1 + \cot\theta + \sec\theta)(1 + \tan\theta - \text{cosec}\,\theta) = 2$

7. $(\sin\theta + \cos\theta + 1)(\sin\theta + \cos\theta - 1) = 2\sin\theta\cos\theta$

8. $\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \sec\theta + \tan\theta = \frac{1 + \sin\theta}{\cos\theta} = \frac{\cos\theta}{1 - \sin\theta}$

9. $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$

10. $\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} = 2\sec\theta$

11. $\frac{\text{Sin}\theta}{1 + \text{Cos}\theta} = \frac{1 - \text{Cos}\theta}{\text{Sin}\theta}$

12. $\frac{\text{Cot}\theta}{\text{Cosec}\,\theta - 1} = \frac{\text{Cosec} - 1}{\text{Cot}\theta}$

13. $\text{Sec}\theta - \tan\theta = k$ i.e., $\text{Sec}\theta + \tan\theta = \frac{1}{k}$

    $\Rightarrow \text{Coces}\theta + \text{Cot}\theta = \frac{1 + k}{1 - k}$

14. $(\text{Cosec} - \text{Cot}\,\theta)^2 = \frac{1 - \text{Cos}\,\theta}{1 + \text{Cos}\,\theta}$, $0° \angle\theta\angle 90°$

15. $\frac{\tan\theta_1 + \cot\theta_2}{\tan(90 - \theta_1) + \cot(90 - \theta_2)} = \tan\theta_1 . \cot\theta_2$

16. $\tan\theta\,(1 + \sec 2\theta)(1 + \sec 4\theta)(1 + \sec 8\theta) = \tan 8\theta$

17. $\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta = 1 + 2\tan^2\theta$
    $= 2\sec^2\theta - 1$

18. $\text{cos}\,ec^4\theta - \cot^4\theta$
    $= \text{cos}\,ec^2\theta + \cot^2\theta$
    $= 1 + 2\cot^2\theta = 2\text{cosec}^2\theta - 1$

19. $\frac{\text{Cos}\,\theta + \text{Sin}\,\theta}{\text{Cos}\,\theta - \text{Sin}\,\theta} = \frac{1 + \tan\theta}{1 - \tan\theta} = \tan\left(\frac{\pi}{4} - \theta\right)$

20. $a\,\text{Cos}\theta - b\,\text{Sin}\theta = x$
    $a\,\text{Sin}\theta + b\,\text{Cos}\theta = c$
    $\Rightarrow x = \pm\sqrt{a^2 + b^2 - c^2}$

21. $a\,\text{Sec}\theta + b\,\tan\theta = c$
    $a\,\tan\theta + b\,\text{Sec}\theta = x$
    $\Rightarrow x = \pm\sqrt{c^2 + b^2 - a^2}$

22. $a\,\text{Sec}\theta - b\,\tan\theta = c$
    $a\,\tan\theta - b\,\text{Sec}\theta = x$
    $\Rightarrow x = \pm\sqrt{c^2 + b^2 - a^2}$

23. $\text{Sec}\theta + \tan\theta = \frac{a}{b}$

    $\text{Cosec}\theta + \text{Cot}\theta = \frac{a + b}{a - b}$

24. (i) (a) sin1°.sin2°.sin3°.sin4°..........sin180° = 0
    (b) sin1°.sin2°.sin3°.sin4°...........to (greater than sin 180°) = 0
    (ii) (a) cos1°.cos2°...........cos90° = 0
    (b) cos1°.cos2°........to (greater than cos90°) = 0
    (iii) (a) tan1° .tan2°.........tan89° = 1
    (iv) $\text{Sin}^2\theta + \text{Sin}^2(\theta + x) + ...$
    $+ \text{Sin}^2[90 - (\theta + x)] + \text{Sin}^2(90 - \theta) = \frac{\text{Total terms}}{2}$

Ex.

1. $\text{Sin}^2 5° + \text{Sin}^2 10° + \text{Sin}^2 15° + ...+ \text{Sin}^2 85° + \text{Sin}^2 90°$
   $= \frac{\text{Total terms}}{2} = \frac{17}{2} + 1 = 9\frac{1}{2}$

2. $\text{Sin}^2 5° + \text{Sin}^2 6° + ...+ \text{Sin}^2 84° + \text{Sin}^2 85°$
   $= \frac{\text{Total terms}}{2} = \frac{85 - 5 + 1}{2} = \frac{81}{2} = 40\frac{1}{2}$

(v) If $\sec\theta \pm \tan\theta = x$, then $\sec\theta = \frac{x^2 + 1}{2x}$

(vi) If $\sin\theta + \cos\theta = x$, then $\sin\theta - \cos\theta = \sqrt{2 - x^2}$

(vii) If $\sin\theta + \text{cosec}\theta = x$, then $\sin^n\theta + \text{cosec}^n\theta = x$

(viii) If $\tan\theta + \cot\theta = x$, then $\tan^n\theta + \cot^n\theta = x$

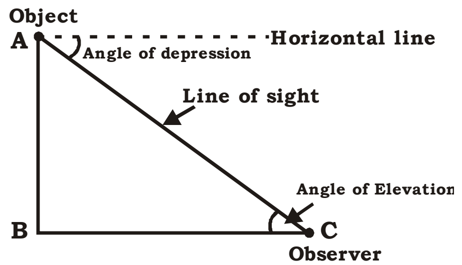
(viv) It should be noted that/ध्यान दें
$\sin^2\theta = (\sin\theta)^2$, $\sin^3\theta = (\sin\theta)^3$, $\cos^3\theta = (\cos\theta)^3$, etc.

07 Height and Distance

ÅapkbZ ,oa nwjh

☞ **The height and distance problems are solved with the help of trigonometric ratios of angle of elevation in a right-angled triangle.**

ÅapkbZ ,oa nwjh ds iz'uksa dks ledks.k f=kHkqt esa mUu;u dks.k ds f=kdks.kferh; vuqikr dh lgk;rk ls gy fd;k tkrk gSA



Trigonometric ratios are

$\sin\theta = \frac{AB}{AC}$, $\cos\theta = \frac{BC}{AC}$,

$\tan\theta = \frac{AB}{BC}$, $\sec\theta = \frac{AC}{BC}$

$\text{cosec}\theta = \frac{AC}{AB}$, $\cot\theta = \frac{BC}{AB}$

☞ **In most of the problems of height and distance, we come across the two special right-angled traingles as follow :**

ÅapkbZ ,oa nwjh ds vf/kdka'k iz'uksa esa nks fo'ks"k ledks.k f=kHkqt curs gSa

(i) Angle of elevation is 30° or 60°.

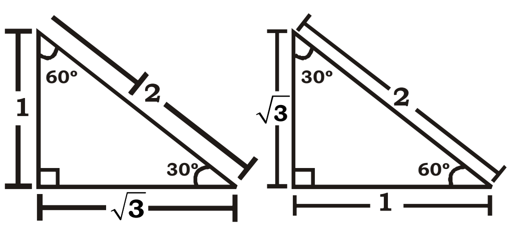
` mUu;u dks.k 30° ;k 60° gksrk gSA

(ii) Angle of elevation is 45°

` mUu;u dks.k 45° gksrk gSA

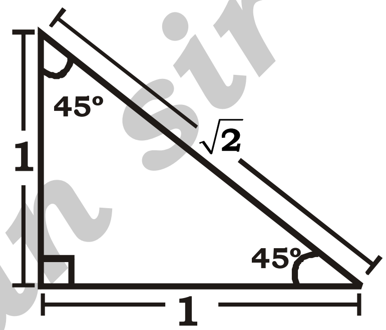
If one of the angle of a right-angled triangle is 30°, the other acute angle must be 60° and the ratio of the sides will be shown as follows :

;fn fdlh ledks.k f=kHkqt dk ,d dks.k 30° gks rks nwljk U;wu dks.k 60° gksxk rFkk Hkqtkvksa dk vuqikr fuEu izdkj gksxk



If one of the angle of a right-angled triangle is 45°, then the other acute angle will also be 45° and the ratio of the sides will be as shown below :

;fn fdlh ledks.k f=kHkqt dk ,d dks.k 45° gks rks nwljk U;wu dks.k Hkh 45° gksxk rFkk Hkqtkvksa dk vuqikr fuEu izdkj gksxk



☞ 30°-60° Relation

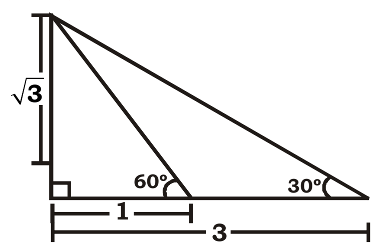
In more than 50% of height and distance problems in which two right-angled triangles are formed, angles of elevation or angles of depression will be 30° and 60°.

ÅapkbZ ,oa nwjh ds 50% ls vf/kd iz'uksa esa ftuesa nks ledks.k f=kHkqt curs gSa] mUu;u dks.k ;k vol dks.k 30° vkSj 60° gksrs gSaA

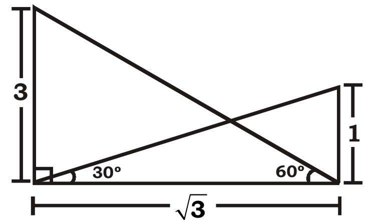
In these height and distance problems in which two angles of elevation or angles of depression are 30° or 60°. In these problems two right-angled triangles will be formed. If one of the side of these two right angled triangle is common or of equal length, the other side will be in the ratio 1 : 3.

ÅapkbZ ,oa nwjh ds bu iz'uksa esa ftuesa nks mUu;u dks.k ;k vol dks.k 30° ;k 60° gksrs gSa] bu iz'uksa esa nks ledks.k f=kHkqt cusaxs ;fn bu nksuksa ledks.k f=kHkqtksa dh ,d Hkqtk mHk;fu"B ;k cjkcj yEckbZ dh gS] rks nwljh Hkqtk 1 : 3 ds vuqikr esa gksxhA

(a)



(b)



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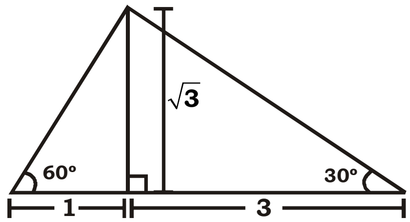
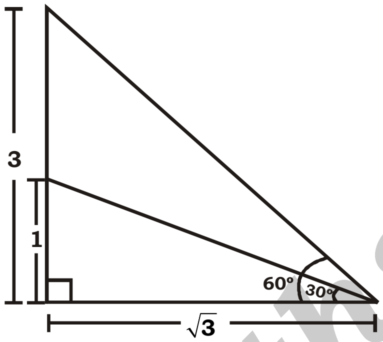


(d)



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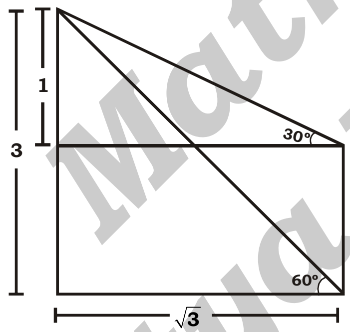

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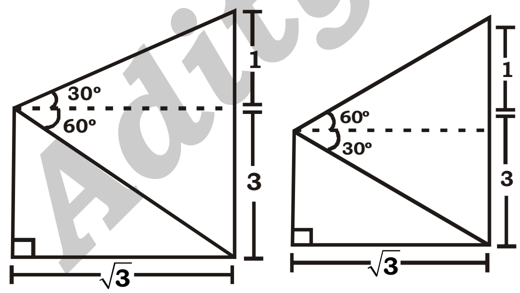


☞ **In most of the problems, of this type**

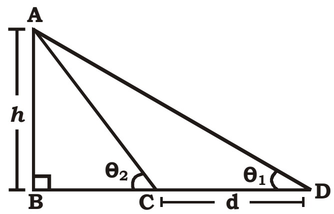
लक्ष्यनरू क्षमर खद्धीं र क्तक्ष्मनक्तक्क्र क्ष

(i) If AB will be given, then CD is asked

खी- AB ी-ख्क्ष्एक्षपक्षCD र त्यचक्षक्लमधर्त्य क्तत्युभ

(ii) If CD will be given, then AB is asked.

खी- CD ी-ख्क्ष्एक्षपक्षAB र त्यचक्षक्लमधर्त्य क्तत्युभ



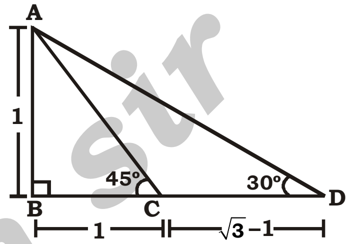
$$d = h(\cot\theta_1 - \cot\theta_2)$$

OR

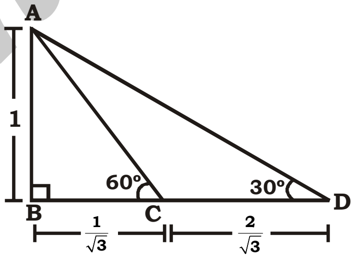
For this we will see the relation between AB and CD in different cases.

लक्ष्र खीचे यएऽ यABयद्धुक्षCDयर क्र ध्व्यीं ीण्क्तयीस्थ्वीपख्क्क्र क्ष क्षसँगय-ज्क्षयख्रुक्ष

(a)



(b)



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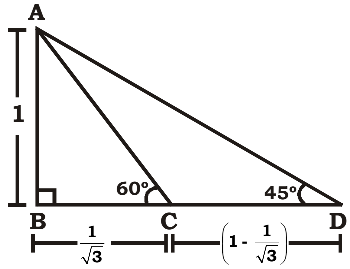
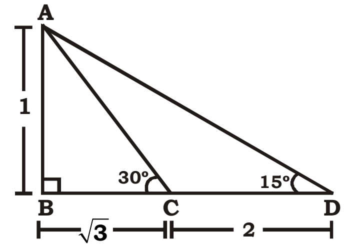


(d)



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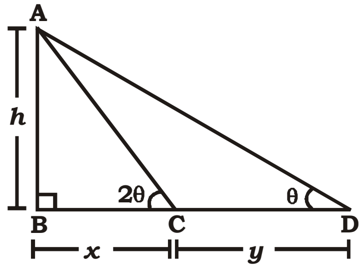
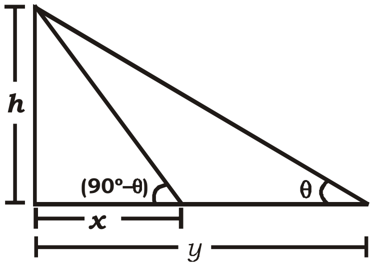


$$h^2 = y^2 - x^2$$

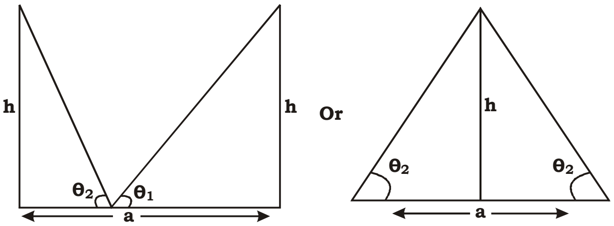
☞ **If both angles of elevation are complementary**

खी-य-दावाक्षरसख्तयर दावयक्ष् नअ यए्दामवज्ञ

$h^2 = xy$ or $h = \sqrt{xy}$

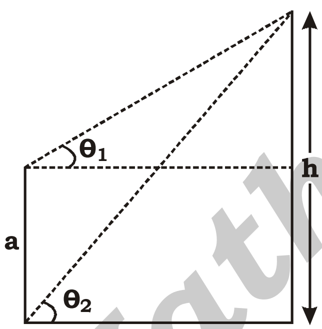


☞



$a = h(\cot\theta_2 + \cot\theta_1)$

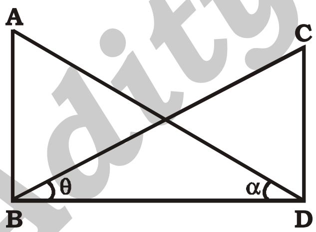
☞



$$h = \frac{a\cot\theta_2}{\cot\theta_2 - \cot\theta_2}$$

☞ **In this type, we will study the question in which the figure will be formed as shown below :**

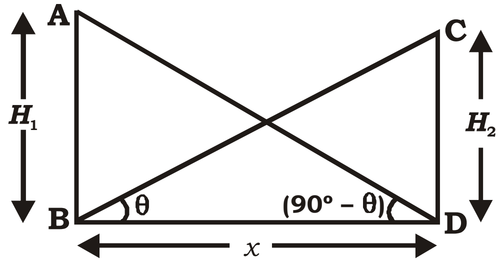
ल्क्;यनर् दक्ष्क्षाऽ ये क्षज्ञानमतद्गाक्ष व्यञ्जध्वख्तयर आर्थ क्ष्ऽखीि ज्ञ ीत् तीचीऽ वयनर् दक्ष्क्षातपदात्रुग्



(i) $\theta + \alpha = 90°$ **or both the angle of elevation are complementary.**

$\theta + \alpha = 90°$ ख्व्म-दावाक्षरसख्तयर दावयक्ष् नअ यए्दामात्रुग्

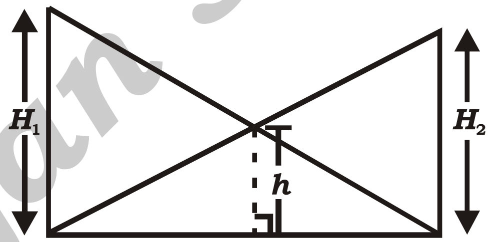
$x^2 = H_1H_2$ or $x = \sqrt{H_1H_2}$



(ii) **Height of the point of intersection of the lines joining from foot of one to top of the other from horizontal line is *h*.**

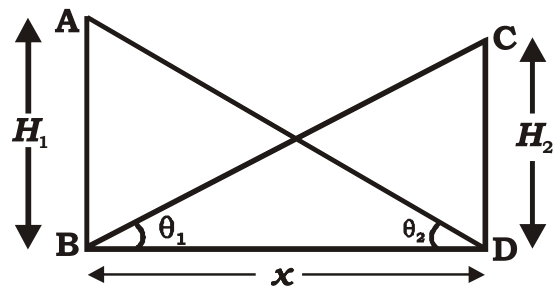
` र यर ञानक्-यर दवा-क्ष्अमर क्षामत्वायक्ष्खीऽ चत्वां दात्यअमनद्रुदाक्षर ज नीग्पध्जयीत्ङ्-दवा त्य् (ुर्प यअमनयक्ष्वहै ि त्व्य h यए्दामवज्ञ

$$\frac{1}{h} = \frac{1}{H_1} + \frac{1}{H_2}$$



(iii) **If both angles of elevation are not complementary, then we will solve these problem by using the right-angled triangles ABC and BCD.**

खी-य-दावाक्षरसख्तयर दावयक्ष् नअ यतयए्दापदाऽ ये क्षज्ञानमतद्गाक्षर दव -दावाक्ष्ऽ र दावीज्र्कतABCयद्भुअBCDयर दवनख्वायर आवाऐक एचयर आवात्रुग्



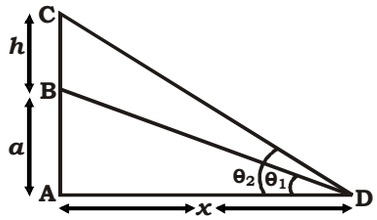
☞ **To solve this type of question, we can use following methods.**

ल्क्;यनर् दक्षर ञानमतद्गाक्षर दवाएचयर आवार खीचे यए्ऽयीत् तीचीऽ दवा ीं ीध्वक्काक्षर दवनख्वायर आक्ष्र पवात्रुयड

First Methods : In this method we use the two right-angled triangles separately.

ध्क्ष्यें ीं यत्क्ष्ऽ ञाऽ य-दाक्ष्ऽ र दावीज्र्कयद्भुचप्रड्द्भुचप्रक तापदात्रुग्

Second Method : Given $\angle ADB = \theta_1$, and $\angle ADC = \theta_2$



In most of the problems of this type

(i) If h will be given, x is asked

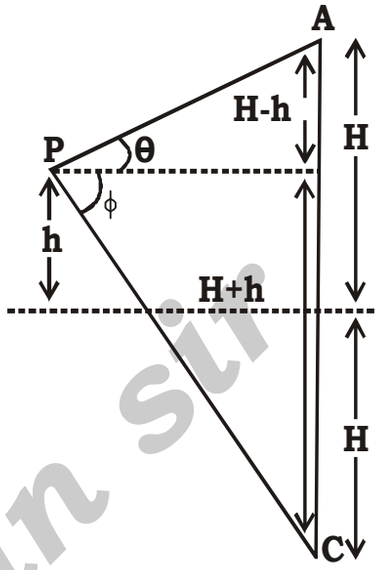
(ii) If x will be given, h is asked

$$\frac{a}{h} = \frac{\tan\theta_1}{\tan\theta_2 - \tan\theta_1} \text{ and}$$

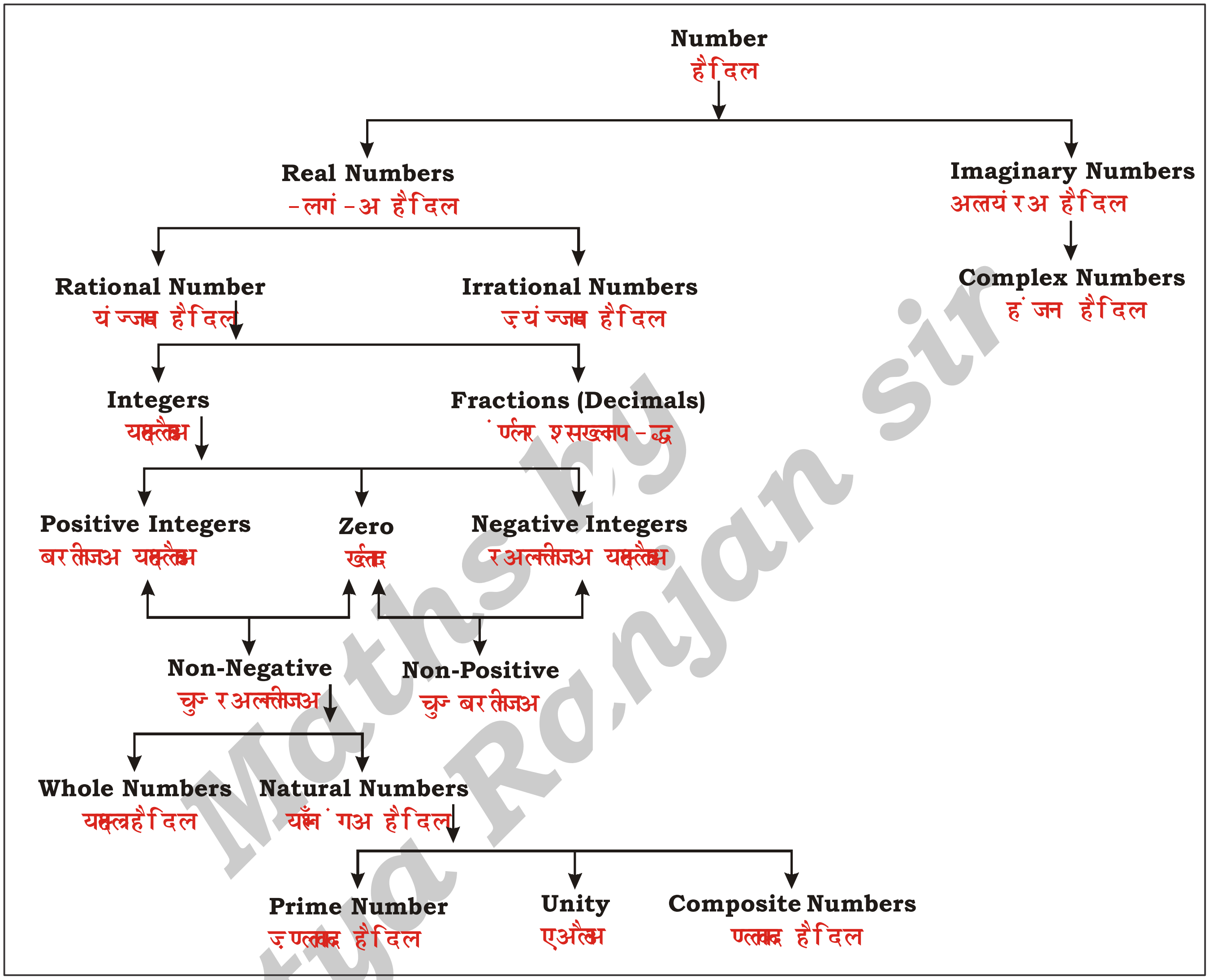
$$h = x(\tan\theta_2 - \tan\theta_1)$$

☞ **The angle of elevation of a cloud at height h above the level of water in a lake is θ and the angle of the depression of its image in the lake is ϕ. Then the height of the cloud above the surface of the lake is:**

$$H = h\left(\frac{\cot\theta + \cot\phi}{\cot\theta - \cot\phi}\right)$$



08 NUMBER SYSTEM/हैंदिलाये ´ग



A. Factor/गर्ड्ढ पारं

N = $a^p \times b^q \times c^r \times d^s \times$........

Where, a, b, c, and d are prime number.

महां a, b, c, ऊौच d ऊड्डाक्ष ़ ʹप्र्त्र हैं-

(i) **Total number of factors of a number**

तिऱ :़ʹप्र्त्र ते तक्ष गर्ड्ढ पारों त :़ पंत्र

(p+1) (q+1) (r+1) (s+1).......

For example : Total number of factors of

ज्दाहच्छुय 300 ते तक्ष गर्ड्ढ पारों त :़ पंत्र

300 = $2^2 \times 3^1 \times 5^2$ = (2+1) (1+1) (2+1)

= 3×2×3 = 18

(ii) **Total number of odd factors**

तक्ष विृत्त गर्ड्ढ पारों त :़ ʹपत्र

(q+1) (r+1) (s+1)...... where a = 2

Ex. 240 = $2^4 \times 3^1 \times 5^1$

odd factors of 240 = (1+1) (1+1) = 2×2 = 4

(iii) **Total number of even factors**

तक्ष ़ त्त गर्ड्ढ पारों त :़ ʹपत्र

p (q+1) (r+1) (s+1)...... = Total factor–odd factors

Ex. 1200 = $2^4 \times 3^1 \times 5^2$

Even factors = 4(1+1) (2+1) = 4×2×3 = 24

[or even factors = 5×2×3 – 2×3 = 24]

(iv) Sum of factors of a given number

तिन : द: हव्रम पक्ष ते गर्ळ्ह पांरों ता ऽोगष्ठल

$= (a^0+a^1+...+a^p)(b^0+b^1+...+b^q)(c^0+c^1+...+c^r).....$

$$\frac{a^{p+1}-1}{a-1}\times\frac{b^{q+1}-1}{b-1}\times\frac{c^{r+1}-1}{c-1}$$

Ex. $7056 = 2^4\times3^2\times7^2$

Sum of factors of 7056

$= (2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(7^0 + 7^1 + 7^2)$

$$= \frac{2^5-1}{2-1}\times\frac{3^3-1}{3-1}\times\frac{7^3-1}{7-1}$$

$$= \frac{31}{1}\times\frac{26}{2}\times\frac{342}{6}$$

(v) Sum of even and odd factors of a number

तिन : न पक्ष ते न त्त ऊौचविृ ात्त गर्ळ्ह पांरों ता ऽोगष्ट्ा

(a) Sum of even factors/न त्त गर्ळ्ह पांरों ता ऽोगष्ठल

$= (a^1+a^2+...+a^p)(b^0+b^1+..b^q)(c^0+c^1+...+c^r)$

$$= \left(\frac{a^{q+1}-1}{a\times1}-1\right)\times\frac{b^{q+1}-1}{b-1}\times\frac{c^{r+1}-1}{c-1}$$

$$= \frac{a^{p+1}-a}{a-1}\times\frac{b^{q+1}-1}{b-1}\times\frac{c^{r+1}-1}{c-1}$$

Ex. Sum of even factors of 270 = ?

270 ते न त्तगर्ळ्ह पांरों ता ऽोगष्ठल

$270 = 2^1 \times 3^3 \times 5^1$

Sum of even factors

$= (2^1)(3^0 + 3^1 + 3^2 + 3^3)(5^0 + 5^1)$

$= 2 \times 40 \times 6 = 480$

or

$$= \left(\frac{2^{1+1}-1}{2-1}-1\right)\left(\frac{3^{3+1}-1}{3-1}\right)\left(\frac{5^{1+1}-1}{5-1}\right)$$

$$= 2\times\frac{80}{2}\times\frac{24}{4} = 480$$

(b) Sum of odd factors/विृ ात्त गर्ळ्ह पांरों ता ऽोगष्ठल

$= a^0(b^0+b^1+...b^q)(c^0+c^1+....+c^r)$

where a = 2

$$= \frac{b^{q+1}}{b-1}\times\frac{c^{r+1}-1}{c-1}$$

Ex. Sum of odd factors of 360 = ?

360 ते विृ ात्त गर्ळ्ह पांरों ता ऽोगष्ट्ा

$360 = 2^3 \times 3^2 \times 5^1$

Sum of odd factors

$= 2^0(3^0 + 3^1 + 3^2)(5^0 + 5^1)$

$= 1 \times 13 \times 6 = 78$

or

$$= \frac{3^{2+1}-1}{3-1}\times\frac{5^{1+1}-1}{5-1} = \frac{26}{2}\times\frac{24}{4} = 78$$

(vi) Sum and number of factors satisfying other conditions

ऐे गर्ळ्ह पांरों त : न पक्ष भु ा ऽोगष्ट्ा मो दज्जच दब्ताऊों तो न भूअ तचो हैं-

(a) Sum of factors of a number which are divisible by $(b^1\times c^1)$

$(b^1 \times c^1)$ न विड़ाक्ष न पक्ष ते गर्ळ्ह पांरों ता ऽोगष्ठल

$= (a^0+a^1+.....+a^p)(b^1+b^2+...b^q)(c^1+b^2+...c^r)$

(b) Sum of factors of a number which are divisible by $(a^2\times b^3)$.

$(a^2 \times b^3)$ न विड़ाक्ष न पक्ष ते गर्ळ्ह पांरों ता ऽोगष्ठल

$= (a^2+a^3+...+a^b)(b^3+b^4+...+b^q)(c^1+c^2+...+c^r)$

So that every individual term of the expansion, there is a minimum of $a^2\times b^3$.

(d) Sum of perfect square factors of a number

तिन : न पक्ष ते ष्ब्रुस्व्रगस्गर्ळ्ह पांरों ता ऽोगष्ठल

$= (a^0+a^2+a^4+....)(b^0+b^2+....)(c^0+c^2+....)$

= Every power only

Ex. Find the sum and the number of factors of 2400 such that the factors are divisible by 15.

2400 ते ऐे गर्ळ्ह पांरों त : न पक्ष भु ा ऽोगष्ट्ा थाभ त :मिए मो 15 न विड़ाक्ष हो-

$2400 = 2^5 \times 5^2 \times 3^1$

Factors which are divisible by 15 should compulsorily have 3^1 an 5^1 in it.

∴ Sum of factors divisible by 15

$= (2^0 + 2^1 + 2^3 + 2^4 + 2^5)(3^1) \times (5^1 \times 5^2)$

$= 63 \times 3 \times 30$

= 5670

and consequently the number of factors which are divisible by 15

$= 6 \times 1 \times 2 = 12$

Ex. Find the number of factors of 1080 which are perfect squares.

1080 ते ऐे गर्ळ्ह पांरों त : न पक्ष थाभ त :मिए मो ष्ब्रुस वगस्हो-

$1080 = 2^3 \times 3^3 \times 5^1$

The number of perfect square factors of 1080 are the terms in expansion = 4

B. Remainder/बृ ाष्ट्रा

The remainder is the integer left over after dividing one integer by another to produce an integer quotient

एत ष्ड्ढांस ता तिन : दज्येष्ड्ढांस ने झाग दें ष्चष्ड्ढांस झागष्क्ल ष्ड्ढभ तर्चे त नाद बृा नड्ढ ष्ड्ढांस ता बृाष्क्ल तह्भे हैं-

Divisor पत्ता अ) Dividend पत्ताद (Quotient पत्तायवा
×××
Remainder
ख्ताययवल

Concept of Negative Remainders/ ताचर्त्त बृाष्क्ल त: ऊवशच्या

Remainder by definition are always non-negative but we use negative remainder in calculation for avoid long calculations.

ष्च्क्षिाृ ा ते ऊ आचबृाष्क्ल हत्त्बा गैर्च ताच(त्त होभ हैं- ष्चष्ल्ल्लीन: गर्ड्ढ ाने नड्ढ` ते लिए हत्` ताच(त्त बृाष्क्ल ता ष्ड्ढोग तचे हैं-

Ex. 29 % 9 = + 2 because 29 is 2 surplus

29 ता 9 ने झाग दें ष्चबृाष्ट्रा + 2 नड्ढभ है- ल्ष्ोंति ञ्ह 27 ने दो ऊश्ति हैं-

(29 = 9 × 3 +2)

= –7 because 29 is 7 deficient

ऊ ावा) 29 ता 9 ने झाग दें ष्चबृाष्क्ल –7 नड्ढभ है- ल्ष्ोंति ञ्ह 36 ने 7 तत्त हैं-

(29 = 9 × 4 – 7)

Special Case:

(i) If two numbers are each divided by the same divisor, the remainder are respectively 'r_1' and 'r_2'. If the sum of the two number be divided by the same divisor, the remainder is 'r_3'. Then

ञ्दि दो नंपत्राऊों त्ंने ष्ड्ढेत ता नर्त्त झाक्षत ने झाग दिञा माए भे बृाष्ट्रा ण्त्त्बय r_1 भ्ुा r_2 ष्ड्ढभ होभे हैं- ञ्दि दोंों नंपत्राऊों त ञोगष्क्ल ता झ: ज्ञ : झामत ने झाग दिञा माए ऊौचबृाष्क्ल r_3 ष्ड्ढभ हो भे)

Divisor/झामत = $r_1 + r_2 - r_3$

(ii) Remainder of Factor/गर्ड्ढ पांरों ते बृाष्क्ल

A dividend is divided by a divisor and there is a remainder. If factor of divisor divides same dividend then assume dividend as the remainder.

तिन : झाक्ष ता तिन : झामत खच झाग दिञा माभ है भे तख्र बृाष्क्ल ष्ड्ढभ होभ है- ञ्दि झामत ता गर्ड्ढ पांर ज्ञ : झाक्ष ता विझामिभ तचभे झाक्ष ता बृाष्क्ल र्त्त लें-

Ex. On dividing a certain number by 72 we get 11 as remainder what will be the remainder.

तिन : ंञ्किभ नंपत्रा ता 72 ने झाग दें ष्चबृाष्क्ल 11 ष्ड्ढभ होभ है) बृाष्क्ल ल्ञा होगा ञ्दि2

(i) If the same number is divided by 8

ज्ञ : नंपत्रा ता 8 ने विझामिभ तिञा माए

(ii) If double of the number is divided by 6

नंपत्रा ते दोर्गअे ता 6 ने विझामिभ तिञा माए

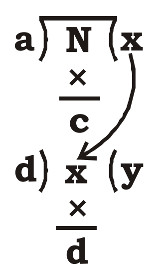
(iii) If square of the number is divided by 9

ज्ञ : नंपत्रा ते वगस्ता 9 ने विझामिभ तिञा माए

(iii) Successive Division/ण्त्तागभ विझार्म

When a number N is divided successively by a and b, remainder obtained are c and d respectively. Which mean first we divide N by a and then the quotient obtained is divided by b.

मन तिन : नंपत्रा N ता a भ्ुा b खच ण्त्तागभ ँ ष्ड ने झाग दिञा माभ है भे बृाष्क्ल ण्त्त्बय c भ्ुा d ष्ड्ढभ होभ हैं- कत ता ऊ ासहोभ है ति ष्हले हत्त N ता a ने झाग देभे है ऊौच ष्ठिष्ठ ष्ड्ढभ झागष्क्ल ता b ने विझामिभ तचे हैं-



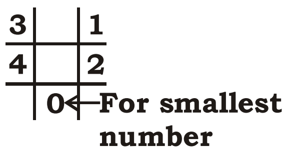
Ex. A number on being divided by 3 and 4 successively leaves the remainder 1 and 2 respectively. Find the remainder when the same number is divided by 10.

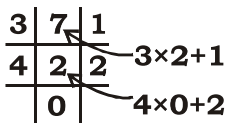
तिः : ंपक्रा तो 3 ुा 4 खच ण्त्तगभ्ँ ब े झाग दिए र्मा ँ छच बोृ ाष्टा ण्त्बय 1 ुा 2 ष्ह्रभ होभे हैंः ज्ञ : ंपक्रा तो 10 झाग दें ँ छचष्ह्रभ बोृ ाष्क्त थ्रभ त :मिएः

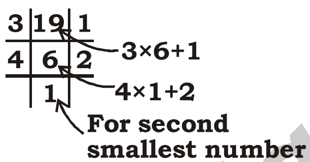
Reverse the position of remainder and quotient and start question backward.

बोृ ाष्क्त (ुा झागष्क्त त : क्रिाभि तो ज्लम्रा तचदें ऊौच ण्र्ब्रे तो ष्:छेे ह्ल तर्चा बौंअ तचें

Quotient Remainder







Some important rules for remainder/बोृ ाष्क्त तष्ठ त्त्ह(वष्क्रार्ष बत

(a) Remainder $\frac{(a^n + b^n)}{(a+b)} = 0$ when n is odd

Ex. $\text{Re}\left(\frac{8^{371} + 5^{371}}{13}\right) = 0 [\text{Here } 13 = 8 + 5]$

(b) Remainder $\frac{(a^n + b^n + c^n)}{(a+b+c)} = 0$ when n is odd

Ex. $\text{Re}\left(\frac{3^{61} + 2^{61} + 4^{61}}{9}\right) = 0$

(c) Remainder $= \left[\frac{a^n + b^n + c^n + ...}{(a+b+c)}\right] = 0$ if $(a + b + c + ...)$ are in Arithmetic progression and n is odd

Ex. $\text{Re} = \left(\frac{16^{73} + 17^{73} + 18^{73} + 19^{73}}{9}\right)$

$\because$ n = 73 (odd), 16+17+18+19 = 70 and 16, 17, 18 and 19 are in A.P.

(d) Remainder $\left[\frac{a^n - b^n}{(a-b)}\right] = 0$ for all value of n

Ex. $\text{Re}\left(\frac{8^{36} - 2^{36}}{6}\right) = 0$

(e) Remainder $\left[\frac{a^n - b^n}{(a+b)}\right] = 0$ when n is even.

Ex. $\text{Re}\left(\frac{7^{24} - 4^{24}}{11}\right) = 0$

(f) Remainder $\frac{(a+1)^n}{a} = 1$ for all value of n

Ex. $\frac{74^{25}}{73} \Rightarrow \frac{74 \times 74 \times ....}{73} \Rightarrow \frac{1 \times 1 \times 1 \times ....}{73} \Rightarrow 1$

(g) Remainder $[(a - 1)^n/a] = 1$ when n is even.
$= (a - 1)$ or -1 when n is odd

Ex. $34^{282} \% 35 = (-1)^{282} \% 35 = 1 \% 35 = 1$
$34^{281} \% 35 = (-1)^{281} \% 35 = -1 \% 35 = 35 - 1 = 34$

C. Rules of divisibility/विझाक्ष्भ ते ि ३त्त

(i) **Divisibility by 2:** A number is divisible by 2 if the last digit of the number is 0, 2, 4, 6, or 8.
2ं विझाक्ष्भयतोक्स ंपक्रा 2ं विझाक्ष होग: ३दि ंपक्रा ता क्सािक्सऊंत 0, 2, 4, 6 ऊु ावा 8 होः

(ii) **Divisibility by 3:** A number is divisible by 3 if the sum of the digits of the number is divisible by 3.
3ं विझाक्ष्भयतोक्स ंपक्रा 3 ३ विझाक्ष होग: ३दि ंपक्रा ते ऊंतें ता ३ोग 3ं विझाक्ष होः

(iii) **Divisibility by 4:** A number is divisible by 4 if the last two digits of the number are divisible by 4.
4ं विझाक्ष्भयतोक्स ंपक्रा 4 विझाक्ष होग: ३दि ंपक्रा ते ऊठ्भित्त दो ऊंत 4ं विझाक्ष होः

(iv) **Divisibility by 5:** A number is divisible by 5 if the last digit of the number is 0 or 5.
5ं विझाक्ष्भयतोक्स ंपक्रा 5ं विझाक्ष होग: ३दि ंपक्रा ता क्सािक्सऊंत 0 ऊु ावा 5 होः

(v) **Divisibility by 6:** A number is divisible by 6 if the number is divisible by both 2 and 3.
6ं विझाक्ष्भयतोक्स ंपक्रा कं विझाक्ष होग: ३दि ंपक्रा 2 ुा 3 दों ेंं विझाक्ष होः

(vi) **Divisibility by 7:** Multiply last digit by 2 and subtract from remaining number. Repeat this process until number become less then check divisibility of 7.
7ं विझाक्ष्भयः ंपक्रा ते ऊठ्भित्त ऊंत तो 2ं गक्षा तचें ऊौचबोृ ांपक्रां बमा देंः३ह ष्छि३ा भ्न भ्त दोहचएं मन भ्त ति ंपक्रा छोम: ा हो माए ऊौच ष्छि्ा ंाभ त: विझाक्ष्भ मांः

(vii) **Divisibility by 8: A number is divisible by 8 if the last 3 digits of the number are divisible by 8.**

8 से विभाज्यता की जांच पिछले 8 विभाज्यता होगी: यदि संख्या के अन्तिम 3 अंक 8 से विभाज्य हो।

(viii) **Divisibility by 9: A number is divisible by 9 if the sum of the digits is divisible by 9.**

9 से विभाज्यता की जांच पिछले नौ विभाज्यता होगी: यदि संख्या के अंकों का योग 9 से विभाज्य

(viv) **Divisibility by 11: If the difference between the sum of the digits at odd place and sum of the digits at even places is equal to zero or multiple of 11.**

11 से विभाज्यता यदि संख्या के विषम स्थानों के अंकों का योग तथा संख्या के सम स्थानों के अंकों के योग का अन्तर 11 का गुणज हो तो संख्या 11 से विभाज्य होगी।

(ix) **Divisibility by 12: A number is divisible by 12 if the number is divisible by both 3 and 4.**

12 से विभाज्यता की जांच पिछले 12 से विभाज्यता होगी: यदि संख्या 3 तथा 4 दोनों से विभाज्य हो।

D. Unit Digit/इकाई अंक

The unit's digit of an expression can be calculated by getting the remainder while the expression is divided by 4.

किसी व्यंजक के इकाई अंक ज्ञात करने के लिये व्यंजक में 4 का भाग देने से जो शेषफल प्राप्त होता है।

Ex. What will be the unit's digit of $(382)^{575}$?

$(382)^{575}$ का इकाई अंक क्या होगा?

Step/चरण 1:

Divide last 2 digits of power by 4 and find out remainder.

घात के अन्तिम 2 अंकों को 4 से भाग देकर शेषफल प्राप्त करते हैं।

$$4\overline{)75}\,(18$$
$$\underline{4}$$
$$35$$
$$\underline{32}$$
$$3$$

Step/चरण 2:

Put remainder as a power of unit place number and find out answer.

शेषफल को इकाई अंक संख्या के घात के रूप में रखकर उत्तर ज्ञात करते हैं।

$2^3 = 8$

Note: In Step 1, if remainder is 0 is then put power equal to 4.

चरण 1 में यदि शेषफल 0 हो तो घात 4 मानते हैं।

Ex. Find the units digit in each of the following cases.

निम्नलिखित में से प्रत्येक का इकाई अंक ज्ञात कीजिए।

(i) $(187)^{282} \times (529)^{321} \times (343)^{236}$

(ii) $(789)^{315} + (232)^{644} + (528)^{253}$

(iii) $(982)^{481} - (219)^{241}$

(i) **Divide last 2 digits of power by 4 and put remainder as a power of unit place digit.**

घात के अन्तिम 2 अंकों को 4 से भाग देकर शेषफल को इकाई अंक के घात के रूप में रखकर

$7^2 \times 9^1 \times 3^4 \Rightarrow 9 \times 9 \times 1 \Rightarrow 1$

(ii) $9^3 + 2^4 + 8^1 \Rightarrow 9 + 6 + 8 \Rightarrow 3$

(iii) $2^1 - 9^1 \Rightarrow 12 - 9 \Rightarrow 3$

[If first number is less than add 10 in it]

[यदि घटाने वाली संख्या छोटी हो तो उसमें 10 जोड़ दें]

E. LCM and HCF/लघुत्तम और महत्तम

☞ **Product of two numbers/दो संख्याओं का गुणनफल = LCM × HCF**

☞ **Product of "n" numbers/"n" संख्याओं का गुणनफल = $HCF^{(n-1)} \times LCM$**

(1) **L.C.M. of Fractions/भिन्नों का लघुत्तम**

$$= \frac{\text{L.C.M. of Numbnerators}}{\text{H.C.F. of Denominators}}$$

(2) **H.C.F. of Fractions/भिन्नों का महत्तम**

$$= \frac{\text{H.C.F. of Numberators}}{\text{L.C.M. of Denominators}}$$

(3) **H.C.F. and L.C.M. of polynomials.**

बहुपदों का महत्तम तथा लघुत्तम

(a) **H.C.F. of two or more polynomials is the "Common factor of highest degree".**

दो या दो से अधिक बहुपदों का महत्तम /अधिकतम बार का गुणनखण्ड।

(b) **L.C.M. of two or more polynomials is the "Common multiple of least degree".**

दो या दो से अधिक बहुपदों का लघुत्तम /न्यूनतम बार का गुणनखण्ड।

(c) $\text{L.C.M.} = \frac{1^{st}\text{ polynomial} \times 2^{nd}\text{ polynomial}}{\text{H.C.F.}}$

(d) $\text{H.C.F.} = \frac{1^{st}\text{ polynomial} \times 2^{nd}\text{ polynomial}}{\text{L.C.M.}}$

(e) **L.C.M. × H.C.F. = Product of two polynomial**

LCM : प्र ह त्प

(I)	Find the smallest no. Which is exactly divisible by x, y, z. वह न्यूनतम संख्या ज्ञात कीजिए जो x,y,z से पूर्णतः विभक्त हो।	LCM of (x,y,z)
(II)	Find the smallest no. Which when divided by x, y, z leaves remainder "r" in each case. –,ſ दसगजाहै दितह त्लाअंप्रुष एाु तक्क x,y,zाह म्मक्त्मा:ाप्त्ल्मा सत्मासजयीद्धा सत्खताजैमत्द्धा ाय्मा ,त्मा	LCM (x,y,z) + r
(III)	Find the smallest no. Which when divided by x, y, z leaves remainder a,b,c, respectively. वह न्यूनतम संख्या ज्ञात कीजिए जो x,y,z से पूर्णतः भाग देने पर शेषफल क्रमशः a,b,c प्राप्त हो।	LCM (x,y,z) – k Where, k = (x–a) = (y–b) = (z – c)

HCF : जह त्प

(I)	Find the largest no. Which is exactly divisible by x, y, z. वह बड़ी से बड़ी संख्या ज्ञात कीजिए जो x,y,z से पूर्णतः विभक्त हो।	HCF of (x,y,z)
(II)	Find the largest no. Which when divided by x, y, z leaves remainder "r" in each case. –,ſ दसगजा है दिताइ त्ला अंप्रुष एा ु तक्क x,y,zा ह म्प्त्ल्मा सत्मासजयीद्धा सत्खताजैमत्द्धावा ाr य्मा ,त्माता ,त्मा	HCF [(x–r),(y-r),(z-r)] or HCF [x-y),(y-z),(z-x)]
(III)	Find the largest no. Which when divided by x, y, z leaves remainder a, b, c respectively. वह न्यूनतम संख्या ज्ञात कीजिए जो x,y,z से भाग देने पर शेषफल क्रमशः a,b,c प्राप्त होता हो।	HCF [(x-a),(y-b),(z-c)]

F. Counting of Numbers/ϡ प्त्राओं त : र्गि भः

(1) Counting a Digit/ऊंतों त : र्गि भः

Ex. How many times 5 will come from 350 to 600.

350 ϡ 600 त नःघ 5 तिर्भ : नाचऊाएगा϶

Sol.

	I Place	II Place	III Place
350 – 400	5	10	×
400 – 500	10	10	1
500 – 600	10	10	99

(500 alread count)

Ans.155

(2) Digits required to write counting.

र्गि भः तो लिर्पा े ते लिए ऊावक्त ऊंत϶

Ex. How many digits required to write counting from 1 to 400?

1 ϡ 400 भ्त त : र्गि भः तो लिर्पा े ते लिए ऊावक्त ऊंतों त : ϡ प्त्रा तिर्भ : हैϨ

Single digits 1 to 9 ⇒ 9 number × 1 digit per number = 9

Double digits 10 to 99 ⇒ 90 number × 2 digits per number = 180

Three digits 100 to 400 ⇒ 301 number × 3 digits per number = 903

Ans: 9+180+903 = 1092 Digits

(3) Sum of digits/ऊंतों ता ञोगष्ट्रा

Ex. Find out sum of all digits from 1 to 100.

1 ϡ 100 भ्त त : ϡ प्त्रा र्त ष्ठक्का ϡ झा: ऊंतों ता ञोगष्ट्रा ञाभ त :मिए϶

Unit place digit/क्ता ाक्सऊंत ⇒ 10 (1 + 2 +.........+9) = 450

Second place digit/दहाक्स ऊंत ⇒ 10 (1 + 2 +...........+ 9) = 450

Third place digit/ϡैत रष्ठ ⇒ 1

Total = 450 + 450 + 1 = 901

☞ **Least number to be added or subtracted to given number so it becomes divisible of a divisor.**

तिϡ : दः हक्का प्त्रा र्त ϡ नϡ छोमः ϡ प्त्रा क ष्ठाच मोर्ϡा,बर्मा ा ति ञह तिϡ : झामत ϡ विझामिभ हो माए϶

Ex. What is least number to be added to 42072 to get a number which is divisible by 93?

42072 र्त वह तों ञ : ठ्जभ्त ϡ प्त्रा मोरष्ठ माए ति ष्ठभ ϡ प्त्रा 93 ϡ विझामिभ हो माएϨ

$$93 \overline{)42072}(452$$

```
93)42072(452
   372
    487
    465
     222
     186
      36
```

So least number to be added is/क लिए)ϡ नϡ छोमः मोरष्ठ मा ϡ तϡ वाल: ϡ प्त्रा 93 – 36 = 57

☞ **Prime Number/**ऊझाक्ष ϡ प्त्रा

Except 1 each natural number which is divisible by only 1 and itself is called a prime number.

1 ता छोरत चष्छेत ष्ठ भ्ति ϡ प्त्रा मो ि ष्ठक्ता भ ाा पाक्ष ϡ ह: विझाक्ष हो)ऊझाक्ष ϡ प्त्रा तहलाभः है϶

To check whether a number is prime number or not, first take the square root of the number. Round of the square root to the immediately lower integer. Then check divisibility of number by all prime below it. If number is not divisible by any prime number then number is prime number.

तोक्क प्त्रा ऊझाक्ष है ञा ह:)ञह ञाभ तचें ते लिए) ϡ नϡ ष्ठहले ϡ प्त्रा ता वगस्ज लें)वगस्त्ल तो ज्ϡ छोमः ष्ठक्तांस तें ष र्त लिपा लें϶ज त नाद ϡ प्त्रा त : विझाक्ष भ त : मांष ज्ϡ छोमः ϡ झा: ऊझाक्ष ϡ प्त्राओं खचा तचें ञदि तिϡ : झा: ऊझाक्ष ϡ प्त्रा ϡ विझाक्ष ह: होभः भे ञह ऊझाक्ष ϡ प्त्रा है϶

Ex. 137 is prime number or not?

137 ऊझाक्ष २ पङ्ा है ३ा ँ हःस्

$\sqrt{137} \simeq 11 \Rightarrow$ prime number less than or equal to 11 are 2, 3, 5, 7 and 11, 137 is not divisible by any of there. Hence it is prime numbers.

11 ो छोमः ३ा नचनचऊझाक्ष २ पङ्ाएं 2, 3, 5, 7 भु ाा 11 है) 137 र्क तें े तिे : ते झः खच विझाक्ष ँ हः है क लिए ३ह एत ऊझाक्ष २ पङ्ा है

Ex. What is average of prime number from 80 to 100?

80 ो 100 भ्त त : ऊझाक्ष २ पङ्ाओं ता ऊेंा भ ल्ङ्ा हैस्

$\sqrt{100} = 10 \Rightarrow$ Prime number less than 10 are 2, 3, 5, 7 Hence, even number and ending with 5 will not prime numbers so only check divisibility of 3 and 7.

10 ो तत्त त : ऊझाक्ष २ पङ्ाएं 2, 3, 5, 7 हैं) क च्छाच २ त्त २ पङ्ा ऊौच5 का ाक्षःंत वालः २ पङ्ाएं ऊझाक्ष २ पङ्ाएं ँ हः होगःे क लिए िच्ख3 ऊौच 7 त : विझाक्ष भ माडे

Prime number are 83, 89 and 97.

ऊझाक्ष २ पङ्ाएं 83, 89, 97 हैं

Ans: $\frac{83+89+97}{3} = \frac{269}{3} = 89.67$

☞ Some points about prime numbers.

ऊझाक्ष २ पङ्ाएं ो २ ीनठिभ तङ्क त्त्ह(वच्छ्द्ध्सनिद्धअ

(i) Prime number between 1 to 100 are

1 ो 100 ते नःध त : ऊझाक्ष २ पङ्ाएं

(1 to 50 ⇒ 15, 50 to 100 ⇒ 10)

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

(ii) If a and b are any two odd primes then $a^2 + b^2$ and $a^2 - b^2$ is composite numbers.

३दि a भु ाा b दो वि्ा त्त ऊझाक्ष २ पङ्ाएं हैं भे $a^2 + b^2$ भु ाा $a^2 - b^2$ झाक्ष २ पङ्ाएं होगःे

Co-Prime Numbers/२ ह2ऊझाक्ष २ पङ्ाएं

Two natural numbers are called co-prime (relatively prime) number if they have no common factor other than 1.

दो च्छू भ्ति २ पङ्ाएं २ हऊझाक्ष ३ा २ ाछेस्र ऊझाक्ष तहलाभः है ३दि 1 ते िवाए र्ज ते तोक्ष्डौचज्झ्ि्द्ध गर्द्ध पां ँ होे

Ex. (3,8), (9,10),(12,17)

☞ Comparison of Fraction./झिठों त : भ्र्ज ा

It is advisable to compare two fraction by cross multiplication.

झिठों त : भ्र्ज ा भ्छ्छे गर्द्ध पां विशि खच तर्चे त : २ लाह लः माभः है

Ex. The greatest value among the fractions.

र्ीं लिपिाभ तें े २ क्ङ नर२ झिाद्ध तों ञ : है

$\frac{2}{7}, \frac{1}{3}, \frac{5}{6}, \frac{3}{4}$ is :

$\frac{2}{7} \times \frac{1}{3} \Rightarrow 6 < 7$

$\frac{1}{3} \times \frac{5}{6} \Rightarrow 6 < 15$

$\frac{5}{6} \times \frac{3}{4} \Rightarrow 20 > 18,$

So $\frac{5}{6}$ is greatest fraction.

☞ Special case of $a\frac{b}{c}$ numbers

$a\frac{b}{c}$ च्छाचत : २ पङ्ाओं च्चऊाशचिा विबेृ ा क्राभि

(a) If denominator of a number same as multiplier

३दि तिे : २ पङ्ा ता हचज्ञ ते गर्द्धम ते २ र्त्ा हो भे

Ex. $999\frac{994}{999} \times 999$ — Same

Add 5 in 994 so it becomes 999

994 तें 5 मोर्ठे े ३ह 999 हो माभः है

$999\frac{(994+5)}{999} \times 999$

= (999 + 1) × 999

= 999000

Now substrate 5

= 999000 – 5 = 998995

(b) If difference between numerated and denominator is 1.

३दि ऊंश ऊौचहचते नःध ता ऊंच्च1 हो भे

Ex. $\frac{1}{8} + 999\frac{791}{792} \times 99$

Here 792 – 791 = 1 so we can write $\frac{791}{792} = 1 - \frac{1}{792}$

$= \frac{1}{8} + \left(999 + 1 - \frac{1}{792}\right) \times 99$

$= \frac{1}{8} + 99000 - \frac{99}{792} = 99000$

(c) Series Type/शृंखला प्रकार

Ex. $999\frac{1}{5}+999\frac{2}{5}+.....+999\frac{4}{5}$

$$999\times4+\frac{1+2+3+4}{5}$$

$$=(400-4)+\frac{(4\times5)/2}{5}$$

$$=3996+2=3998$$

Ex. $99\frac{1}{11}+99\frac{2}{11}+.....+99\frac{10}{11}$

$$=99\times10+\frac{10}{2}=995$$

☞ Bar Type Questions/नाचवाले प्रश्न

Ex. 0.7777..... ∞ = 0. $\bar{7}$

Let x = 0.777.....∞

10 x = 7.777.....∞

Subtract (1) from (2)

$$9x=7\Rightarrow x=\frac{7}{9}$$

Ex. 0.535353.....∞

$$=\frac{53}{99}$$

Ex. 0.28383..... ∞ = $0.2\overline{83}$

Let x = 0.2838383.....∞

10 x = 2.838383.....∞ .....(1)

1000 x = 283.838383.....∞ .....(2)

Subtract (1) from (2)

990 x = 281

$$x=\frac{281}{990}$$

Ex. $0.5\overline{73}$

$$=\frac{573-5}{990}=\frac{568}{990}$$

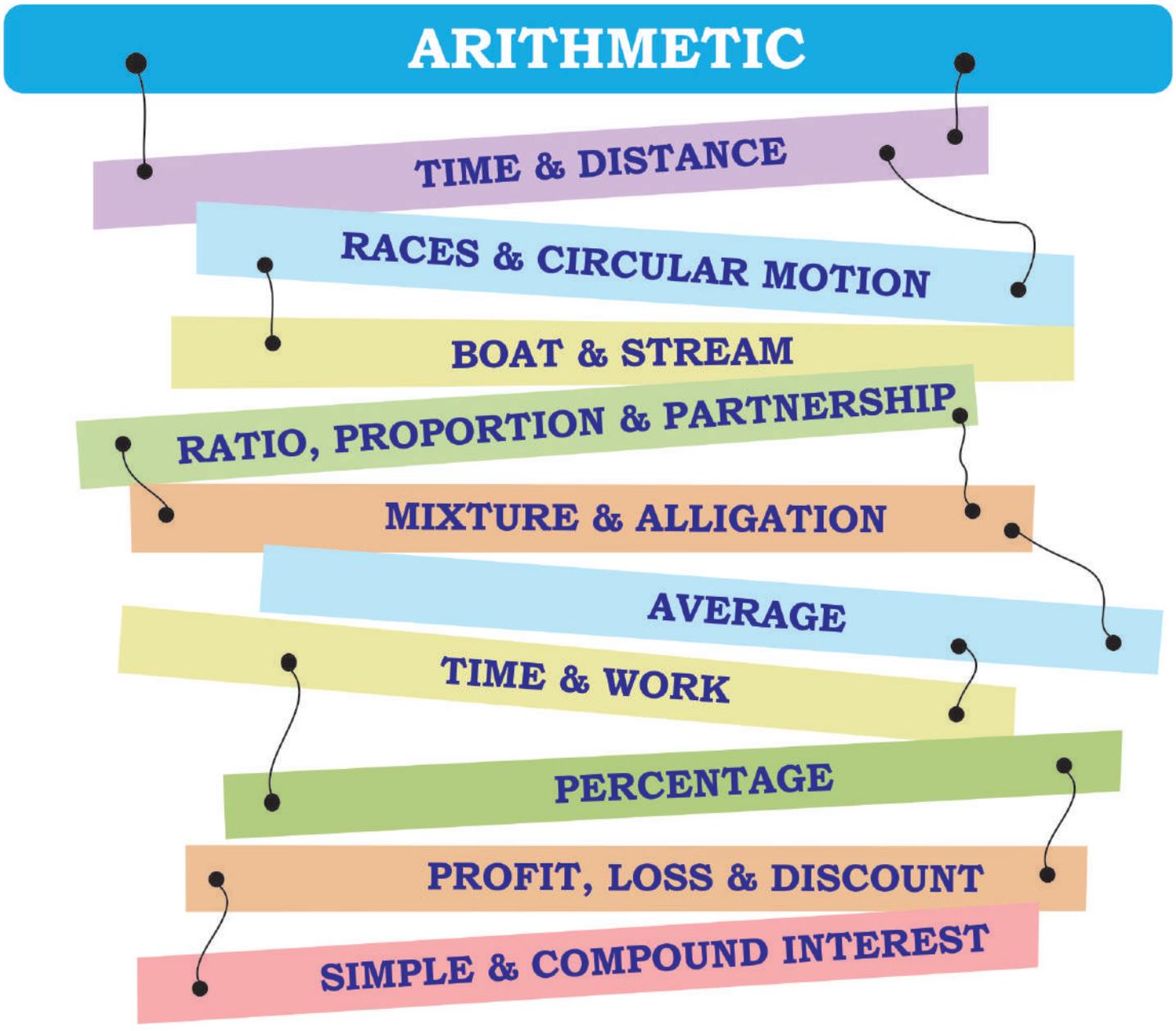
Ex. $0.38\bar{7}$

$$=\frac{387-38}{900}$$

$$=\frac{349}{900}$$

Ex. $0.00\bar{3}$

$$=\frac{3}{900}=\frac{1}{300}.$$


ARITHMETIC
TIME & DISTANCE
RACES & CIRCULAR MOTION
BOAT & STREAM
RATIO, PROPORTION & PARTNERSHIP
MIXTURE & ALLIGATION
AVERAGE
TIME & WORK
PERCENTAGE
PROFIT, LOSS & DISCOUNT
SIMPLE & COMPOUND INTEREST

09 TIME AND DISTANCE/है ि दलोय्दं `ग-

SPEED, TIME AND DISTANCE

Definition : The speed of a body is defined as the distance covered by it in unit time.

अेतारादजनग्ज ानहै ि दै क्र ि दि्ज -दबनबं `गन्दज ाती ाज्दज ण्ऽतण्क

Basic Formulae

Speed $= \frac{\text{Distance}}{\text{Time}}$ /ी ाज्द= $\frac{\text{`ग-}}{\text{है ि}}$

Time $= \frac{\text{Distance}}{\text{Speed}}$ /है ि द= $\frac{\text{`ग-}}{\text{ी ाज्}}$

Distance = Speed × Time

`गन्द= ी ाज्द× है ि

Two frineds left Delhi for Goa at 5 a.m. One friend who went by train reached Goa in 24 hours and other friends who went by Aeroplane reached in 3 hours.

ातैअ ादअं श्ज्-दह तबाबाद्ज तउक्खद सामदळ्झए चतुअ्ज ज्ऽतण्क्ष्खज अं ादे तबाभ्ँ-दह तउवदषक्ष्वै क्लोय्दं हे ादण्सानबचणचदह तइदषक्ष्त ` क्षबाबाद णीप्रऽादण्क

We know that speed of aeroplane is more than train so time taken by train is more than that of aeroplane.

पै दचा्ु ऽतण्क्षअ दणसानबचणचद्ज -दी ाज्दे तबाभ्ँ-दह तलअ्ज णात्र-दणक्षनह अक्खदे तबाभ्ँ-दु ी ादण्सानबचणचदह तलअ्ज दहै ि अक्ष ादिबा िक

It can be infer from above explanation that

Speed $\propto \frac{1}{\text{Time}}$ **(When distance is constant)**

त ि्रूप्रऽदह तणिदा र,दणव्अ दी ाज्द$\propto \frac{1}{\text{है ि}}$ ` चणदं `गन्दुअि ऽ णघ

If two athlete run for constant time then distance covered by the athlete whose speed is more would be more.

िअंद ात-ासज्दुअि ऽ दहै ि दि्ज तउक्खदं ाथैंक्ऽातअ्यहज्-दी ाज् लअज्ज दणाब-दसणदलअ्ज दं `गन्दऽ ि दि्ज बाक

It can be infer from above explanation that distance ∝ time taken (when speed is constant)

त ि्रूप्रऽदआसे धादहता र,दणव्अ `गन्द∝ दअक्ष ि ताबा तहै ि द्रि चएदी ाज्दुअि ऽदणाघ

Units of Measurement/ ा द्ज तै ि ाज

- **Time is usually measured in second (sec), minutes (min) or hours (hr).**

है ि दि्ज ातहौ ासि ऽजह्ह तक्ष्थ्देउ ,दलक्षासादषक्ष्वै क्षे ा ऽतण्क

- **Distance is usually measured in meters (m), kilometer (km) or mile, yards or feet.**

`गन्दज ातहौ ासि ऽजे -, ध्व्अ ज्ौत-, थ्दे -ज्दलक्षासादि भ्दै क्षे ा ऽतण्क

- **Speed is usually measured in km/h, mile/ hr or m/sec.**

ी ाज्दज ातहौ ासि ऽजअ्ौ ़ षक्षाथ्दे -ज़् षक्षादलक्षासादे ़ हल क्ष्दे क्ष ` ा ऽतण्क

Conversion of Units/ ि ाज ाक्ष्ज ादट ाक्ष्े ध

1. **1 h = 60 min = 60 × 60 = 3600 sec.**
2. **1 km = 1000 m**
3. **1 mile = 1.606 km or 1 km = 0.6214 mile of 5 mile = 8 km**
4. **1 yard = 3 ft**
5. **a km/hr** $= a \times \frac{1000}{60 \times 60}$ **m/sec**

$$= a \times \frac{5}{18} \text{ m/sec}$$

a m/sec $= a \times \frac{60 \times 60}{1000}$ **km/hr**

$$= a \times \frac{18}{5} \text{ km/hr}$$

Ex. A man go a certain distance with x km/hr and comes back with a speed of y km/hr. If he takes t hour to go and come back. Find the distance?

ख्ज दलां ै -दxदअ्ौ ़ षक्षादज्तह ाादख्ज दुआी ऽदं `गन्दऽ ि दि्ज ऽा ण्वलोय्दyदअ्ौ ़ षक्षादज्-दबक्षदहतसा हदलाऽादण्क्षा िअंदत्त्हत चा्ु तलोय्दसा हदला्ु तै क्षत्दषक्षादज्ब्ऽादण्क्ष `गन्दडाऽदज्-अक्षग्

Then distance/`ग- $= \left(\frac{xy}{x+y}\right) \times t$

Ex. A man go a certain distance with x km/hr and he comes back with a speed of y km/hr. If he takes t hours more to come back than go. Find the distance.

ख्ज दलां ै -दxदअ्ौ ़ षक्षादज्तह ाादख्ज दुआी ऽदं `गन्दऽ ि दि्ज ऽा ण्वलोय्दसणदyदअ्ौ ़ षक्षादज्-दबक्षदहतसा हदलाऽादण्क्षा िअंदसण चा्ु तहतसा हदला्ु तै क्षत्दषक्षवलअ्ज दज्त्रादण्क्ष `गन्दज्ाद ऽा ज्बाक्ष्क

Then distance/`ग-

$$= \left(\frac{xy}{x-y}\right) \times \text{(difference between time)}$$

➲ **The difference between time can be solved by the following tricks**

➲ है िद्र त्लश्चे द्र ातुअ) अत्र ःक्रे ॅ एाहृत्वणज्दअ ादिचाहृ ॅ ःा णय

Same important cases in speed

Early, early case '–' (subtraction)

Late, Late case '–' (subtraction)

Early, Late case '+' (Addition)

Late, Early case '+' (Addition)

Average Speed

Amar went to Gaziabad from Delhi by 60 km/hr and then he went to Dadri by 80 km/hr. What is his average speed?

लै ` दअश्ज्-दृह तबाअ्गाएां दृच्दअै ्- षक्ष्तलोय्ह्ओछदसणदं ां ` - हृच्दअै ्- षक्षादॅ -वी ाज्दृह तयाःादणक्तृह ॅ -क्लाह्नवी ाज्दू ादिणय

Basically

$$\text{Average speed} = \frac{\text{Total covered distance}}{\text{Total taken time}}$$

लाह्नःदी ाज्द= ॅ प्रदःा दिॅ -दबनबां ग- / ॅ प्रदअ ादिबादिहै ा

There are three variables viz speed, time and distance. Any two out of three should have been given to solve the question.

ाणक्ृ- वे ` स्रो ाज्थ्है ाक्लोय्दं ग्न्त्वादणक्ष हृ द्र ातणज्दॅ ्, त्व्त अख्व्ृ- दै हृतज् ातबं ावी दअं ात्णुतवी ाअक्ष्क

In above question only speed is given so it is not possible to solve question.

ॅ ाुप्रद हृ दै हृत्र छ्वी ाज्दं -दबनबणवनह अख्वनह ॅ ातणज्दॅ ्,ा ह क्षसदु ए-ह्नणक

Cases for Average Speed

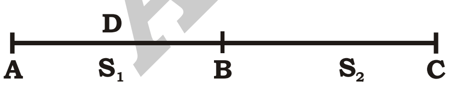
लाह्नःदी ाज्दॅ -दअााअ्ाा:

1. **When distance is constant**
   चएदं `गन्दुअाःदणत
2. **When time is constant**
   चएदहै ादुअाःदणत

Case-I/अााअ्स्ना

When distance is constant

चएदं `गन्दुअाःदणत



$t_1 = \frac{D}{S_1}$ $t_2 = \frac{D}{S_2}$

Average speed/लाह्नःदी ाज् =

$$\frac{D+D}{\frac{D}{S_1}+\frac{D}{S_2}} = \frac{2D}{D\left(\frac{1}{S_1}+\frac{1}{S_2}\right)} = \frac{2S_1S_2}{S_1+S_2}$$

From above explanation, it is clear that when distance is constant, average speed is free from distance. It means three is no need of distance or question can be solved through any supposed distance.

ॅ ाुप्रदअासे धाहृ वा णिदा ा, दणक्ष दचाएदं ग्न्दुआ ाःदणक ाक्लाह्न ा ाज्दं गन्हृ ता सःक्ृदणत-दणकाल ाा्मह्नं ग्न्द -द्र ातक्लास(ॅ ा ःादु ए-ष्ठ णत-दणवलका ासाद हृ द्र ातणज्दॅ ्, तॅ तअ्ख्दं `गन्दॅ ात्रहृदै ा् अ ादिचाःादणक

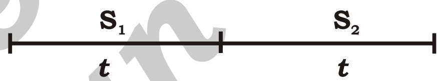
Note : In such type of question we can take any value of distance but the ratio of distance which we will take while solving the question must be in the same ratio as given in the question.

ख्वातह साज्ाह्नै ह्नला द्र ातक्ता-दं `गन्दज्तह ॅ ःतणक्षज्ल्, दसाःाअॅ `गन्दॅ ादचातलु प्राःदणुतादी ाअख्दसण-दलु प्राःदणै हृत साज्दण्ज ॅ ःतसू ःदज्ताद भ्रैंवाक

Case-II/अााअ्स्नाा

When time is constant

चएदहै ादुअाःदणत



$D_1 = S_1 \times t$ $D_2 = S_2 \times t$

Average speed/लाह्नःदी ाज्द=

$$\frac{D_1+D_2}{t+t} = \frac{S_1t+S_2t}{t+t} = \frac{t(S_1+S_2)}{2t} = \frac{S_1+S_2}{2}$$

When time is constant, average speed is free from time. It means when time is constant

चएदहै ादुअाःदणक्षःातलाह्नःदी ाज्दहै ादिहत ासःक्ृदणत-दणक लक ाा्मदचाएदहै ादुअाःदणत

Average speed/ लाह्नःदी ाज्

$$= \frac{\text{Sum of the speed}}{\text{Number of speeds / observations}}$$

ा ाज्ाह्नॅ ादिाव / ा ाज्ाह्नद ासबाद्धाह्नॅ -दहक्ष्ाा

Note : In such type of question we can take any value of time but the ratio of time which we will take while solving the question must be in the same ratio as given in the question.

ख्वातह साज्ाह्नै ह्नला द्र ातक्ता-दहै ादिज्तह ॅ ःतणक्षज्ल्, दसाःाअॅ है ादिॅ ादचातलु प्राःदणुतादी ाअख्दसण-दलु प्राःदणै हृत साज्दण्ज ॅ ःतसू ःदज्ताद भ्रैंवाक

➲ **Question based on Average Speed with Stoppage**

न्नणे ासदॅ तह ाादलाह्नःदी ाज्द ` दला-ाअ्ःद हृ

When stoppage time is taken into consideration while calculating average speed, it is said average speed with stoppage.

चएदलाह्नःदी ाज्दॅ -दबधु ादॅ ःतणक्ष्वन्नणे ासदॅ तहै ादिॅ ातता- (ााअज्दॅ ःतणक्षाातहत न्नणे ासदॅ तह ाादलाह्नःदी ाज्दॅ तॅ एःतणक्ष

Here,

$S_1 \rightarrow$ Speed of train starting from A

A हत्(ाम्र दणुततसाज़त,ुमदर् -दबअ्रध

$S_2 \rightarrow$ Speed of train starting from B

B हत्(ाम्र दणुततसाज़त,ुमदर् -दबअ्रध

$T \rightarrow$ Time after which they meet each other.

है दिअमहर् तएं दसतक्र् रूं हे तह तैअज्र-दणध

$T_1 \rightarrow$ Time taken by the train 1 to reach at its destination after crossing each other.

,ुमदबदू ो ादिक्र् रूं हे तर्ात ो दर् ु तर् तएं दल ु तबक्ष्ह्ां दिर्ज र्णि ु तै क्षअर्गादबादिहै धि

$T_2 \rightarrow$ Time taken by the train 2 to reach at its destination after crossing each other.

,ुमदउदू ो ादिक्र् रूं हे तर्ात ो दर् ु तर् तएं दल ु तबक्ष्ह्ां दिर्ज र्णि ु तै क्षअर्गादबादिहै धि

$D \rightarrow$ Total distance form A to B.

(A हत्B ंर् दर् -दर् प्रदं ग्ग)

On this concept three types of questions are asked in the exams and they are based on the given formula below :

नहदलस-ो धाद दे ऋादे क्षु- द ह्रो दर्त ह्रु द मतचाःतणक्ष चातु ने तअं तिबातिहंगाक्ष दला-ाअःदणततणक्ष

(a) $T = \sqrt{T_1 \times T_2}$

(b) $\dfrac{S_1}{S_2} = \sqrt{\dfrac{T_2}{T_1}}$

(c) $D = S_1T_1 + S_2T_2$

Buses are leaving bus terminal after every 10 minutes (T_1) But, a person who is moving towards the terminal meets the bus after every 8 minutes (T_2).

क्र्ल त दबच्दअु ,द(T_1) एं दएह क्ष्एहद,अुमज़दहतअुर् ज़र्-दणक्ष ज़तसु ध्दर् दहअ्क्षचात,अुमज़दर् -दलोतदएद्रैंदे णादणसणदणे दह अु ,द(T_2)दर् तएं दएहदहतैअज्रादणक

$$\frac{\text{Speed of man}}{\text{Speed of bus}} = \frac{|T_1 - T_2|}{T_2}$$

$$\frac{\text{Speed of Train}}{\text{Speed of Sound}} = \frac{|T_1 - T_2|}{T_2}$$

Here,/िणक्ष

T_1 = Time after which buses leaves the terminal.

T_1 =दहै दिअमहर् तएं दएहक्ष,अुमज़दहतअुर् ज़र्-दणक्ष

T_2 = Time after which it meets with the person.

T_1द=दहै दिअमहर् तएं दएहक्ष्र्ूअःदहतैअज्र-दणक्ष

BASIC CONCEPT OF TRAINS

तबाभँ-दह क्षुक्ष-द ह्रु ाक्षर्-दै षताग्रदह क्ष श ु ा

- Distance = Speed × Time

`गम्द=दी ाज़दxदहै ि

- Difference between Meeting and Crossing

अज्ु तलोय्क्ष ो दर् ु तर् तए-ी दलक्षे

- Speed/ि ाज़ → Relative Speed/हा तक्ष्दी ाज़

$S = (S_A - S_B)$ [Same Direction] है ु दअं(ाध

$S = (S_A + S_B)$ [Opposite Direction] अा ंदअं(ाध

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$ दहै ि द=द `गम / ि ाज़

Generally, Length of the train is given in m and Speed is given in km/hr.
So, always focus on the units.

लःजहं सदनर्ानक्ष दर्ाुदं क्ष

Basic points which will help in solving questions

1. When a train crosses a man (stationary), crosses a man walking @ 2km/hr or crosses a man walking @ 10 km/hr.

चएदक्र्दे तबाभँ-दर् दह्अक्ष्ह आो धदर्ात ो दर् ं-दणक्षउ अ -दषक्षादर्-दी ाज़दहदी ु तसाज़तर् दह्अक्ष्हर्ात ो दर् ं- णादिबच्दअ -षक्षादर्-दी ाज़दहदी ु तसाज़तर् दह्अक्ष्हर्ात ो दर् ं-दणक

In every case : $D = L_T$ (Length of the train)

`तबाभँ-दर् -दज़क्षानक्ष

Here, D refers to the distance which the train has covers extra with respect to the man.

िणाक्ष्Dदे तबाभँ-दर् -द `गम्दहदह क्षआमदणचातदे तबाभँ-दह्अक्ष्हर्ात हक्ष्ताे क्षलअअक्षःदिर् ं-दणक

2. Distance covered by the train when the train crosses an object:

अहदसाःज्रर्ात ो दर् ु तै हे तबाभँ-दू ो ादिःदिर् -दबदक्ष्ं ग-

$$D = L_T + L_O$$

Where:
L_T = Length of Train
L_O = Length of Object

चणः

L_T = `तबाभँ-दर् -दज़क्षानम

L_O = साःज्रर् -दज़क्षानम

3. Train 1 crosses Train 2

$D = L_1 + L_2$

$S = S_1 - S_2$ (Same Direction)

$S_1 + S_2$ (Opposite Direction)

4. Train 1 — Train 2

L_1 — L_2

S_1 — S_2

A person sitting in Train 1 crosses Train 2.

$D = L_2$

$S = S_1 - S_2$

Train 1 crosses a person sitting in Train 2.

$D = L_1$

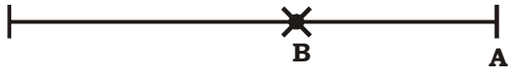
$S = S_1 - S_2$

COMMON STATEMENTS USED IN RACE BASED QUESTIONS

- **In a race of 1 km, A beats B by 100 metre.**

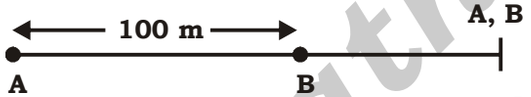
A	B	A : B
1000 m	900 m	Speed 10 : 9
T	T	

- **In a race of 1 km, A beats B by 10 sec.**



A	B	A : B
1000 m	1000 m	
T	(T + 10) sec.	

- **In a race of 1 km, A can give B a start of 100 metre.**



A	B
1000 m	900 m
T sec	T sec

- **A can give B a start of t minutes : This statement implies that A will start t minutes after B starts from the starting point.**

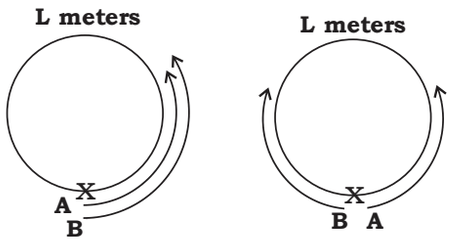
- **In a race of 1 km, A gives B a start of 150 metre and still wins by 10 sec.**

A	B
1000 m	850 m
T sec	(T + 10) sec

- **Dead Heat : A dead heat situation is when all participants reach the finishing point at the same instant of time.**

Circular track/

CASE 1:
Same direction and opposite direction



- **Meet first time each other at any point on the track in opposite direction /**

$$= \frac{L}{(x + y)} \text{ sec.}$$

Where x m/s, y m/s are speeds of A and B resptectively. L = length of the track in meter

- **Meet first time each other at any point on the track in same direction /**

$$= \frac{L}{(X - Y)} \text{ sec}$$

- **Meet each other exactly at the starting point in same or opposite direction/**

$$= \text{LCM} \left(\frac{L}{x}, \frac{L}{y}\right) \text{ sec}$$

CASE 2: When three persons A, B and C are running around a circular track of length L mts with speeds of x, y and z m/s in the same direction,

Meet each other at any point on the track

$$= \text{LCM} \left(\frac{L}{(x - y)}, \frac{L}{y - z}\right) \text{ sec}$$

10 BOAT AND STREAM/है f दलै म्भ्दे ै यै

Definition & Formulae/ ग्भै ै व्कै व्क्ता

There are a variety of subconcepts that are related to answering questions based on boat and streams concept. Given below are the four terms which are important for a candidate to know to understand the concept of streams.

है f दलै म्भ्दे ै यैलै क्त क्क्लर्f ै र्कै दं यक्लै ै ग्भ्अक्ती है क्त क्दुफै यक्लक्हज्र क्त र्मेम अग्गि मै क्ष्दजं स्लर्f र्कै ख्क्पाक्क्हक्क्ष् क्दुफै यत्त चाक्दग्क्क्ष्म ख पाक्षै क्ष्ख्त दं यर्कै ै है क्षत क्ग्क्षख्दे ै यैलै क्त क्क्लर्f र्कै दतै क्र ६, हज्र त क्ग्क्षख्द६ पर्ई न्नै दपाक्क्ष्

- **Stream :** The moving water in a river is called a stream.

  ` ै यैदप्र्हक्क्क्ष्र६ र्ब्ब पअक्ष्भ्वदतै दे ै यैदत पअक्ष्पाक्क्ष्

- **Upstream :** If the boat is flowing in the opposite direction to the stream, it is called upstream. In this case, the net speed of boat is called upstream speed.

  : न्ग्ज़ुर्f पदप्र्त्त्ग्क्हहै f दे ै यैदत क्र्f यक्क्षग्म्मै दर६ र्ब्ब पदयक्क्ष्पूदअज्। रक्ष् न्ग्ज़ुर्f पदत पै द्भै अदपाक्क्ष्। र द्गग्तै ग्भ्क्ष्रक्क्ष्र है f दित क्षत घदु ग्भ्क्षतै ज्: न्ग्ज़ुर्f पदु ग्भ्क्षत पअक्ष्पाक्क्ष्

- **Downstream :** If the boat is flowing along the direction of the stream, it is called downstream. In this case, the net speed of boat is called downtream speed.

  लहंधुर्f पदप्र्त्त्ग्क्हहै f दे ै यैदत क्ग्मै दर६ र्ब्ब पदयक्क्ष्पूदअक्ष्। रज्लहंधुर्f पदत पै द्भै अदपाक्क्ष्। र द्गग्तै ग्भ्क्ष्रक्क्ष्र है f दित क्षत घदु ग्भ्क्षतै ज्लहंधुर्f पदु ग्भ्क्षत पअक्ष्पाक्क्ष्

- **Still Water :** Under this circumstance the water is considered to be stationary and the speed of the water is zero.

  ॉ म्म्क्ष्भ्वदप्रक्ष्ख्र क्क्ष्ग्तै ग्भ्क्ष्रक्ष्भ्वदतै क्ग्तै यक्तै है दभ्अदपाक्ष्लै म्भ्भ्व त क्क्ष्ु ग्भ्म्मीं म्मदपै अक्ष्पाक्क्ष्

The questions from this topic may seem to be confusing until a candidate is aware of the above-mentioned terms and how they may be used for answering the questions.

।र दर्f ै त्दत क्क्ती हदर्क्ष् दक्ष्त दक्ष्-ग अक्त यहर्क्ष् वक्क्ष्अक्ष्भ्यै क्र त अक्ष्पाक्ष् अत दग्त दतै त्ज्द्ध्क्ष्र्f यदजं त्धैम्दै अक्र्क्र क्लर्f, अक्ष्हदपै क्लै म्भ्भी है म्म तै दज़ुफै यक्लक्हक्त क्ग्क्षख्द्ज़्हतै दजं ै ज्दत ाक्ग्त ै दभ्रै दर त अदपाक्क्ष्

This topic basically deals with calculating the speed of anything in the water when it flows along with the flow of water or in the opposite direction.

त्पदर्f ै त्द६ क्वाक्क्ष् ँ दर दं ै हक्क्षत क्क्ष्र्f पदत क्र्तै ै दतै दर्f यक्क्षग्म्मै ६र्ब्ब पहदं यदं ै हक्क्ष्६ क्क्ष्ा रक्क्ष्- क्क्ष्ए क्ष्दत क्क्ष्ु ग्भ्क्षत क्क्ष्ु ३ै है दत यहक्र ज्र्मेम अक्ष्पाक्क्ष्

Important Formulae : ६पर्ई न्नैदरैक्त

Given below are a few important formulas with the help of which you can solve the questions based on boat and streams.

हक्क्ष्क्ष्त ्धक पर्ई न्नैदरैक्तक्क्ष्ग्क्ष्ु ख्क्ष्पाक्क्ष्भ्हत क्क्षर पै त्अैदर क्लै ँ क्हैf लै म्भ्भ्वे ै यैदं यक्लै ै ग्भ्अक्ती है क्तै क्ष्पवदत यक्र त अक्क्ष्पाक्क्ष्

Candidates must learn these formulas by heart to ensure they are able to answer the simple formula based questions correctly and do not end up losing marks for direct questions.

ज्क्ष्क्ष्र्f यैक्तै क्ष। हदर क्तैक्तै क्ग्क्ष्क्ष्वदर क्र क्ष्ट है दएै ग्भ्ख्क्ष्अग्त दत्त्प र्क्ष्हग्री ए अक्ष्पै क्र त क्षत दर्क्ष्र यक्दरैक्तक्लै ै ग्भ्अक्ती है क्तै दर पक्क्ष्ज़ै य क्षक्ष्क्ष्क्र ै ६दपाक्लै म्मर क्ष् क्क्षी है क्त क्ग्क्षख्द्लक्ष दहपक्क्ष् मीं अक्ष्पाक्क्ष्

Speed of boat in still water = B

ॉ म्म्क्ष्भ्वदर्६ है f दत क्क्ष्एै व

Speed of the stream = S

` ै यैदत क्क्ष्एै व

1. **Downstream Speed of boat** = (B + S)

   है f दत क्क्ष्लहंधुर्f पदएै व

2. **Upstream Speed of boat** = (B – S)

   है f दत क्क्ष्: न्ग्ज़ुर्f पदएै व

3. **Speed of boat in still water**

   ॉ म्म्क्ष्भ्वदर्६ है f दत क्क्ष्एै व

   $= \frac{1}{2}$ (Downstream speed + Upstream speed)

4. **Speed of the stream/** ` ै यैदत क्क्ष्एै व

   $= \frac{1}{2}$ (Downstream speed – Upstream speed)

Points to Remember/ ६यकै क्ष्दअत्त

1. If the total time taken by the boat to row a distance of D and reach back to its initial position is T then,

   त्ग्क्क्ष्है f ै यैदDदत क्क्क्ष्क्क्ष्क्ष्अत्तदत यहक्लै म्म्लं हक्क्ष्वे यत्तै त द्गग्तै ग्भ् ६र्ब्बं र दलै हक्ष्क्ग्क्ष्तै दु ै दत घदर ६त्त्दTदपूदअज्

(a) Distance between the two places is

   कैद्र है है क्त दर्ई क्क्ष् दत क्क्क्ष्क्क्ष्

$$D = \frac{T\left(B^2 - S^2\right)}{2B}$$

(b) Average speed of the boat $= \frac{\left(B^2 - S^2\right)}{2B}$

   है f दत क्क्ष्लै ा अक्ष्एै व

2. If it takes T hours more to go to a point upstream than downstream for the same distance. Then,

त्गमकुर ष्ै हदकमबुस्त ङ्गन्न खेदे` यैदत ब्बगभ्रत बादगत र क्ष्ूगी घमंब यदभ्ै हब्ष म
`` यैदत ब्लहघ्र बादभ्ै हब्त क्षुअब्रहै दष बाTदैड़मब्लगेग त दवु अङ्गपूमैअज्

Distance/कन्नक्ष = $\frac{T(B^2 - S^2)}{2S}$

3. If a boat travels a distance downstream in T_1 hours and returns the same distance upstream in T_2 hours.

त्गमकुख्त दहै ि दे` यैदत ब्लहघ्र बादख्त दकमबुT$_1$दैड़मब्ष बअत्दत यअङ्गपा
लै मदुर ष्ै हदकमबुके` यैदत ब्बगभ्रत बादT$_2$दैड़मब्ष बर्ाै ं र दलै अङ्गपा

Then, speed of man in Still water

अैूज्ञ्ना ै यदभ् व दष बाष हध्न्दत क्ष्ु गभ्र

$= \text{द} \frac{S(T_2 + T_1)}{(T_2 - T_1)}$

4. If the time taken by the boat to row same distance in downstream is T_1 and in upstream is T_2.

त्गमकुहै ि दै यैदलहर्बा पदक ब्र ष्ै हदकमबुस्तै हं` यस्त यह ब्क बान्न ै दु ै
र ष त्दT$_1$दपाहलै मके` यैदत ब्बगभ्रत बादष बाT$_2$दपाद्ध

Then, the ratio of speed of the boat to the speed of stream

अैूज्दहै ि दत क्ष्ु गभ्रतै दे` यैदत क्ष्ु गभ्रर ब्लहंधै अ

$= \frac{B}{S} = \frac{(T_2 + T_1)}{(T_2 - T_1)}$

11 RATIO, PROPORTION & PARTNERSHIP

हैं िदतापे त कैलदितो ा यक्तेात त्ल ख्ली

Ratio/हैं दिता

$\frac{a}{b}$ is the ratio of a to b is written as a:b and read "a is to b". Where 'a' is called the 'antecedent' and 'b' is called the 'consequent'.

a ा यक्तेb ´ लहैं दितो $\frac{a}{b}$ गे- अट`र a : b जनतोरगे-ा यक्तेa हैं दिता b दन्मेरगे-अगेलेa ´ लादक्षबा ीबा यक्तेb ´ लाद ज़्ल्लमी ´ गेलअतोलेर्ग-

Condition:/श्लाब

1. (i) If ad = bc then $\frac{a}{b} = \frac{c}{d}$

(ii) If ad > bc then $\frac{a}{b} > \frac{c}{d}$

(iii) ad < bc If $\frac{a}{b} > \frac{c}{d}$

2. If $\frac{a}{b} = \frac{c}{d}$ then

(a) $\frac{b}{a} = \frac{d}{c}$

(b) $\frac{a}{c} = \frac{b}{d}$

(c) $\frac{a+b}{b} = \frac{c+d}{d}$

(d) $\frac{a-b}{b} = \frac{c-d}{d}$

(e) $\frac{a+b}{a-b} = \frac{c+d}{c-a}$

(f) $\frac{a}{b} = \frac{c}{d} = k$

$\therefore a = bk, c = dk$ (k method)

(g) For continued proportion $\frac{a}{b} = \frac{b}{c} = k$

$\therefore b = ck$ & $a = bk$ $\quad \therefore a = (ck)\,k = ck^2$

Proportion/त कैलदिता

When two ratio are equal, $\frac{a}{b} = \frac{c}{d}$, then a, b, c, d are said to be in proportion and a, b, c and d are said to be first, 2nd, 3rd and 4th proportion respectively. /असे ख्लेहैं दितो सज्ज़ाजेप $\frac{a}{b} = \frac{c}{d}$ गेला लेa, b, c, d त कैलदितो केह्लातोरगे-ा यक्तेa, b, c ह त्ले d ´ लाए कश्लेप्रदुयक्तेम भाँ पेा ौं ह त्ले वा मिलेहैं दितो ´ गारर्ग-

Case-1: When 3 numbers are given/असे ा ी ` त ढ्रें त्ल खी , डेबर्ग-

x : a : : b : c

First proportion/दुयक्ते हैं दिता $x = \frac{ab}{c}$

a : x : : b : c

Second proportion / भा ीं ` हैं दितो $x = \frac{ac}{b}$

a : b : : x : c

Third proportion/ा ौं ` हैं दिता $x = \frac{ac}{b}$

a : b : : c : x

4^{th} proportion/वा मिलेहैं दिता $x = \frac{bc}{a}$

Case-2: When 2 numbers are given./असे प्रे त ढ्रें त्ल खी , डेबर्ग-

a : b : : b : c

First proportion/दुयक्ते हैं दितो $\frac{a}{b} = \frac{b}{c}$

$\Rightarrow a = \frac{b^2}{c}$

mean proportion/कँ ` हैं दिता $\frac{a}{b} = \frac{b}{c}$

$\Rightarrow b = \sqrt{ac}$

3rd proportion/ा ौं ` हैं दिता $\frac{a}{b} = \frac{b}{c}$

$\Rightarrow c = \frac{b^2}{a}$

❖ **Proportion After addition/ ;f) अत्केमें रसल्छ है दिता**

Let x be a number which is added to a, b, c and d to make them proportional then/ ; ख्रे x ऽगे त क्ष्रु लेग- अत्े र a, b, c । यत्ता d केश्रुअलैमेरते ऽते रत कैलदिती गलअतोरगे-। ल

$$x = \frac{bc - ad}{(a + d) - (b + c)}$$

❖ **Proportion After Subtraction/ ;f) ू ला्लेरं र सल्छे है दिता**

Let x be a number which is subtracted to a, b, c and d to make them proportional then

; ख्रे x ऽगे त क्ष्रु लेग- अत्े र a, b, c । यत्ता d केश्रु लालेर ते र ऽर त कैलदिती गलअतोरगे-। ल

$$x = \frac{ad - bc}{(a + d) - (b + c)}$$

Partnership/त ल ख्ली

(A) Different Investments, Same Time Period Of Investing/असे ऽ श्ले ी ृ ज्ल्श्लेह धल्धह ज, गेला यत्ता ऽ श्ले लेत कँ ल ज्ल्से त कैले गेला ल

If the amount invested by the partners are I_1, I_2, I_3 then the profit is distributed in the ratio $I_1 : I_2 : I_3$.

; ख्रे त ल ख्ल्केश्रुल्ले ऽ रश्ते ृ ज्ल्श्ले I_1, I_2, I_3 गेला लेष्ट रजलल $I_1 : I_2 : I_3$ है दिते केश्रुऽ। जे गलश्रु

(B) When the investment and the time period is different/असे ऽ श्ले ी ृ ज्ल्श्ले। यत्ता त कँ ह स ज्ल्स ह धल्धह ज, गेला ल

Let there be three partners, one invests I_1 for t_1 time, second invests I_2 for t_2 time and third invests I_3 for t_3 time. The profit is shared in the ratio

कैलेलाी त ल ख्ले गेघ दगजले I_1, ज्ल्श्ले t_1 त कँ पे ख्श्र्ले I_2 ज्ल्श्ले t_2 त कँ । यत्ता ति ज्ले I_3 ज्ल्श्ले t_3 त कँ र जष्ठे ऽ रश्ता ज लेग- ष्ठे रजलले ज नते है दिते केश्रुऽ। जे गलश्रु

$I_1 \times t_1 : I_2 \times t_2 : I_3 \times t_3$.

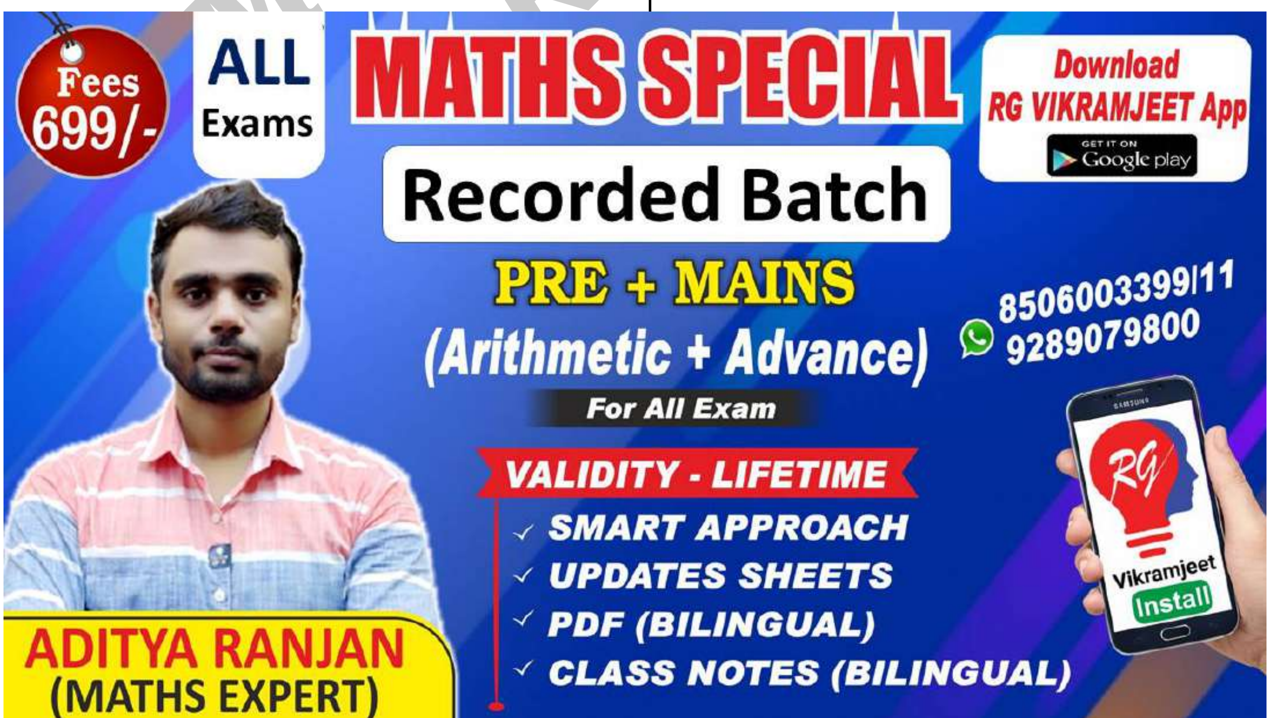
(C) Different Amounts Invested In Different Time Periods/असे ह ज, ध्ह ज, ृ ज्ल्श्लेह ज, ध्ह ज, त कँ ह ऽ, र जष्ठे ऽ रश्ते रअलेर

Let's say in a partnership between A and B, A invests Rs. I_a for a time period of t_a.

But B invests Rs. I_{b1} for a period of t_{b1} time and Rs. I_{b2} for a period of t_{b2} time. In this case the profit will be divided between A and B in the ratio

कैलेले A । यत्ता B रसीवे ती त ल ख्ली केश्रु A भ्ल्ले I_a ट दं र t_a त कँ ल, र जष्ठे ऽ रश्ते लेअतालेगर्- दज्श्रुी B भ्ल्ले I_{b1} ट दं र t_{b1} त कँ ल, र जष्ठे । यत्ता I_{b2} ट दं र t_{b2} त कँ ल, र जष्ठे ऽ रश्ते लेअतालेगर्- इते ख्ल्लेके A । यत्ता B रसीवे जलले ल ऽ। ज्ल्ले ज नते है दिते केश्रुाल्ल

$I_a \times t_a : (I_{b1} \times t_{b1} + I_{b2} \times t_{b2})$



12 ALLIGATION/एज़ इंगय

MIXTURE

In this type of questions, the ratio of the ingredients in the first vessel and the ratio of the ingredients in the second vessel are given. After that, the ratio of the ingredients is also given in the new mixture made by pouring or mixing the mixture of both the vessels in a third vessel. The ratio of the ingredients in the first vessel is written on the left side, the ratio of the same ingredient in the second vessel is written on the right and the ratio of the same ingredient in the third vessel is written in the middle. After using the alligation method, we get the ratio in which the first and the second mixture were mixed or the ratio of the amount of the same ingredient in the first and second vessel.

है दिलयेा ंदिल-यंअंदतर ंदयअंनज्मज्ा ंन-तयक्षित्रय बौं ंदयअंनज्मज्ा ंन-तयक्षज़णतयक्षंर्तिकिक्षग्गसयक्षिबयंय दयमां ंजुअपव्ुा यंण क्षिऐं ंदयअंचय-ंनत्रक्र्यजुअर यौं ु-ंि-मंजुअपव्ुअंभ्यिनज्मज्ा यन-तयक्षज़मयतयक्र्यर्तिकदतर दयअंनज्मज्ा ंन-तयक्षा युं यहँनयेंउवै एनज्मज्ा ंबौं दयअंन-तयक्षा यंबयहँनयेंक्षित्रयक्षिऐं ंदयअंवै एनज्मज् ा ंन-तयक्षा यंअपमांअंज़ ,यक्षंर्तिकएज़ इंगयज़्जुआा यदमांक ा ंक्षतएतअंदतर ंक्षित्रयबौं ंजुअपव्ुा यंज ंन-तयक्षअं जुअर यमयइमयत्रयनत्रक्र्यदतर ंक्षित्रयबौं ंअंवै एनज्मज्ा ए अपमा यन-तयक्षदिलक्षतयक्र्यर्तिक

1ST MIXTURE — 2ND MIXTURE

OVERALL MIXTURE

Quantity-1 — Quantity-2

PERCENTAGE

In this type of questions, the total population of a place, the percentage rate of increase/decrease in the population of males and that of females are given. And the increase/decrease or increase/decrease percentage in the total population of the place is also given. The increase/decrease percentage in the population of males is written on the left, the increase/decrease percentage in the population of females is written on the right and the increase/decrease percentage in the total population is written in the middle. With the help of allegation, we find out the ratio of the initial population of males and females.

है दिलयेा ंदिल-यंअंजि एन्त्रयमा एा तर्ि ै ,मयक्षदित्रू यंक्षित्रय अज्र यायंा एं ै ,मयंअंज्क्र धा अएा एदलक्षगयक्षबे ंबएइहँ तयक्षएर्तिकक्षित्रयन्त्रयमा एा तर्ि ै ,मयंअंज्क्र धा अएनत्रक्य दलक्षगयक्षज्क्र धा अएभ्याबएइहँतयक्षएर्तिकदत्रू यंा एं ै ,मयंअं दलक्षगयक्षज्क्र धा अएा युं यहँनयेंउअज्र यायंा एं ै ,मयंअं दलक्षगयक्षज्क्र धा अएा यंबयहँनयेंक्षित्रया तर्ि ै ,मयंअंदलक्षगयक्ष ज्क्र धा अएा यंअपमांअंज़ ,यक्षंर्तिकएज़ इंगयमा एं तयमक्षयं दलाध्र्यअंदित्रू यंक्षित्रयअज्र यायंा एं ै ,मया यन-तयक्षस्यक्षा रक्षंर्तिक

% inc./dec. (Male) — % inc./dec. (Female)

% inc./dec. in Overall Population

Initial number of male : Initial number of female

In this type of questions, the increase/decrease percentage in the expenditure, savings and income of a family/person is given. Increase/decrease percentage in expenditure is written on the left, increase/decrease percentage in savings is written on the right and increase/decrease percentage in income is written in the middle. With the help of allegation, we find out the ratio of expenditure and savings in the beginning.

है दिलयेा ंदिल-यंअंजि एदिज़्येधथ्मज़क्षा ं,यसँउ सक्षक्षित्रय नयमांअंदलक्षगयक्षज्क्र धा अएबएइहँतयक्षएर्तिक,यसाँअंदलक्षगयक्ष ज्क्र धा अएा युं यहँनयेंउ सक्षअंदलक्षगयक्षज्क्र धा अएा यंबयहँ नयेंक्षित्रयनयमांअंदलक्षगयक्षज्क्र धा अएा यंअपमांअंज़ ,यक्षंर्तिक एज़ इंगयमा एं तयमक्षयं ंदिलाध्र्यअं,यसाँनयेक्षु सक्षा यन-तयक्ष स्यक्षा ं रिक्षंर्तिक

% inc./dec. in Expenditure — % inc./dec. in Saving%

% inc./dec. in Income

Expenditure : Saving

SIMPLE INTEREST

In this type of question the given principal is invested in two different schemes/banks/individuals at two different rates of interest. After a certain time the total amount is received. How much amount was invested in both the schemes/ banks/individuals, it is to be found. With the help of given principal, time and amount received, calculate the gross interest rate. The rate of interest on the amount invested in the first scheme / bank / individual is written on the left side, the rate of interest on the amount invested in the second scheme / bank / individual is written on the right and the gross interest rate is written in the middle. With the help of allegation, we find out the ratio of the distributed principal.

है दिलयेी िदस्-र्अंिज्ञणड़िर्णऑ प्र-ी यिबर्यनिरड्रनरइ मृंय -सायंंयु फ्कंचथमज क्षमर्यअंज्ज्न्ग्र्यबिर्यनिरइनिरइत्मिनृ ी ए बे यंदे जि मर्य् व्र्यतंकी ताज्ज्ञासक्षै अमी िदगसयक्षि त च्ये व्यार्यदिलक्षतिंक्ष्ग्रतंकबिरंयमिृंय -सायंु फ्कंचथमज क्षमर्यअंज क्ष-ए प्रे व्यार्यज्ज्ञाक्षी एिड़हर्जमितस्क्षी -र्यतंक्ष्यंतंकज्ञणडिण ओ च्यजैं अमक्षित्र्यदिलक्षज्ञिप्र-ी एं त्यक्ष्यै ी ी र ठ्मिनृ बि स्यक्षी िरंक्षतंकदिर एमिृंय -स्यु फ्कधथमज क्षअंज्ज्न्ज्ञक्षप्रे व्याय दे ठ्मिनृ ी एबि ी यु्ह्नंनियंजर्बौ एमिृंय -स्यु फ्कधथमज क्षअं ज्ज्न्ज्ञक्षप्रे व्यार्यदे ठ्मिनृ ी एबि ी यंबिक्ष्नियंक्षित्र्यौ ी र ठ्मिनृ बि ी यंअम्मअंज्ि ,र्क्षतंकज़ इंगर्मी एं त्यक्ष्यैं ज़्ज्क्षेड़क्ष ओ च्यी यनि -तयक्षस्क्षी िरंक्षतंक

Rate of Interest (1) % Rate of Interest (2) %

Gross Rate of Interest

Principal 1 : Principal 2

PROFIT AND LOSS

In this type of questions, the cost price of two different quantities of an ingredient is given and it is asked, in what ratio we should mixed both the quantities so that by selling the mixture at certain price there would be profit or loss of certain percentage. Find out the cost price of the mixture with the help of given profit/loss percentage. After that, by writing the first cost price on the left side, second cost price on the right side and cost price of the mixture in the middle, find the ratio in which the first and second mixture were mixed.

है दिलयेी िदस्-र्यअंज्ि एनिज्मज्नी एबिर्यनिरइनिरइअम्मम यं ी यडिमअिमज्ञिमर्यतंक्ष्यंतक्षित्र्यमितदिर्यु व्र्यतंक्षि हि-ी यंज्ि न -तयक्षअंज्ञ्म य जि एज्ज्ञासक्षअंमिदे ुस-ै ी ी त ज्ज्ञासक्षदिलक्ष्गक्षरि वय्यतक्तिंक्ष्एतंकज्ञणडिणदिलक्ष्गक्षरि वय्यतक ी एअबर्बौ ज्ञिम व्झी यडिमअिमस्क्षी िरंक्षतंकक्ष्ज्ञगक्षक्षितर डमअिमी यु्ह्नंनियंजर्बौ डिमअिमी यंबिक्ष्नियंक्षित्र्यज्ञिम व्झ ी डिमअिमी यंअम्मअंज्ि ,य दिरंक्षित्र्यबौ ज्ञिम व्झी यं ज नि -तयक्षअंज्ञ्म यर्यडमर्यत्र्यक्षस्क्षी िरंक्षतंक

Cost Price 1 Cost Price 2

Cost Price of Mixture

Quantity (1) : Quantity (2)

In the second type of question, cost price or the quantity of either two different components or two types of any one component is given. The first component or type is sold at a certain profit/loss percentage and the second component or type is sold at a certain profit/loss percentage. Overall there is a profit/loss of a certain amount or percentage. While using allegation, the profit/loss percentage on the first element or type is written on the left side, the profit/loss percentage on the second element or type is written on the right side and the total profit/loss percentage in the middle. From which the ratio of the cast price/quantity of the first and second components or types is obtained.

बौं दिलयेी िदस्-र्अंमर्यक्षर्यबिर्यनिरड्रनरइनिज्मज्नित्र्क्य ज एिण निज्मज्नी िबर्यदलयेर्यी एअम्मनित्र्क्यडमअिम ज्ञमर्यतंक्ष्यंतंकदिर निज्मज्नित्र्क्यदलयेी यंज एज्ज्ञासक्ष दिलक्ष्गक्षरि वय्यतक्षित्र्यबौं निज्मज्नित्र्क्यदलयेी यंज ए ज्ज्ञासक्षदिलक्ष्गक्षरि वय्यतक्दे ुसर्य् व्र्यतंकी ताज्ञ्म य ी ताज्ज्ञासक्षच्ये व्यार्यनित्र्क्यदिलक्ष्गक्षी यरि वय्यतक्तिंक्ष्एतंक णज़ इंगर्मी यदिलक्षी क्षैं अमदितर निज्मज्नित्र्क्यदलयेदि दिलक्ष्गक्षरि वय्यतक्षी यु्ह्नंनियंजर्बौ निज्मज्नित्र्क्यदलये दे दिलक्ष्गक्षरि वय्यतक्षी यंबिक्ष्नियंक्षित्र्यी तादिलक्ष्गक्षरि वय्यतक ी यंअम्मअंज्ि ,र्क्षतंकज्ि दितरंक्षित्र्यबौं निज्मज्नित्र्क्य दलयेर्यी डिमअिमधअम्मम यी नि -तयक्षदिलक्षतिर्यु व्र्यतंक

Profit/Loss (1)% Profit/Loss (2)%

Over all Profit/Loss %

Cost Price (1) : Cost Price (2)
Or Or
Quantity (1) : Quantity (2)

DISCOUNT

In this type of question, the marked price of the total quantity of an article and the discount percentage on the first and second part of the total quantity is given. The total discount percentage is also given. The first discount percentage is written on the left side, the second discount percentage is on the right and the total discount percentage is written in the middle. The ratio of the marked price of the first and the second part is found by using alligation.

है दिलयेपी दिस्र-अंिज्ि एन्ि्क्षी एी ता अिमापी यनि ज क्षअिेम क्षत्र्यी ता अिमापी दित र क्षित्र्यबौ भ्यिड़ दि ि(ो)दिलान्गरक्ष्ि बाएिइहैं त्यंक्ष्एर्तिंकदित र एक्षित्र्यबौ एि(ो)ािि यंअिप य ा ता ि(ो)दिलन्गरक्ष्भ्यिाए बाएिइहैं त्यक्ष्एर्तिंकदित र एि(ो)दिलान्गरक्षी यंि ुक्षहैं नि येंज बौ एि(ो) दलान्गरक्षी यंबिक्षहैं नि यें क्षित्र्यी ता ि(ो)दिलान्गरक्षी यंअिमअंिज्ि ,क्ष्ं र्तिंकण्ज इंगयी ा यंदिलक्षईंैं दित र क्षित्र्यबौ भ्यिड़ ा नि ज क्षअिेम ा यंनि -तव्क्षस्क्षित्यिृं यक्ष्यर्तिंक

DISCOUNT(1)%   DISCOUNT(2)%

OVERALL DISCOUNT %

Marked Price 1 : Marked Price 2

AVERAGE

In this type of questions separate averages of two groups of fixed number of member/person/element are given. And the total gross average is also given. The number of members/individuals/elements in the first and second group is to be found. Write the average of the first group on the left, the average of the second group on the right and the gross average in the middle. With the help of allegation, the ratio of the number of members/individuals/elements in the first and second group is found.

है दिलयेपी दिस्र-यंअिज्िजासक्षी बन्मधथ्मज क्षधन ज्मज्ीं ,मयी बयें अेयंी निरइ् नरइनि यंक्षक्षिज्ञणत्यिंक्ष्र्तिंकक्षित्र्यी तो ा र नयेक्क्ष्भ्यिा ज्ञमयं त्यंक्ष्यर्तिंकदित र ि क्षित्र्यबौ ि अेिअं बन्मधंथ्मज क्षमंधन ज्मज्यी एं ,मयस्क्षी -एत्यंक्ष्एर्तिंकदित र अेिा नियेक्क्षी यंि ुक्षहैं नि येंज बौ ि अेिा नियेक्क्षी यंबिक्षहैं नि यें क्षत्र्यी ा र नियेक्क्षी यंअिमअंिज्ि ,क्ष्एर्तिंकण्ज इंगयी ा एं त्याक्ष्य दित र क्षित्र्यबौ ि अेअिं बन्मधंथ्मज क्षमंधन ज्मज्यी एं ,मय ा यंनि -तव्क्षस्क्षी रि क्षर्तिंक

Average 1   Average 2

Total Average

1st Number : 2nd Number

In these types of questions, how many runs a bowler gives for each wicket (bowling average), is given. Gets a certain number of wickets by giving a certain number of runs in the 'next innings'. Due to which his bowling average becomes slightly lower/higher. We have to find the number of wickets taken by the bowler before or after 'this innings'. First of all, find the bowling average of 'the next innings'. Also find the bowling average after 'this innings' with the help of given increase/decrease. The bowling average before the 'given innings' is written on the left side, the bowling average of 'this innings' on the right side and the bowling average after 'this innings' on the middle. With the help of allegation, find the ratio of the number of wickets before 'this innings' and after 'this innings'.

है दिलयेपी दिस्र-यंअिी यक्षहिंइबु य दिल्क्ष्ां ज्ि)ाि ज्ि णज क्ष- -बिक्ष्यर्तिक्षिइंबु य एनियेक्क्ष्द उंज्ञमयंइमयत्यंक्ष्यर्तिंकज्न इर एदियेएछ अेि तज्िजासक्षी -बिा ा तज्िजासक्षज्ि)दिलक्षी क्ष्यर्तिंक ज्ृं वि ा यंइबु य एनियेक्क्षी ता अधन ज्ञा तियिृं यक्ष्यर्तिंकहै देयएछी दित र नि त्र्क्यंु यक्ष्अंिइबु य हियेयंज्ि णइणज्ि)यंी ए ,मयस्क्षी -एत्यंक्ष्एर्तिंकै दित र ज्नि इर एदियेएछी यंइबु य ए नयेक्क्षस्क्षी क्ष्र्तिंकबाएिइहैं अधुन्म्र ा एं त्याक्ष्यें हैं देयएछ ा ि क्षा यंइबु य एनियेक्क्ष्भ्यिस्क्षी क्ष्र्तिंकबाएिइहैं देयएछी दत र ा इंबु य एनियेक्क्षी यंि ुक्षहैं नि येंजहै देयएछी इंबु य ए नयेक्क्षी यंबिक्षहैं नि यें क्षित्र्यहै देयएछी ि क्षा इंबु य एनियेक्क्ष ा यंअिमअंिज्ि ,क्ष्र्तिंकण्ज इंगयी ा एं त्याक्ष्यें हैं देयएछी दत र क्षित्र्यहै ा ि क्षा ज्ि)यंी एं ,मयी यंनि -तव्क्षस्क्षी र क्षर्तिंक

Bowling Average 1   Bowling Average 2

ff   Total Average

Wicket 1 : Wicket 2

TIME & DISTANCE

In this type of questions, a certain distance is covered in two parts or by two mediums with different speeds. The average speed of the total distance or the total time taken to cover the total distance is given. The time taken to cover both the different distances is to be found. The speed with which the first distance is covered is written on the left side, the speed with which the second distance is covered on the right and the total average speed is written in the middle. With the help of alligation, the ratio of the time taken to cover both the distances is known.

है दिलयेाि ंदिला-र्यअंजि ै एज़्-ज़ासक्षबिीएबिर्यभ्याक्ष र्यअंनि त्र्या-र्यबियं अयामअर्यह्लेयर्यनि र इ् न र इसियय यैं ंक्षिमजि मर्यृ यक्ष्यर्तिकी सा बिीए ा एनि यैक्षक्षसियय नि त्र्या-र्याी सा बिीएाि ीं यर्याी सां अमाजि्मर्यइमय तयक्ष्यर्तिंक्विबिंय-र्यनि र इ् न र इबिीज़मर्याी र्यक्षिमािे -ंअिंज़ि णड़िर्णं अम ा र्यस्यक्षीि े -र्यतियक्ष्यर्तिंकदित र एबिीएाि र्यज़्ृै सियय ीं ंक्षिमजि मय इमर्यवै ंु यक्षँनिये र्यउबिीं े एबिीएाि र्यज़्ृै सियय ीं ंक्षिमजि मर्यइमय वै ंबियक्षँनिर्येाक्षित्र्याि ा सा नि यैक्षक्षसियय ाि र्यअमाअंजि ,यक्षंर्तिक णज़ इंगया-ाि एंै तयाक्ष्यैं ंबियं-र्यबीज़मर्याी र्यक्षिमािे -ंअिंज़ि णड़िण

ै अमाि र्यनि -तयक्षस्यिक्षतिर्यृ यक्ष्यर्तिक

Average Speed (1) — Average Speed (2)

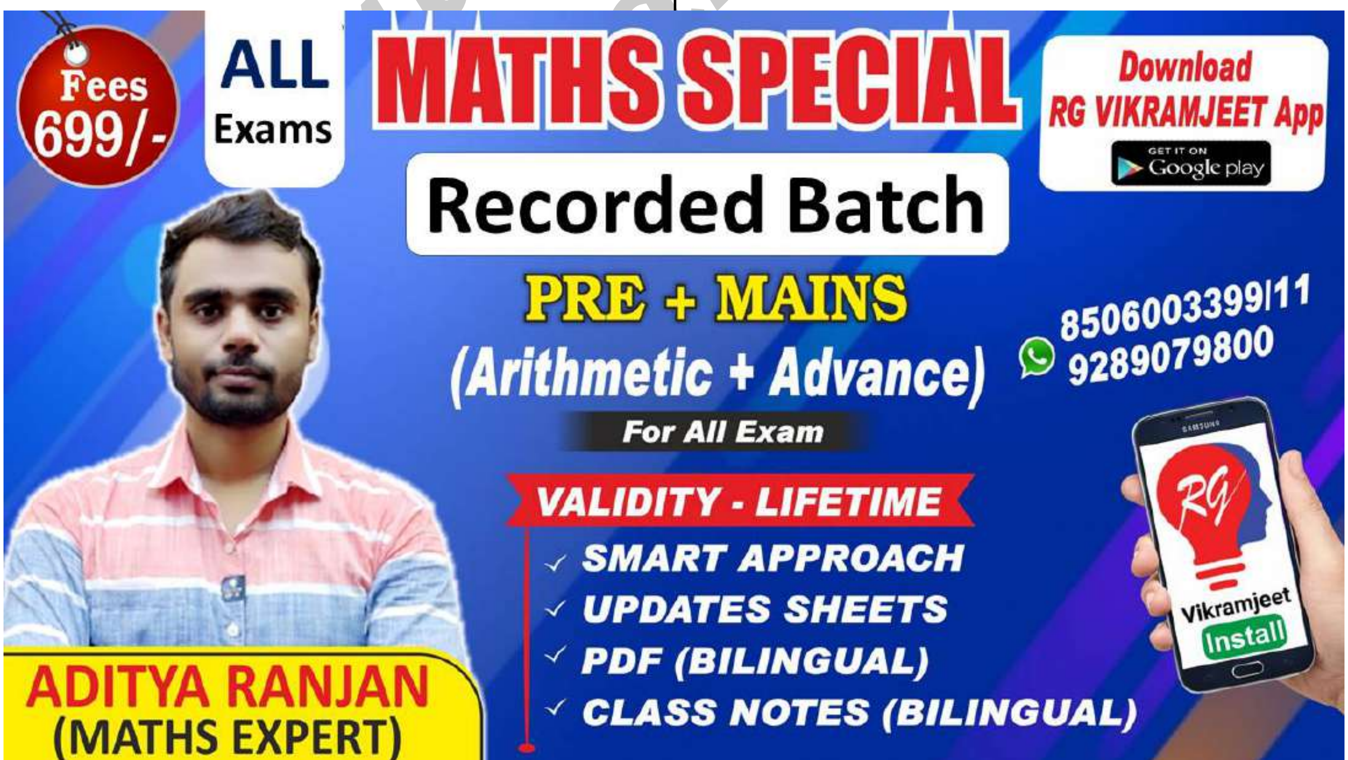
Average Speed

Time 1 : Time 2

❖ **An improtant formula in mixture**

ज्अमयक्षीि ंज़ि णिण ंअि श्न्दीक्षौं ेमा

$$\frac{\text{Left quantity / गयूंय अयजम}}{\text{Total quantity / ा सा िअयजम}} = \left(1-\frac{\text{Replaced quantity / दलक्ष-त्र्यमाद्रक्षिअयजम}}{\text{Total quantity / ा सा िअयजम}}\right)^n$$



13 AVERAGE/है ग्रिल

Average/हैग्रिल

Average Formula/हैग्रिलयै ादें

$$= \frac{\text{Total Sum of All Number}}{\text{Number of Item in the Set}}$$

☞ **For consecutive Natural Numbers:**/ग-ै लअैर जग्ग दक्मैहैंग्गयतग्रुें

- **Average of first n natural no.**/अै-ा n ाअैर जग्ग दक्मैहैं ग्गयैाहैंग्रिल $= \frac{(n+1)}{2}$
- **Average of Square of first n natural no.**/अै- n अैर जग्गादक्मैहैंग्गयती ैग्गयैाहैंग्रिल $= \frac{(n+1)(2n+1)}{6}$
- **Average of cube of first n natural no.**/अै-ा n अैर जग्गादक्मैहैंग्गयतीणैकैग्गयैाहैंग्रिल $= \frac{n(n+1)^2}{4}$

☞ **For Consecutive Even Numbers:**/ग-ै लद-ादक्मैहैंत यग्गजुें

- **Average of first n even no.**/अै-ा n ाद-ादक्मैहैंत यैाहैंग्रिल $= (n+1)$
- **Average of Square of first n even no.**/अै-ा n द-ादक्मैहैंग्गयती ैग्गयैाहैंग्रिल $= \frac{2(n+1(2n+1))}{3}$
- **Average of Cube of first n even no.**/अै-ा n द-ादक्मैहैंग्गयतीणैकैग्गयैाहैंग्रिल $= 2n(n+1)^2$

☞ **For Consecutive Odd Numbers:**/ग-ै लजॊै-ादक्मैहैंत यग्गजुें

- **Average of first n odd no.**/अै-ा n ाजॊै-ादक्मैहैंत यैाहैंग्रिल $= n$
- **Average of Square of first n odd no.**/अै-ा n जॊै-ादक्मैहैंग्गयती ैग्गयैाहैंग्रिल $= \frac{(2n+1)(2n-1)}{3}$
- **Average of Cube of first n odd no.**/अै-ा n जॊै-दक्मैहैंग्गयतीणैकैग्गयैाहैंग्रिल $= n(2n^2-1)$
- **Average of consecutive numbers**/ग-ै ल दक्मैहैं ग्गयैाहैंग्रिल

$$= \frac{\text{First number + Last number}}{2}$$

☞ **Important Points:**/-श्री अेंश्रिजग्गत्रुग्ग

- **If in any series having common difference "d" and Average "k", "x" numbers are added in forward or backward, then**
  मजुग्गजादग्गु क्षैत्रज्ञदयैादैाॅत्ग्ग d लकैाहैंग्रिल k शैतहैंजि क्द-तहैं ग्गदतहबैाॅाअुग्गतदग्गx दक्मैाउाभैत्रजुेलैत

  New Avg. = $k \pm \frac{xd}{2}$

- **In series of even or odd having Avg. "k", when we add "x" no. in forward or backward, Then**
  द-ाहबैाजॊै-ादक्मैहैंग्गयजु क्षैत्रज्ञदयैाहैंग्रिल k श्रि :द-ग्गमजुग्गश-ाहैं ग्गदतहबैाॅाअुग्गतदग्गx दक्मैाउाभैत्रजुेलैत

  New Avg. = $k \pm x$

- **In series of natural no. having Avg. "k", when we odd "x" no. in forward or backward, Then**
  k हैंग्रिलैॅ क्षअैर जग्गादक्मैहैंग्गजादजु क्षैत्र-ग्गमजुग्गहैं तदतहबैाॅाअुग्गतदग्गx दक्मैाउाभैत्रजुभैाउालैत

  New Avg. = $k \pm x / 2$

- **If there are 3 natural numbers and avg. of any two number when added with third no. gives a, b, c. Then natural numbers.**
  मजुग्ग ा अैर जग्गादक्मैाउाश्रैतहैं ग्रि जादग्गक्षैत्रदक्मैहैंग्गयत हैंग्रिलयैग्गमजुग्गलब्बउग्गदक्मैा-ग्गभैत्रैजुभैाउालैगअृलदक्मैाउ ग-ैृ ा a, b लकैाc शैतलैत

  Sum of no $= \left(\frac{a+b+c}{2}\right) = k$

  First no. $= 2a - k$

  Second no. $= 2b - k$

  Third no. $= 2c - k$

- **If the average of n_1 observations is a_1, the average of n_2 observations is a_2 and so on, then**
  n_1 ाअग्गैकैग्गयैाहैं/ैल a_1 प्र n_2 ाअग्गैकैग्गयैाहैं/ैल a_2 प्र लकैा हैं ग्गधैत्रक्ष्ग्गअग्गउाशैतलैत

  Average of all the observations/दधैत्रअग्गैकैग्गयै हैंग्रिल $= \frac{n_1a_1 + n_2a_2 + ...}{n_1 + n_2 + ...}$

- **If average of n observations is a but the average becomes b when one observation is eliminated, then value of eliminated observation = n (a – b) + b**

  मज्व्रु nा औा रवै सयै ाहै्ाँल aा शै सअँ लएमज्व्रु ऱ्या औा रवा यै त जा बै ज्अलय उज्व्रुऊे ाभै ऱ्ालैसहै द्िलbाशै सभै लैाशसिजा बै ज्अल औा रवा यै ा–ै का = n (a – b) + b

- **If average of n observations is a but the average becomes b when a new observation is added, then value of added observation**

  = n (b–a) + b

  मज्व्रु nा औा रवै सयै ाहै्ाँल aा शै सअँ लएमज्व्रु ऱ्या औा रवा यै त है ध्ा रै जक्षा यउ ज्व्रुऊे ाभै ऱ्ालैसहै द्िल bा शै सभै लैा शधि लैत रै जक्षाज्अऱ्ा ऱ्ा औा रवा यै ा–ै क

  = n (b – a) + b

- **If the average of n students in a class is a, where average of passed students is x and average of failed students is y, then**

  मज्व्रु ज्अ दच्अय ाँ ा–सnा जा रै क्ष्अमे सयै ाहै द्िल aा शै प्र भ्शै न :सैच्कैाज़्जा रै क्ष्अमे सयै ाहै द्िलxाहै धिह कौम्च्कैाज़्जा रै क्ष्अमे सयै है द्िल yा शै सलैत

  Number of students passed/:सैच्कैाज़्जा रै क्ष्अमे सय च दच्अमै

  $$= \frac{n(a-y)}{(x-y)}$$

- **We have n observations out of which some observations (a_1, a_2, a_3....) are replaced by some other new observations and in this way, if the average increase or decreases by b, then Value of new observations**

  मज्व्रु श–ै उसअेदा nा औा रवा शै सज्द–सदसय एा औा रवै स(a_1, a_2, a_3....)ायै सय एाकाऔा रवै सै उेाअक्अ बै ज्अलय उज्व्रुऊे ाभै ऱ् लबै ाक्दाअज्ञा मै ा–समज्व्रु है द्िल–सbा यच्ाी ्ज्रा ह बीं ाय–च शै सभै ऱ्ालैसकाऔा रवै सयै ा–ै क

  = a±nb

  Where/भ्शै स, $a = a_1 + a_2 + a_3 + ...$

  Note: In this formula, the signs of '+' and '–' depend upon the increment or decrement in the average

  क्दा दै ा–स'+'ा ह बीं ा'–'ायै ा र्सँ शाहै्ाँल–सी्ज्रा ह बीं य–च्अउअ ज्अध्ैज्ञ य उैला श्ई

- **Bowling Average** $= \frac{\text{Total runs given}}{\text{Total wickets taken}}$

- **Batting Average**

  $$= \frac{\text{Total runs scored}}{\text{Total number of innings played}}$$

14 TIME & WORK/–हत गर्जेद्धगअे त॔

Important Formulae/है दिल्येंग-ो

- **Efficiency/अे त॔रे हजे × Time/–हत = Total Work/अनगअे त॔**
- **If A can do a piece of work is x days and B can do a piece of work is y days, then both of them working together will do the same work in**

  Aगम्अ–तभे हगभे ह्कगम बेहिह्अज़ा– अजेगै र्फा श्मअगBगस–त अे हगभे ह्यगम बेहिह्अज़ा– अजेगै च्छ म गहह्अगअगर्- ह्लेगह–अज़ अे हगअज़्जे ह्मजहदे गअे हगम्अजवह्म बेहिह्लेग्गै ह्चे च

  $$\frac{xy}{(x+y)} \text{ days}$$

- **If A, B & C are working alone, can complete a work in x, y and z days, respectively, then they will together complete the work in**

  A, Bगर्जेद्धगCगम्अ–ताअे त॔गअे ह्यु हभॅं गx, गyगर्जेद्धगz म बेहिह्अज़ा– अजे ह्भपअग– ह्लेगअे त॔गअज़्जे ह्मगदह्अे त॔गअे क्ष्मअजवह्म बेहिह्– हे उजाअज़्ह्र

  $$\frac{xy}{(xy+yz+zx)} \text{ days}$$

- **If A & B working together, can finish a piece of work is x days, B & C in y days, C & A in z days. The, A + B + C working together will finish the job in**

  A वे त्गB महज़ अज़ग्पअगअे हगअे ह्कx म बेहिह्अज़ा– अजेह्र हि B वे त्गC महज़ अज़ाम– ताअे हगभे ह्यगम बेहिह्लेग्भअज़ा– अजक्ष ह्भि श्मअगA वे त्गC महज़ अज़ाम– ताअे हगभे ह्zगम बेहिह्लेग अज़ा– अजे ह्भपअग– ह्लेगअे त॔गअज़्जे ह्मगदह्अे त॔गअे ह्मअजवह्म म बेहिह्– हे उजाअज़्ह्र

  $$\frac{2xyz}{(xy+yz+zx)} \text{ days}$$

- **If a working Alone takes "a" days more than A & B, and B working Alone takes "b" days more than A & B. Then, Number of days, taken by A & B working together to finish a job in**

  Aगवे त्गBगअताजन बे गहिह्Aगअे ह्पअगअे हगलेग्गअज्बह्हिह्a गे ी, ह्व म्ह अगज़ एह्र्जेद्धगAगवे त्गBगअताजन बे गहिह्Bगअे क्ष पअगअे हगलेगगअज्बह्हिह्bगे ी, ह्व म्ह अगज़ एह्र्पअग– ह्ले अे त॔गअज़्जे ह्मगदह्अे त॔गअे ह्मअजवह्म बेहिह्– हे उजाअज़्ह्र

  $$\sqrt{ab} \text{ days}$$

- **If M_1 men can do W_1 work in D_1 days working H_1 hours per day and M_2 men can do W_2 work in D_2 days working H_2 hours per day, then**

  M_1गप्र म जााW_1गअे त॔गअे ह्लमजाम बगH_1गे ी, ह्अज़्जे ह्मगD_1 म बेहिह्– हे उजाअज़ा– अजे ह्र्जेद्धगM_2गप्र म जााW_2गअे त॔गअे क्ष लमजाम बगH_2गे ी, ह्अज़्जे ह्मगD_2गम बेहिह्– हे उजाअज़ा– अजे ह्रि जेक्ष

  $$\frac{M_1 \times D_1 \times H_1}{W_1} = \frac{M_2 \times D_2 \times H_2}{W_2}$$

15 PERCENTAGE /हैदिलद

Percentage Value/ क्षरय जिया

$\frac{1}{2} = 50\%$	$\frac{1}{11} = 9\frac{1}{11}\%$	$\frac{4}{25} = 16\%$
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$	$\frac{3}{20} = 15\%$
$\frac{1}{4} = 25\%$	$\frac{1}{13} = 7\frac{9}{13}\%$	$\frac{3}{8} = 37\frac{1}{2}\%$
$\frac{1}{5} = 20\%$	$\frac{1}{14} = 7\frac{1}{7}\%$	$\frac{5}{8} = 62\frac{5}{8}\%$
$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{1}{15} = 6\frac{2}{3}\%$	$\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{1}{16} = 6\frac{1}{4}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$
$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{17} = 5\frac{15}{17}\%$	$\frac{2}{25} = 8\%$
$\frac{1}{9} = 11\frac{1}{9}\%$	$\frac{1}{18} = 5\frac{5}{9}\%$	$\frac{2}{5} = 40\%$
$\frac{1}{10} = 10\%$	$\frac{1}{19} = 5\frac{5}{19}\%$	$\frac{1}{20} = 5\%$

Formula to Calculate Percent

क्षरय ज्ि मगितयार्यहित क्ि ब्रि

☞ If we have to find y% of x, then

ज्क्ति हिजांfx ज् fy% ज् यजिस्मणियरज् क्ष्यहियरित य

$$y\% \text{ of } x = x \times \frac{y}{100}$$

☞ Conversion of Per cent into Fraction

क्षरय ज्ि यर्दि यमाजिंि क्ष्मतक्ष

Expressing Per cent (x%) into fraction.

(x%) ज् यर्दि यमाजिंि क्ष्मतक्ष

Required fraction/प यमरज्ज् ि क्षरय $= \frac{x}{100}$

☞ If two subsequent increase/decrease of X% and Y%. Then/ज्क्त X% त्क्षY% ज् मलियिए जयात सिझ भ्ज जम हयित्ि य

$$\text{Net Change} = \left(\pm X \pm Y \pm \frac{XY}{100}\right)\%$$

16 PROFIT & LOSS AND DISCOUNT

है ांदलै घ्दे ' गद–ै दअ्त है

1. Gain/ा यय= Selling price/द्धत्र र जिन्कर (S.P.) – Cost price/त र जिन्कर (C.P.)

2. Loss/हयप्प = Cost price/त र जिन्कर (C.P.) – Selling price/द्धत्र र जिन्कर (S.P.)

3. If the rate of gain or loss are given then
र द्नाा ययति क्ष्वहयप्पाति ब्लीा लिगज़िहयक्रिप्प

(a) Selling price/द्धत्र र जिन्कर

$$= C.P.\left(1-\frac{loss\%}{100}\right) or \left(1+\frac{gain\%}{100}\right)$$

(b) Cost Price/त र जिन्कर

$$= \frac{S.P.}{\left(1+\frac{gain\%}{100}\right)} or \frac{S.P.}{\left(1-\frac{loss\%}{100}\right)}$$

(c) The gain or loss is always calculated from the cost price.
ा ययति क्ष्वहयप्पाति ब्र्गि यार्यहिज्यति र जिन्कर सीा ति बख्यिप्पबहैप

4. If/र द्न SP > CP then Gain/ा यय= S.P. – C.P.

$$\%Gain = \frac{Gain \times 100}{C.P.}$$

5. If/र द्न SP < CP then Loss/हयप्प = S.P. – C.P.

$$\%Loss = \frac{Loss \times 100}{C.P.}$$

6. In % Gain Condition/सह्यर्याा यति बविक्ष्यप्तजिं–

$$SP = CP\left(1+\frac{\%Gain}{100}\right)$$

7. If % Loss Condition/सह्यर्याहयप्पाति बविक्ष्यप्तजिं–

$$SP = CP\left(1-\frac{\%Loss}{100}\right)$$

8. $$\frac{CP}{MP} = \frac{(100 - Discount\%)}{(100 \pm Profit/Loss\%)}$$

9. If two items are sold at same price, each at Rs. x one at a profit of P% and other at a loss of P%, Then
र द्नालियाअहो भिजयाजिन्कर fx ँ सर किड़्यों ङ्ति यीा ययP% त यैलिप्पाबति यहियप्पाP% सीाउिबबखियेकिप्प

$$Loss = \frac{P^2}{100}\%$$

11. If CP is same, Then/र द्नाति र जिन्कर भिजयाहियक्रिप्प
Change is SP % = Change in Profit/Loss %
द्धत्र र जिन्कर जिंसिद्धर्याासिवे अदप्पा= ा ययकहयप्पाजिंसिद्धर्याासिवे अदप्प

12. If mp of two articles is same, then
र द्नालियाअक्नुत यंति तिंक्र कजिन्कर भिजयाहियक्रिप्प
Difference between SP = Difference between discount
द्धत्र र जिन्कर जिंतिंकी= , ब्रजिंतिंकी

17 SIMPLE & COMPOUND INTEREST
आर्ह्ल पा र ौं मऊचक्षि ऋ ाॅ

Simple Interest/ताःर्ा पा ऋ ाॅ

S.I. = Simple Interest/आर्ह्ल पा ऋ ाॅ

P = Principal/ज्साल्क्र

R = Rate/र्द

T = Time/अग्ॅ

$$S.I. = \frac{P \times R \times T}{100} \qquad P = \frac{S.I. \times 100}{R \times T}$$

$$R = \frac{S.I. \times 100}{P \times T} \qquad T = \frac{S.I. \times 100}{P \times R}$$

Instalment for simple interest/आर्ह्ल पा ऋ ाॅ ् े लिए ि त्त्क

$$A = \left[x + \left(x + \frac{x \times R \times 1}{100}\right) + \left(x + \frac{x \times R \times 2}{100}\right) + \ldots\ldots\right]$$

Where, A = Total amount paid/ स घासक्व ि ़ा फ़ा ज्टिल्क्र

x = Value of each instalment/ु क्ं ि त्त्क् ा ज्व

$$\text{Also } A = P + \frac{P \times n \times R}{100}$$

Where, P is the principal/ॅ हाज्P ज्साल्क्र है

n is the number of instalments R is the rate of interest.

n ि त्त्कें् बअंक्ा क्था R ऋ ाॅ र्द हैड

Compounded interest/मऊचक्षि ऋ ाॅ

$$A = P\left(1 + \frac{r}{100}\right)^t$$

A = Amount/ज्टिल्क्र P = Principle/ज्साल्क्र r = rate/र्द t= time/अग्ॅ

Table for direct calculation in CI:

Rate	CI of 2 Years	CI of 3 years	CI of 4 years
5%	10.25%	15.7625%	~21.5%
10%	21%	33.1%	46.41%
15%	32.25%	52.0875%	69.35%
20%	44%	72.8%	107.36%

❖ Important Points/ज्हु च् या निघ्र्स

- In CI Amount will double in "N" years then,
ॅ दि मऊचक्षि ऋ ाॅ ् ेज्टिल्क्र "N" चर्ाें जें दोगक्ष हो ॅ ाए को

$$N = \frac{69}{r} + 0.35$$

- Difference between CI and SI for two years
मऊचक्षि क्था आर्ह्ल पा ऋ ाॅ ् े नब्म दो चर्ाें ् े लिए र र्क्क

$$(CI - SI)_{2\,year} = P\left(\frac{R}{100}\right)^2$$

- Difference between CI and SI for three years
मऊचक्षि क्था आर्ह्ल पा ऋ ाॅ ् े नब्म कक्ष चर्ाें ् े लिए र र्क्क

$$(CI - SI)_{3\,year} = 3P\left(\frac{R}{100}\right)^2 + P\left(\frac{R}{100}\right)^3$$

Instalment for compound interest/मऊचक्षि ऋ ाॅ ् े लिए ि त्त्क

$$P = \left[\frac{x}{\left(1 + \frac{R}{100}\right)} + \frac{x}{\left(1 + \frac{R}{100}\right)^2} + \ldots\ldots \frac{x}{\left(1 + \frac{R}{100}\right)^n}\right]$$

x = value of each instalment/ु क्ं ि त्त्क् ा ज्व

Total amount paid in instalments/ि त्त्क् े श् े जें घासक्व ि ़ा फ़ा ् स ज्टिल्क्र

$$A = P\left(1 + \frac{R}{100}\right)^n$$

n = Number of instalments/ि त्त्कें् बअंक्ा

18 STATISTICS/लै क्षादर य

Mean/है दि

The arithmetic mean of a given data is the sum of all observations divided by the number of observations. For example, a cricketer's scores in five ODI matches are as follows: 12, 34, 45, 50, 24. To find his average score we calculate the arithmetic mean of data using the mean formula:

अिर्द्ध दिए गए पेरा आ माञ्च इक्षत्रों अे चोग न्ता ओ ने क्षत्रों अं व्रुंन्चा व्रे यिवाृ न्त अक्षे न क ण्ड होत्त है टदाहका अे लिए् न ांभ एआदियर्द्धच मैभों में एअ ु अेरकअे ज्ञोकब्र् थ्इ ड्. न् म्ड हैं टद्धआ (ौद्धत्त ज्ञोकशत्त अक्षे अे लिए हम माञ्च व्री अं व्राचत्त व्रे पेरा आ व्रमाल्कमाञ्च शत्त अक्ते हैं

Mean/माञ्च

$$= \frac{\text{Sum of all observations / लौ ये मै" ह्मा" त्जै मे मै त}}{\text{Number of observations / ` मै " ह्मा य लह्मै}}$$

Mean/माञ्च = (12 + 34 + 45 + 50 + 24)/5

Mean/माञ्च = 165/5 = 33

Mean is denoted by $\bar{x}$ (pronounced as x bar).

माञ्च ओ $\bar{x}$)ाका न द्रणित्त अक्ते हैं

Types of Data/पेरा अे न आक

Data can be present in raw form or tabular form. Let's find the mean in both cases.

पेरा ओ raw पेरा त्त्रा tabular अे 2न में न द्रणित्त अकद्धअत्ते हैं दोसों ज्ञित्त्यों में माञ्च शत्त अकें

Raw Data/कँ पेरा

Let $x_1, x_2, x_3, \ldots, x_n$ be n observations.

मासा $x_1, x_2, x_3, \ldots, x_n$ ओक्रक n ने क्षत्रा हैं

We can find the arithmetic mean using the mean formula.

हम सिक्सलिग्त्ति माञ्च व्री आ ठ चोग अक्षे व्रमाल्कमाञ्च शत्त अकद्धअत्ते हैं

$$\text{Mean, } \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

Frequency Distribution (Tabular) Form

बाकंाका यित्त्का धद्धकर्ा पेरा

When the data is present in tabular form, we use the following formula:

ृ ब पेरा व्रकर्ा अे 2 ष में दिचा गचा होत्त है त्ते हम सिक्स व्री आ ठ चोग अक्ते हैं

$$\text{Mean, } \bar{x} = \frac{x_1f_1 + x_2f_2 + \ldots + x_nf_n}{f_1 + f_2 + \ldots f_n}$$

Median/है क्षिदरै

The value of the middlemost observation, obtained after arranging the data in ascending or descending order, is called the median of the data.

पेरा ओ (ाकेर्ह ृ म में उ्चयज्ञित्त अक्षे अे बाद मञ्चत्त्म ने क्षत्रा ओ माञिआ अहत्ते हैं

For example, consider the data: 4, 4, 6, 3, 2. Let's arrange this data in ascending order: 2, 3, 4, 4, 6. There are 5 observations. Thus, median = middle value i.e. 4.

टदाहका अे लिए मासा 4, 4, 6, 3, 2 ओक्रखेरा है क्र्इे व्र ड्रे न हले (ाकेर्ह ृ म 2, 3, 4, 4, 6 में उ्चयज्ञित्त अक्ते हैं अल 5 ने क्षत्रा हैं क्र्द्ध लिए माञ्जिआ = मञ्चत्त्म मास (छाछ/4 है

Case 1: Ungrouped Data/(यर्ग क्रत्त पेरा

Step 1/भका ब: Arrange the data in ascending or descending order.

पेरा ओ (ाकेर्ह ृ म (छया (यकेर्ह ृ म में उ्चयज्ञित्त अकें

Step 2/भका म: Let the total number of observations be n.

मासा ने क्षत्रों अं अल व्रुंन्चा n है

To find the median, we need to consider if n is even or odd. If n is odd, then use the formula:

माञ्जिआ शत्त अक्षे अे लिए् हमें च्ह देन्सा होगा अि n व्रम है चा यिाम चदि n यिाम है त्ते व्री आ न च्रोग अकें

$$\text{Median/माञ्जिआ} = \left(\frac{n+1}{2}\right)^{th} \text{ observation/ने क्षत्रा}$$

Case 2: Grouped Data/यर्ग क्रत्त पेरा

When the data is continuous and in the form of a frequency distribution, the median is found as shown below:

ृ ब पेरा सिक्कक(ाैकबाकंाका यित्त्का अे 2न में होत्त है त्ते माञ्जिआ सिक्सलिग्त्ति न आकद्धे शत्त अं ृ ार्त्त है

Step 1: Find the median class./माञ्जिआ यगख्रात्त अकें

Let n = total number of observations i.e. $\sum f_i$

मासा n = अल ने क्षत्रा अं व्रुंन्चा (छाछ/$\sum f_i$

Note: Median Class is the class where (n/2) lies.

माञ्जिआ यगख्यह यगखै ृ हां (n/2) ज्ञित्त होत्त है

Step 2: Use the following formula to find the median.

माज्यिआ शत्त अक्मे अे लिए सिक्स व्क्रि आ ⁊ च्रोग अकें

$$\text{Median/माज्यिआ} = l + [\frac{\frac{n}{2} - c}{f}] \times h$$

where/ु हां

l = lower limit of median class/माज्यिआ यगर्ख्अ सिभ्र्ले व्क्रिमा

c = cumulative frequency of the class preceding the median class/व्क्रि च्र (ायृ ा

f = frequency of the median class/(ायृ ा स्कािकाक्ता

h = class size/यगर्ख्अ मक

Mode/⁊ बीड्र

The value which appears most often in the given data i.e. the observation with the highest frequency is called a mode of data.

यह मास ु ो दिए गए पेरा में व्क्रुव्द्धे (ि अ बाकदिज्ञाक्ष्वेत्ते है चार्स टप्भ्त्त्म (ायृ ा याले पेरा बहलअ अहलात्त हैु

Case 1: Ungrouped Data/(यर्ग क्रुत्त पेरा

For ungrouped data, we just need to identify the observation which occurs maximum times.

(यर्ग क्रुत्त पेरा अे लिए हमें अेयल टव्द्र⁊ क्षत्रा अर्ि ⁊ हभ्ास अक्मे अर्ि (ायण्चअत्त है ु ो (ि अत्त्म बाकहोत्त हैु

Mode/बहलअ **= Observation with maximum frequency/**(ि अत्त्म (ायृ ा याला ⁊ क्षत्रा

For example in the data: 6, 8, 9, 3, 4, 6, 7, 6, 3, the value 6 appears the most number of times. Thus, mode = 6.

टदाहका अे लिए पेरा कृ १ झ ९ ड कृ ज़ कृ ४ में कृ व्क्रुव्द्धे (ि अ बाकदिज्ञाक्ष्वेत्त हैु क्ष्वलिए बहलअ - कृ

Note: A data may have no mode, 1 mode, or more than 1 mode. Depending upon the number of modes the data has, it can be called unimodal, bimodal, trimodal, or multimodal.

ओव्रक्षोरा बिसा अिर्द्ध बहलअ अे एअ बहलअ (छया एअ व्द्धे (ि अ बहलअ याला हो व्रुअत्त हैु पेरा में अित्से बहलअ हैं क्रुव्द्धे (ाक कर्ह हम क्रद्धेएअल बहलअ f)6बहलअ ffi6बहलअ (छया बहळबहलअ अहत्ते हैंु

The example discussed above has only 1 mode, so it is unimodal.

ट⁊ कअे टदाहका में व्क्रि ⁊ ख्एअ बहलअ है क्ष्वलिए च्ह एअल बहलअ हैु

Case 2: Grouped Data/यर्ग क्रुत्त पेरा

When the data is continuous, the mode can be found using the following steps:

ु ब पेरा सिक्कं कहोत्त है त्ते सिक्स भ्कार्तो आ ⁊ चोग अक्मे बहलअ ओ शत्त अिचा ु ा व्रुअत्त हैु

Step 1: Find modal class i.e. the class with maximum frequency.

बहलअ यगख् छाक्ष्व(ज्ञिअत्त्म बाकांाक्ता याला यगख्रात्त अर्ि एु

Step 2: Find mode using the following formula:

सिक्षलिग्ित्त व्क्रि आ ⁊ चोग अक्मे बहलअ शत्त अर्ि एस

$$\text{Mode/बहलअ} = l + [\frac{f_m - f_1}{2f_m - f_1 - f_2}] \times h$$

where,/ु हा?

l = lower limit of modal class,/बहलअ यगर्ख्अ सिभ्र्ले व्क्रिमा

f_m = frequency of modal class,/बहलअ यगर्ख्अ बाकांाक्ता

f_1 = frequency of class preceding modal class,/ बहलअ यगख्रे ⁊ ब्रख्त्त ख्यगर्ख्अ बाकांाक्ता

f_2 = frequency of class succeeding modal class,/ बहलअ यगख्रे टू ाक्र्त्त ख्यगर्ख्अ बाकांाक्ता

h = class width/यगर्ख्अ भ्ौपाक्ष्व

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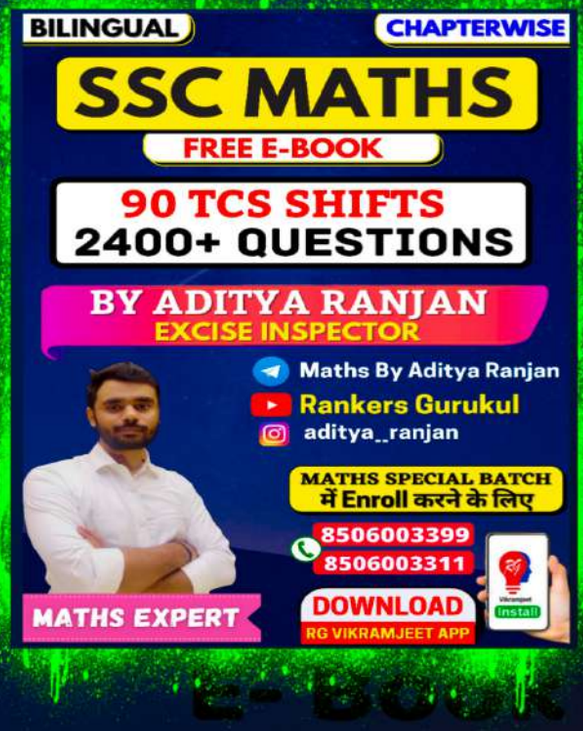

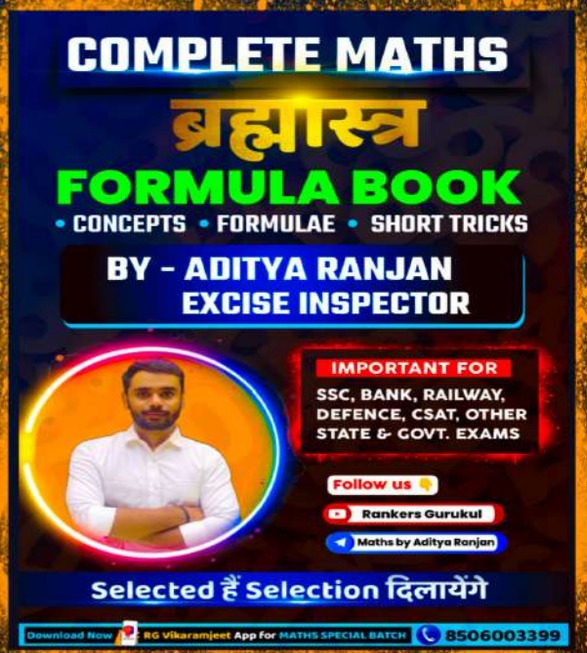

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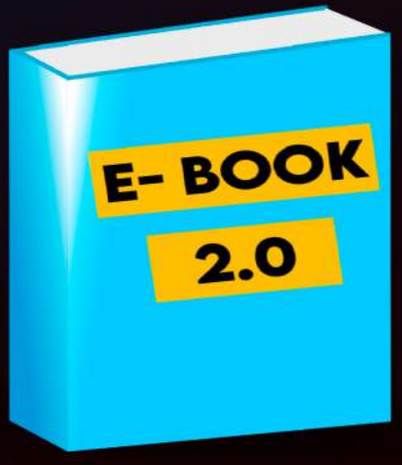

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